Chapter 6: Work & Energy

Most of the time, it is challenging to tackle a quite complicated dynamical problem using Newton's 2^{nd} law. This needs very careful analysis for the vectors involved and you may not even know all of the forces! So, different simpler approach becomes mandatory.

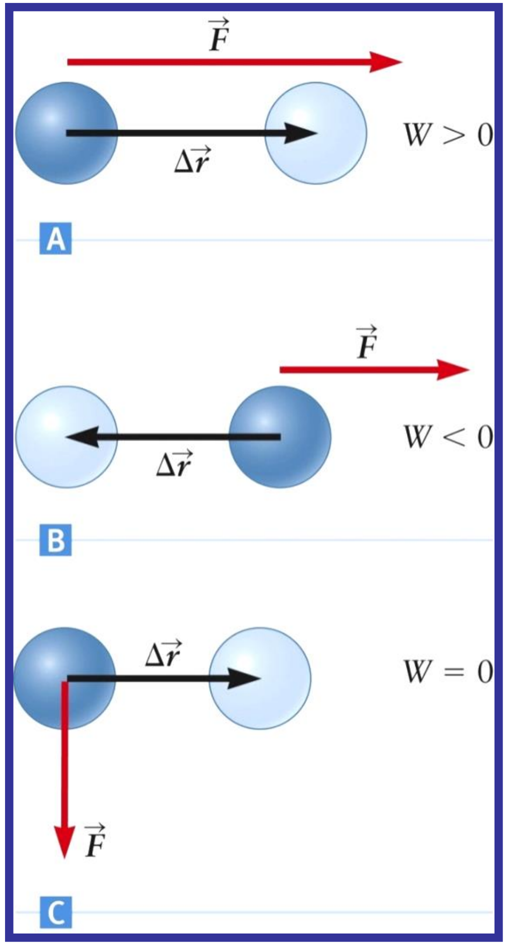
This is the motivation for the **Work-Energy Theorem**.

Work and Directions

- The component of **F** along the displacement direction is:

$$F\cos\theta \equiv F_{\parallel}$$

- When $F_{\parallel}$ is ***parallel*** to the displacement, ***the work is positive***.
- When $F_{\parallel}$ is ***anti- parallel*** to the displacement, ***the work is negative***.
- When the component of the force is ***perpendicular*** to the displacement, ***the work is zero.***



$$W \equiv F_{||}d = Fd\cos\theta$$

- Its possible to exert a force & yet do no work!
- Could have $\mathbf{d = 0 \Rightarrow W = 0}$
- Or could have $\mathbf{F \perp d}$

$$\Rightarrow \theta = 90^{\circ}, \cos\theta = 0$$

$$\Rightarrow W = 0$$

- **Example,** walking at constant speed **v** with a grocery bag.

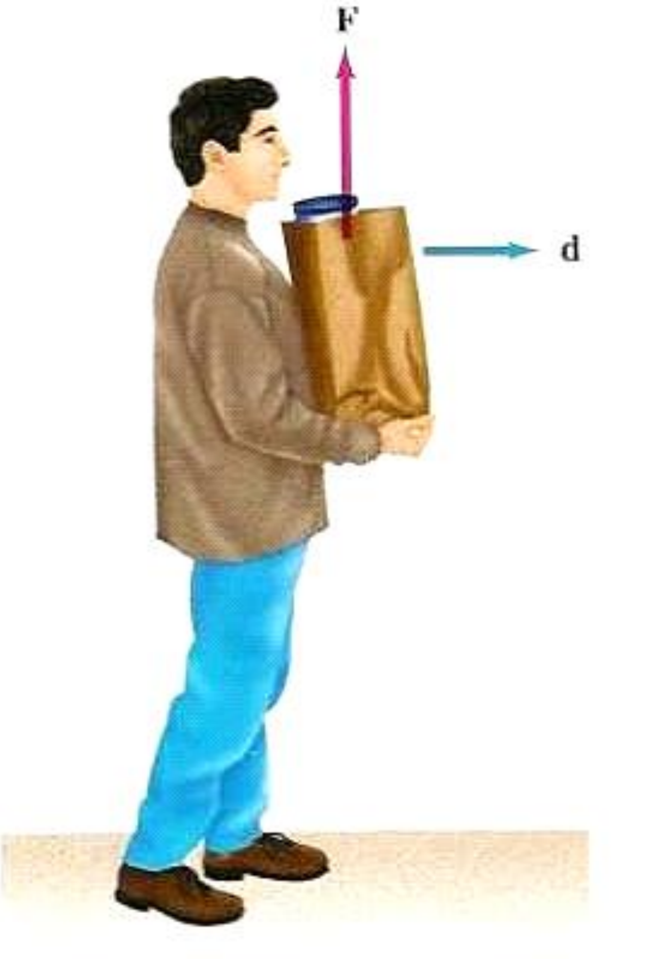
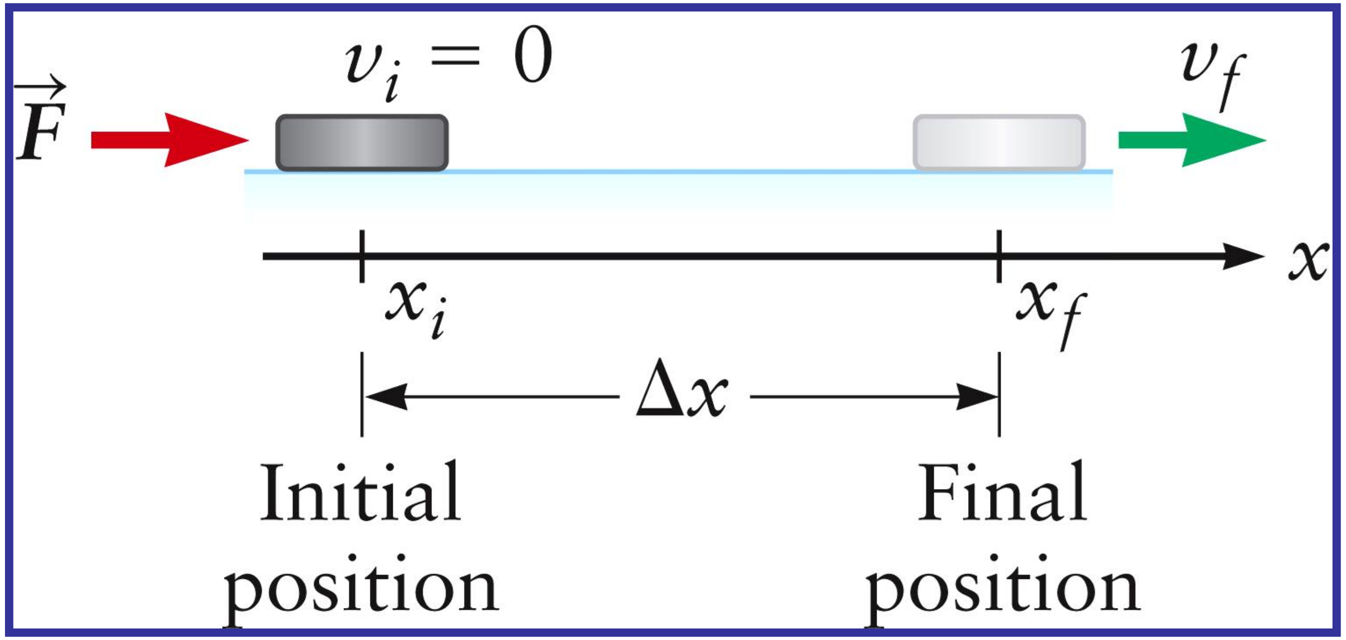


FIGURE 6–2 Work done on the bag of groceries in this case is zero since **F** is perpendicular to the displacement **d**.

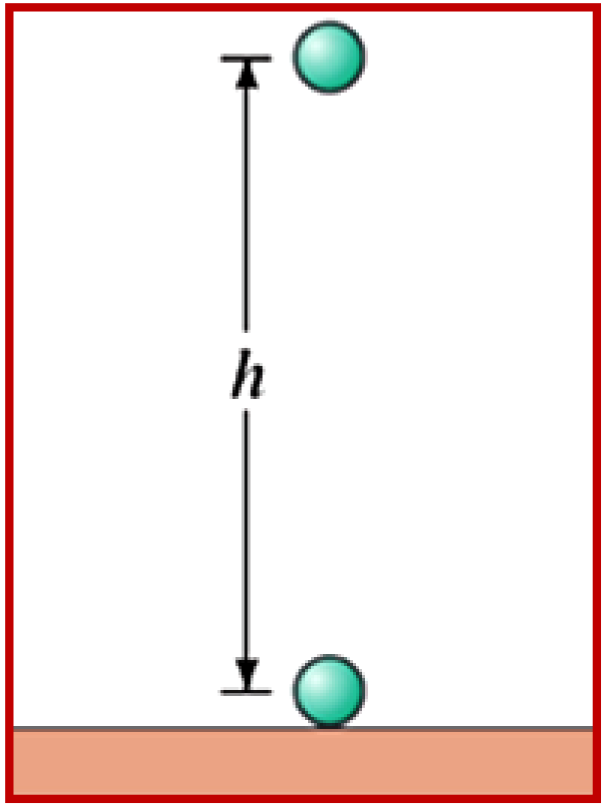
Work

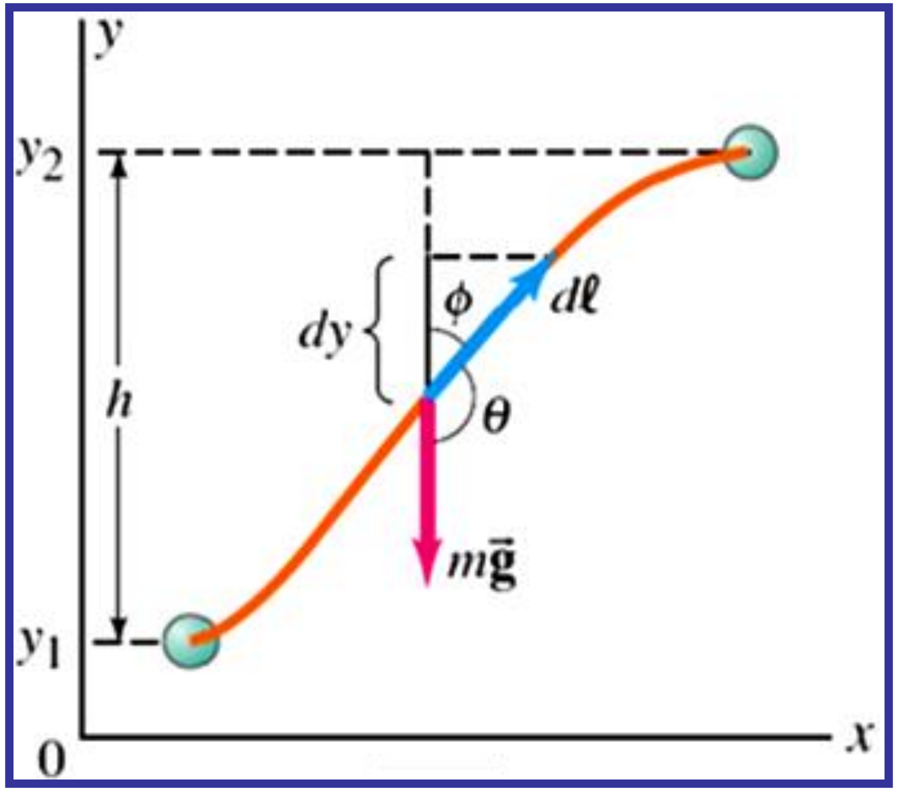


- Experiments have verified that although various forces produce different times & distances, the product of the force & the distance remains the same.
- To accelerate an object to a specific velocity, you can exert a large force over a short distance or a small force over a long distance

Definition: A force is **conservative** if & only if ***the work done by that force on an object moving from one point to another depends ONLY on the initial & final positions of the object, & is independent of the particular path taken.***

Example: gravity.





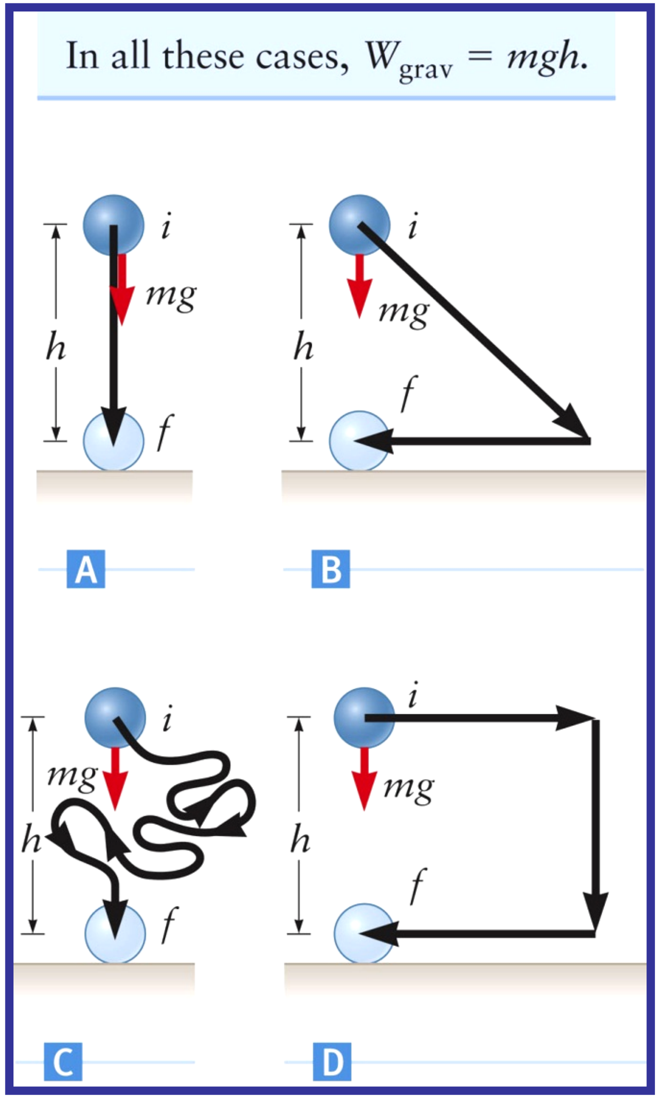
Gravitational PE = U

- The **work done by the gravitational force** as the object moves from its initial position to its final position is

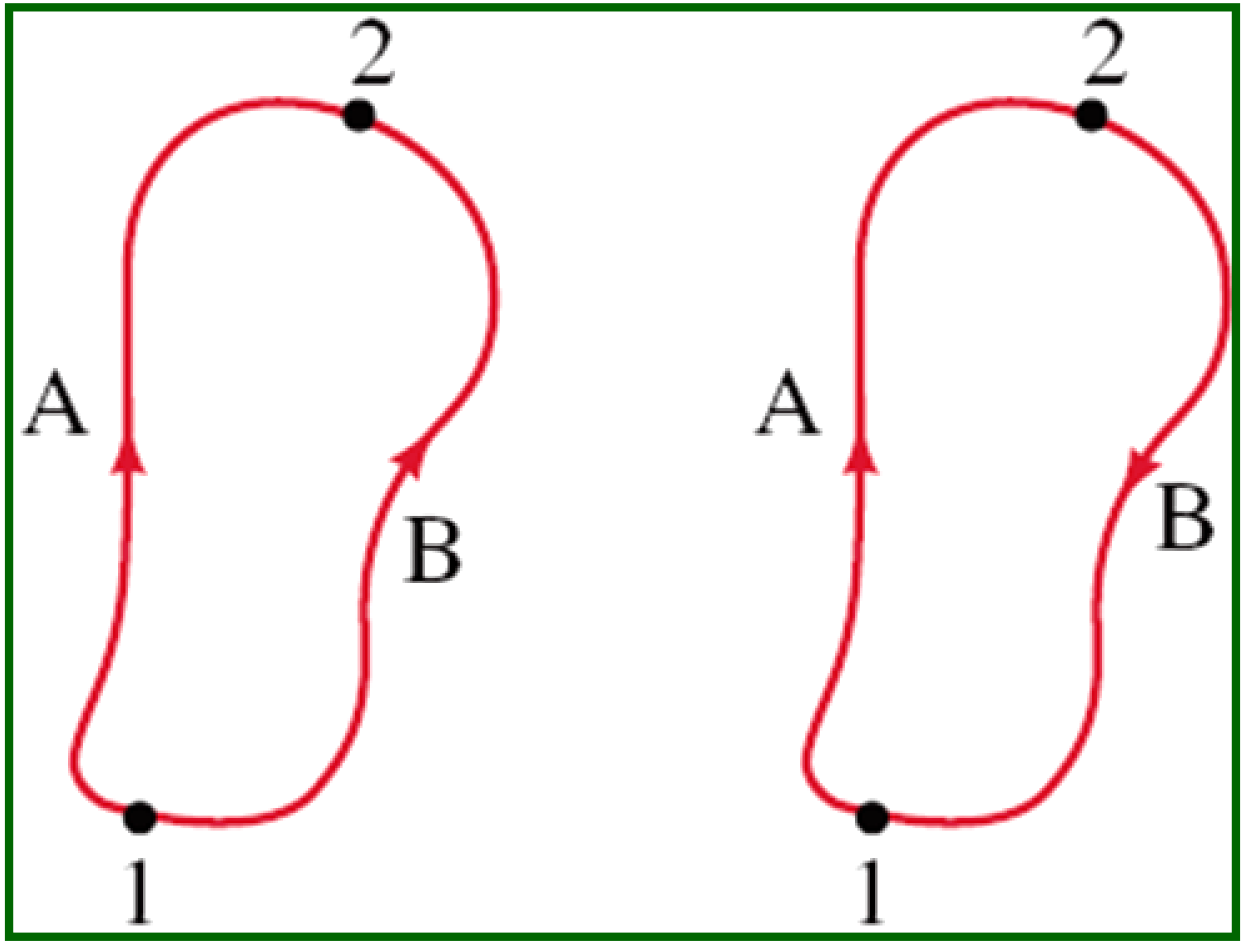
Independent of the path taken!

- Because of this property, the gravitational force is called a

Conservative Force.

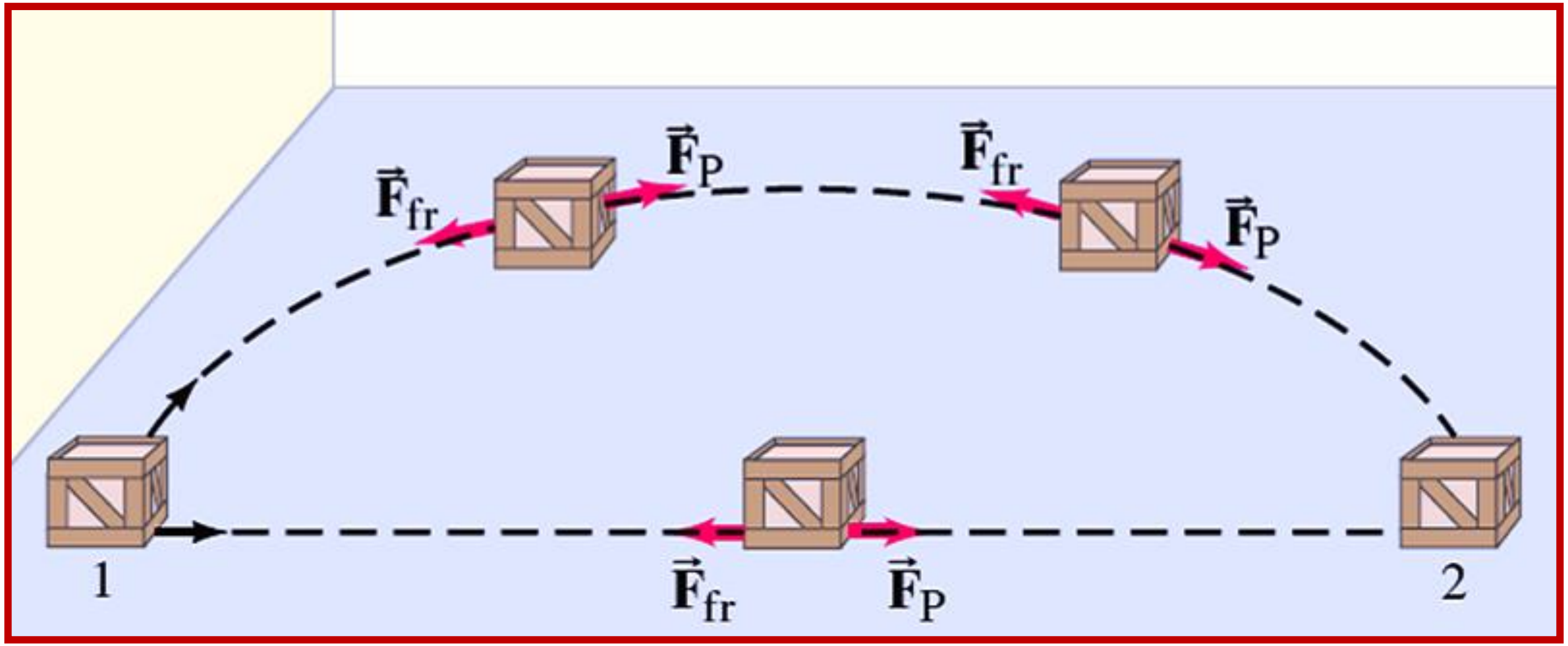


Conservative Force: Another definition: ***A force is conservative if the net work done by the force on an object moving around any closed path is zero.***



If **friction** is present, the work done depends not only on the starting & ending points, but also on the path taken.

Friction is a non-conservative force!



Friction is non-conservative!!!

The work done depends on the path!

Example: Roller Coaster

- ***Mechanical energy conservation!*** **(Frictionless!)**

$$\Rightarrow (½)m(v_1)^2 + mgy_1 = (½)m(v_2)^2 + mgy_2$$ **(Mass cancels!)**

Only height differences matter! **Horizontal distance doesn't matter!**

- Speed at the bottom?

  $y_1 = 40$ m, $v_1 = 0$

  $y_2 = 0$ m, $v_2 = ?$

  Find: $v_2 = 28$ m/s

- What is **y** when

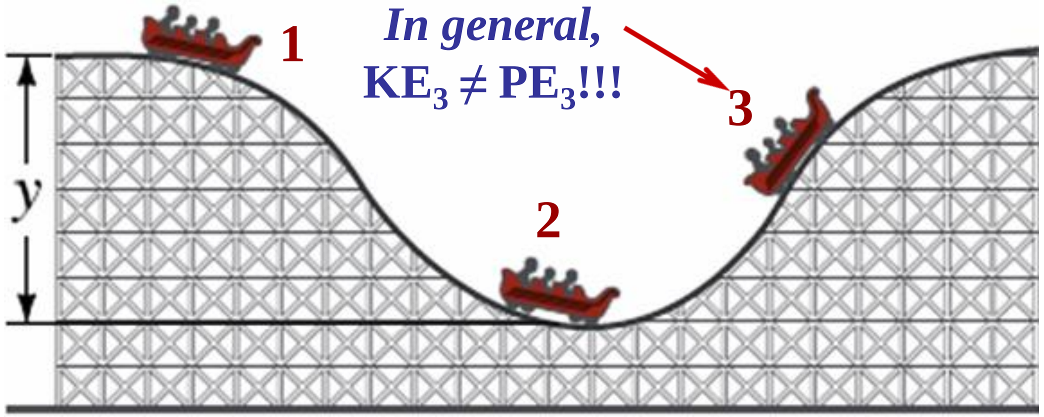
  $v_3 = 14$ m/s?

Use: $(½)m(v_2)^2 + 0$

$= (½)m(v_3)^2 + mgy_3$

Find: $y_3 = 30$ m

Height of hill = **40 m**. Car starts from rest at top. Calculate: **a.** Speed of the car at bottom of hill. **b.** Height at which it will have half this speed. Take **y = 0** at bottom of hill.



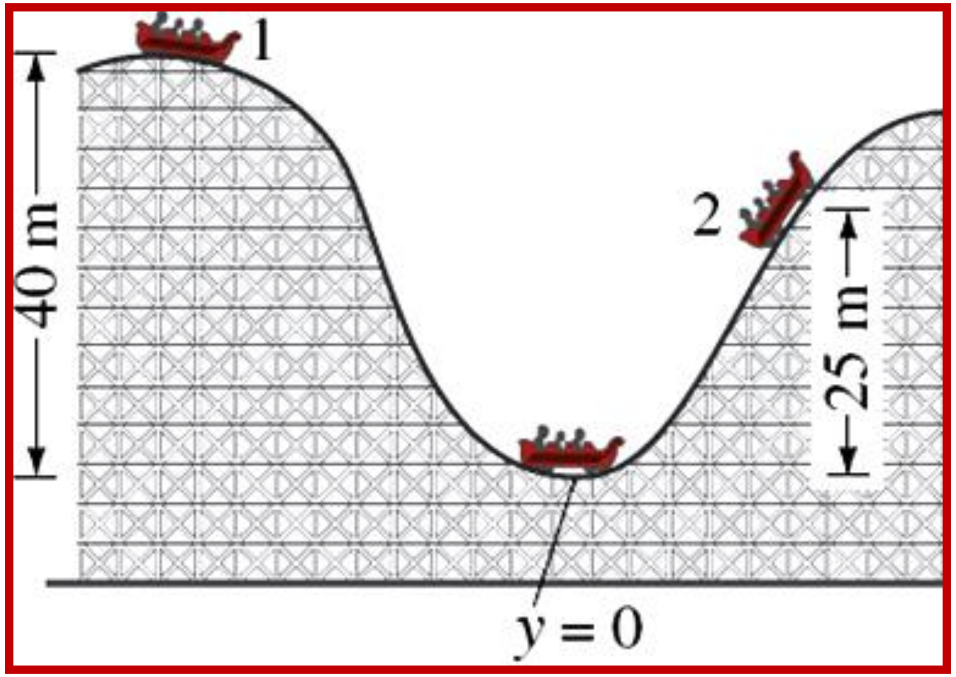
In general,
$KE_3 \neq PE_3$!!!

A very common error!
WHY????

NOTE!! *Always* use $KE_1 + PE_1 = KE_2 + PE_2 = KE_3 + PE_3$ *Never* $KE_3 = PE_3$!

Example: Roller Coaster with Friction

A roller-coaster car, mass **m = 1000 kg**, reaches a vertical height of only **y = 25 m** on the second hill before coming to a momentary stop. It travels a total distance **d = 400 m**.



Calculate the average friction force on the car.

$m = 1000$ kg, $d = 400$ m, $y_1 = 40$ m, $y_2 = 25$ m, $v_1 = y_2 = 0$, $F_{fr} = ?$

$$\Rightarrow \quad F_{fr} = 370 \text{ N}$$