Statistics, lecture 20: 25+26 كاملة
يوتيوب بالترتيب

e.g) let $X \sim \chi^2(10)$, Find:

a) the 10^{th} Percentile of X

b) the 95^{th} Percentile of X

c) the 99^{th} Percentile of X.

soll)

a) $P(X < P_{10}) = 0.10$

$\alpha = 0.9$, $d.f = 10$ من الجدول، التقاطع بين

$= 4.685$

b) $P(X < P_{95}) = 0.95$

$P_{95} = 18.307$

c) $P(X < P_{99}) = 0.99$

$P_{99} = 23.209$

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*The distribution of the difference between 2 sample means:

If $X_1 \dots X_n \sim n(\mu_1, \sigma_1^2)$ and

$Y_1 \dots Y_m \sim n(\mu_2, \sigma_2^2)$ then

$$\bar{X} \sim n(\mu_1, \frac{\sigma_1^2}{n})$$

$$\bar{Y} \sim n(\mu_2, \frac{\sigma_2^2}{m})$$

$$\therefore \bar{X}-\bar{Y} \sim n(\mu_1-\mu_2, \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}) \text{ or}$$

$$Z = \frac{(\bar{X}-\bar{Y}) - (\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} \sim n(0,1)$$

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Provided that σ_1 and σ^2 are known

*If $\sigma_1 = \sigma_2 = \sigma$ (unknown), then

$$T = \frac{(\bar{X}-\bar{Y}) - (\mu_1-\mu_2)}{S_p \sqrt{\frac{1}{n} + \frac{1}{m}}} \sim t(n+m-2)$$

where $S_p^2 = \frac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}$

" The pooled variance "

Note: If $n, m \geq 30$, then

ومش شرط يكون التوزيع Normal

$$Z = \frac{(\bar{X}-\bar{Y}) - (\mu_1-\mu_2)}{\sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}} \dot{\sim} n(0,1)$$

لو سألك عن نوع التوزيع تباحكيله t ، وهنا
مجرد تقريب لل Normal ليس توزيع

e.g) Suppose that the grades of female and male students in calculus 101 are normally distributed with means 70 and 65 respectively and standard deviations 8 and 10 respectively. In samples of 15 female and 20 male students, find the Prob. that the female students will have an average more than male students average.

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Sol) $X_1, X_2 \dots X_{15} \overset{r.s}{\sim} (70, 8^2)$
$Y_1, Y_2 \dots Y_{20} \overset{r.s}{\sim} (65, 10^2)$

$\bar{X} \sim n(70, 4.27)$
$\bar{Y} \sim n(65, 5)$

$\bar{X} - \bar{Y} \sim n(5, 9.27)$

$P(\bar{X} > \bar{Y}) = P(\bar{X} - \bar{Y} > 0)$
$P(Z > \frac{0-5}{\sqrt{9.27}}) = P(Z > -1.64)$
$= 1 - P(Z \leq -1.64)$
$= 0.9495$

*The distribution of the difference between 2 sample proportions:

$$\hat{P_1} - \hat{P_2} \dot{\sim} n(P_1 - P_2, \underbrace{\frac{P_1 q_1}{n} + \frac{P_2 q_2}{m}}_{m,n \text{ قيم كبيرة}})$$

$$\text{or } Z = \frac{(\hat{P_1} - \hat{P_2}) - (P_1 - P_2)}{\sqrt{\frac{P_1 q_1}{n} + \frac{P_2 q_2}{m}}} \dot{\sim} n(0,1)$$

e.g) suppose that 50% of population A own cars and 35% " " B own cars, If a sample of size 100 is drawn from population A and a sample of size 80 is drawn from population B, what is the Prob that the difference between sample proportions $\hat{P_A} - \hat{P_B}$ will be between 0.1 and 0.2 ?

Sol)

A	B
$P_1 = 0.5$	$P_2 = 0.35$
$n = 100$	$n = 80$
$q_1 = 0.5$	$q_2 = 0.65$

$\hat{P_1} - \hat{P_2} \dot{\sim} n(0.15, 0.00534)$

$P(0.1 < \hat{P_1} - \hat{P_2} < 0.2) =$
$P(\frac{0.1 - 0.15}{0.073} < Z < \frac{0.2 - 0.15}{0.073}) =$
$(-0.68 < Z < 0.68) =$
$P(Z < 0.68) - P(Z < -0.68) =$
$0.7517 - 0.2483 =$
0.5034