Statistics, lecture 24

لقاء مباشر 16 "ن ر"

* $X_1, X_2, X_3 \ldots X_{n_1} \overset{r.s}{\sim} n(\mu_1, \sigma_1^2)$
* $Y_1, Y_2, Y_3 \ldots Y_{n_2} \overset{r.s}{\sim} n(\mu_2, \sigma_2^2)$

Independent

"العينة الاولى مش مرتبطة بالثانية ولا بأي شكل من الاشكال"

$\bar{X} \sim n(\mu_1, \sigma_1^2/n_1)$

$\bar{Y} \sim n(\mu_2, \sigma_2^2/n_2)$

$$\bar{X}-\bar{Y} \sim n\left(\mu_1-\mu_2, \frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)$$

Test for $\mu_1-\mu_2$ → مثلاً بدي اعرف هل همّة متساويات او مختلفات

$H_0: \mu_1-\mu_2 = k$ vs $H_a: \mu_1-\mu_2 \neq k$

or $H_0: \mu_1-\mu_2 \leq k$ " $H_a: \mu_1-\mu_2 > k$

or $H_0: \mu_1-\mu_2 \geq k$ " $H_a: \mu_1-\mu_2 < k$

عادةً بيختاروا قيمة k صفر عشان يعرفوا اذا متساويات او لا.

1. Standardized test statistic is:

a) $$Z = \frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)_0}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$ if σ_1 and σ_2 are known

b) $$t = \frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)_0}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}}$$ σ_1 and σ_2 are unknown $\sigma_1 \neq \sigma_2$

$d.f = \min\{n_1-1, n_2-1\}$

c) $$t = \frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)_0}{sp \cdot \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$ σ_1 and σ_2 are unknown

$\sigma_1 = \sigma_2$

Recall:

$$S_p = \hat{\sigma} = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{\underbrace{n_1+n_2-2}_{d.f}}}$$

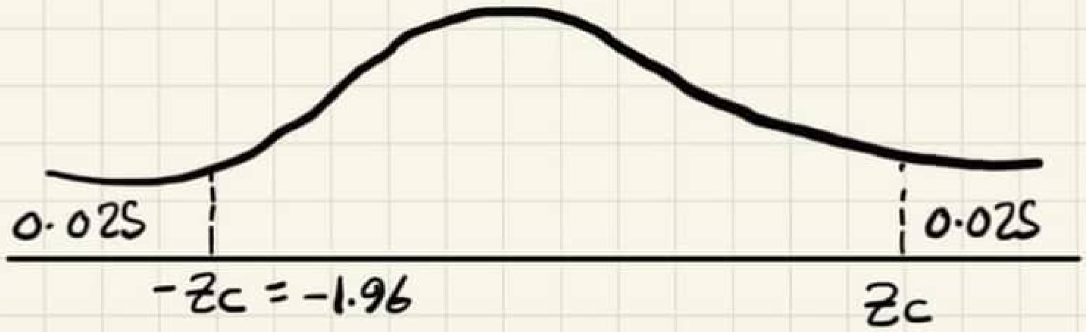
* طريقة الحل وحساب tc او zc نفس الي قبله ما في اختلاف.

***Example 2, book, P444:**

$H_a: \mu_1 \neq \mu_2$ "claim" $\quad H_0: \mu_1 = \mu_2$ $\quad$ 2 tailed test

$\mu_1 - \mu_2 = 0$ ↳ k

$$Z = \frac{(\mu_1-\mu_2)-(\mu_1-\mu_2)_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = 1.85$$



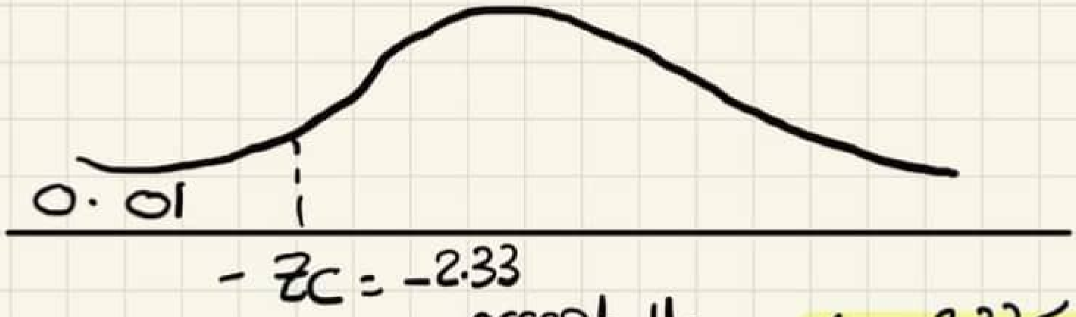
Fail to reject H_0 $\Leftarrow$ $-1.96 < 1.85$

***Solve example 3, book, P445: ^^**

$H_a: \mu_1 < \mu_2$ "claim" $\quad H_0: \mu_1 \geq \mu_2$ $\quad$ left tailed test

$\mu_1 - \mu_2 \geq 0$

$$Z = \frac{0-6}{6.87} = -0.87$$



accept H_0 $\Leftarrow -2.33 < -0.87$

طريقة ثانية للحل باستخدام P-value :

$0.1922 > \alpha \Rightarrow$ accept H_0

Example 1, book, P452 :

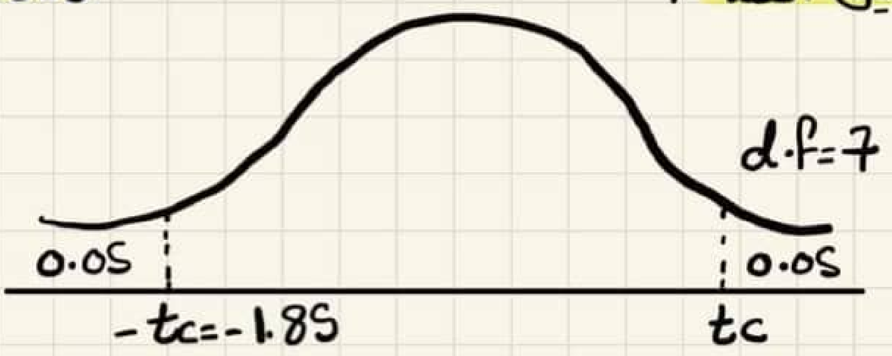
$H_a: \mu_1 \neq \mu_2$ "claim" $H_0: \mu_1 = \mu_2$ two tailed test

$\mu_1 - \mu_2 = 0$

σ_1, σ_2 unknown , $\sigma_1 \neq \sigma_2$

لما تبحث عن قيمة t، بتروح عالجدول one-tail هيك اتفقنا .

$$t = \frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = 0.922$$



$0.922 > -1.85$
Accept H_0

ملاحظة: ما في يطلب نحسب P-value في حالة t او chi ، فقط في حالة Z لكن
عادي ممكن يعطينا اياها جاهزة و يعطينا α و احنا نقارن

Example 2, book, P 453:

Ha: $\mu_1 < \mu_2$ "claim" Ho: $\mu_1 \geq \mu_2$

$\mu_1 - \mu_2 \geq 0$

σ_1^2, σ_2^2 are unknown, $\sigma_1^2 = \sigma_2^2$

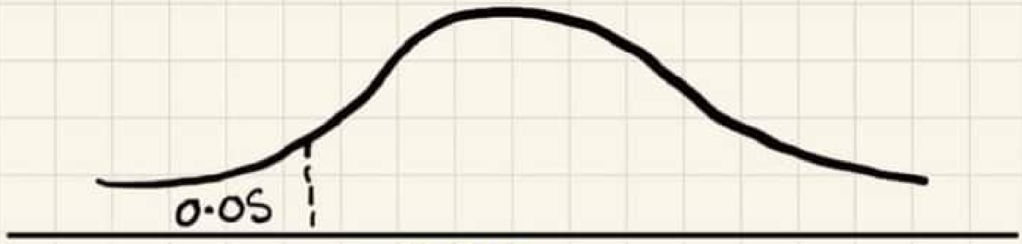
$$t = \frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{sp \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

.... recall $S_{\bar{x}_1} - S_{\bar{x}_2} = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$\simeq 0.0155$

$t = -1.930$

$d.f = 60$



$-t_c = -1.671$

$-1.930 < -1.671 \Rightarrow$ reject Ho

Test for $P_1 - P_2$

Ho: $P_1 - P_2 = K$ vs Ha: $P_1 - P_2 \neq K$

Ho: $P_1 - P_2 \leq K$ vs Ha: $P_1 - P_2 > K$

Ho: $P_1 - P_2 \geq K$ vs Ha: $P_1 - P_2 < K$

standardized statistics:

$$Z = \frac{(\hat{P}_1 - \hat{P}_2) - (P_1 - P_2)_0}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$\bar{P} = \frac{x+y}{n_1+n_2}$ $\bar{q} = 1 - \bar{P}$

n_1, n_2 large, Independent

$n_1 P_1 \geq 5$, $n_2 P_2 \geq 5$

$n_1 q_1 \geq 5$, $n_2 q_2 \geq 5$

Example 1, book, P 471: ^^

$H_0: P_1 = P_2$ "claim"

$P_1 - P_2 = 0$

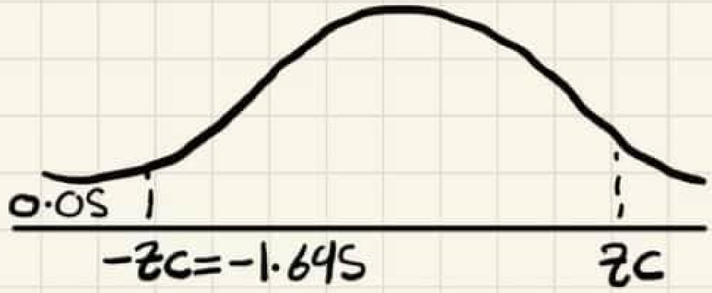
$H_a: P_1 \neq P_2$

* ضروري احقق الشروط *

$$\bar{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{182 + 208}{450} \simeq 0.8666$$

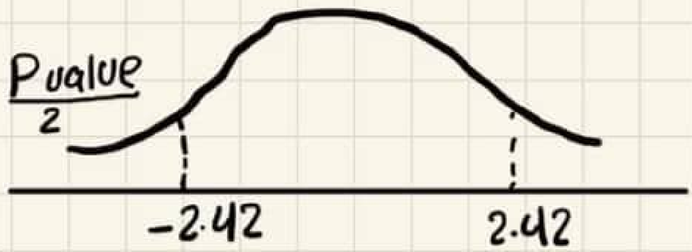
$$\bar{q} = 0.1334$$

$$z = \frac{(\hat{P}_1 - \hat{P}_2) - (P_1 - P_2)_0}{\sqrt{\bar{P}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 2.42$$



$2.42 > +1.645 \Rightarrow$ reject H_0

الحل باستخدام P-value :



$$\frac{P_{value}}{2} = 0.0078$$

$P_{value} = 0.0156 < \alpha$ reject H_0

Test of paired data "dependant samples"

كيف ممكن العينتين يكونوا معتمدات على بعض ؟ زي لما نجرّب دواء معين على مجموعة من النـاس و نشوف التأثير قبل وبعد ، الي قبل باعتبار عينة والي بعد عينة ثانية رغم انهم نفس الاشخاص بالظبط

الشروط :

randomly selected, dependent, normally distributed or $n \geq 30$

Paired t-test:

x_i	y_i	$d_i = x_i - y_i$	d_i^2
		Σd_i	Σd_i^2

$\bar{d} = \frac{\Sigma d_i}{n}$ $\qquad$ $s_d = \sqrt{\frac{\Sigma d_i^2}{n-1} - \frac{(\Sigma d_i)^2}{n(n-1)}}$

C.I for μ_d:

$$\bar{d} \pm t_c \cdot \frac{s_d}{\sqrt{n}}$$

Test statistics is: $t = \frac{\bar{x} - \mu_d}{s_d/\sqrt{n}}$ $\quad$ $d.f = n-1$

Example 1, book, P 461:

$H_0: \mu_d \geq 0$ $\qquad$ $H_a: \mu_d < 0$ "claim"

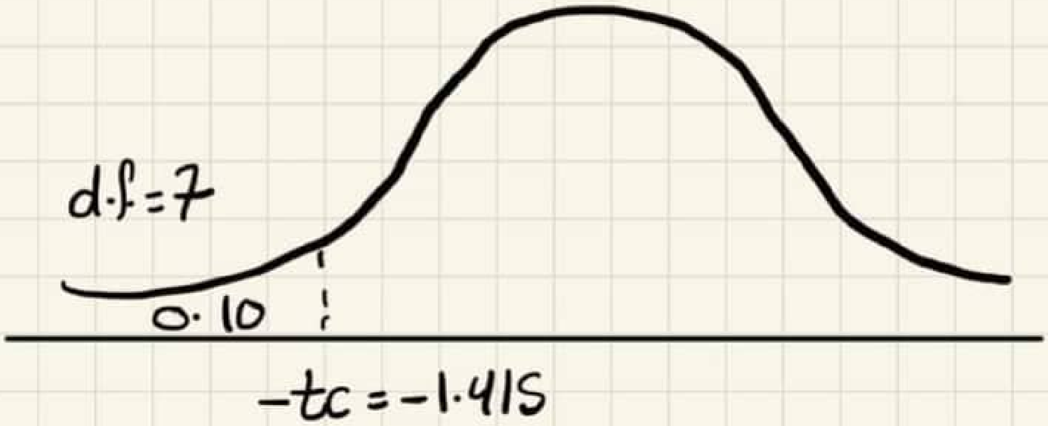
$d_i(x_i - y_i) = -2, -3, 0, -1, 2, -2, -5, -3 \Rightarrow \Sigma d_i = -14$

$d_i^2 \quad = 4, 9, 0, 1, 4, 4, 25, 9 \Rightarrow \Sigma d_i^2 = 56$

$\bar{d} = \frac{\Sigma d_i}{n} = \frac{-14}{8} = -1.75$

$s_d = \sqrt{\frac{\Sigma d_i^2}{n} - \frac{(\Sigma d_i)^2}{n(n-1)}} = 2.1213$

$t = \frac{d - \mu_d}{s_d/\sqrt{n}} = -2.333$



$-2.333 < -1.415 \Rightarrow$ reject H_0

solve example 2, book, P 462 :

solve Q 23, book, P 468 :