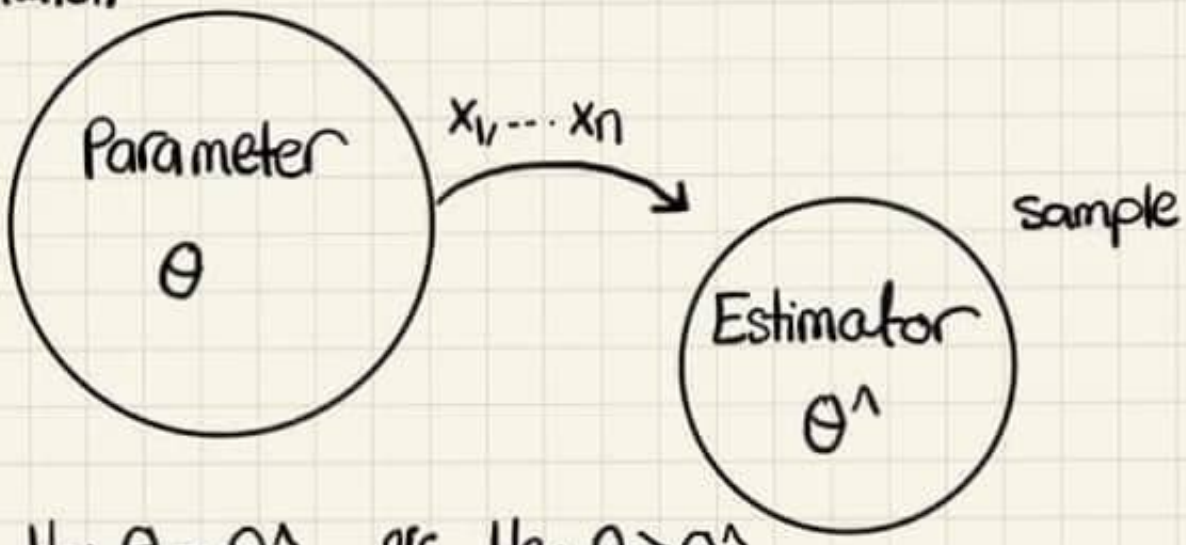
Statistics, lecture 23

19_يونيو

*Null hypothesis, H_0:
A statement about a population parameter that is assumed to be true until it is declared false.

Alternative Hypothesis, H_a:
A statement about a population parameter that will be true if the null hypothesis is false

Population



$H_0: \theta = \theta^\wedge$ vs $H_a: \theta > \theta^\wedge$
or $\theta < \theta^\wedge$
or $\theta \neq \theta^\wedge$

عادي ممكن كمان تكون
$\geq$ ، $\leq$

Test statistics: a function of $x_1, x_2, x_3 \ldots x_n$

e.g) The mean cholestrol levels in a general population are normally distributed. A sample of 16 persons is taken under a test with mean $\bar{x} = 220$ mg/dL and standard deviation $S = 25$ mg/dL. Test at 1% significance level that the mean cholestrol level is less than 230 mg/dL

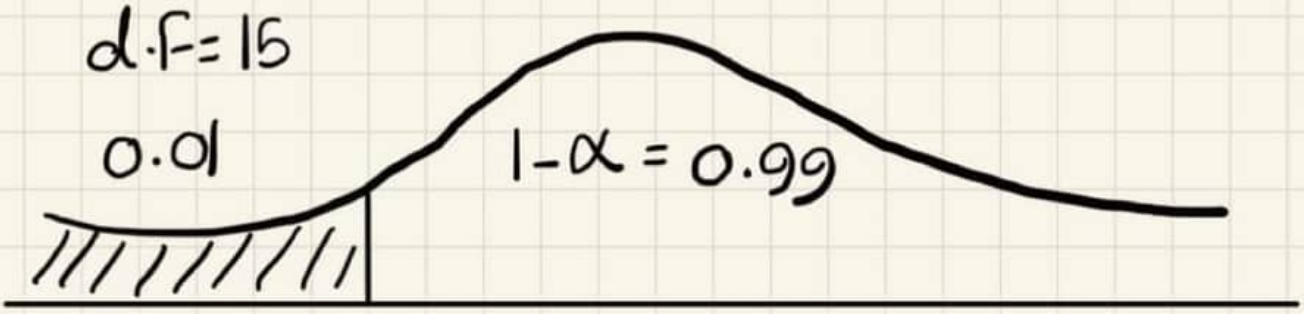
①

Sol) $n=16$ $\bar{x}=220$ $S=25$ $\alpha=0.01$

$H_a: \mu < 230$ "claim" $H_o: \mu \geq 230$

$$t=\frac{\bar{x}-\mu_o}{S/\sqrt{n}}=-1.6$$

left tailed test لهذا " < " ال بدل



$t_c=-2.539$

حسبتها بالاستفادة من التماثل زي ما درسنا قبل هيك

Fail to reject H_o بالتالي $-2.539 < -1.6$

e.g) A random sample of 400 people with a proffesional degree taken showed that their monthly mean salary is 450 JDs with a standard deviation of 100 JD. Test at 5% significance level that the monthly mean salary is different from 460 JDs.

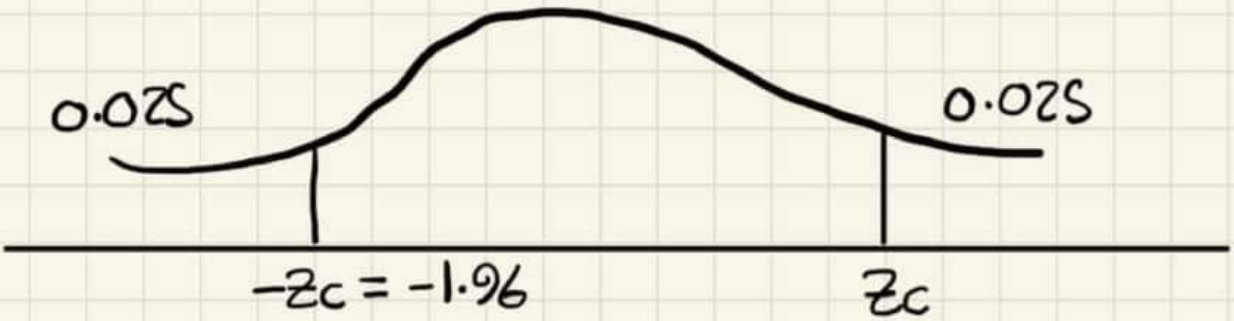
$n=400$ $\bar{x}=450$ $S=100$ $\alpha=0.05$

$H_a: \mu \neq 460$ "claim" $H_o: \mu = 460$

الي بيسأل عنه تبعله في Ha

sol) $Z = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = -2$

two tailed test بما انه " ≠ " اذاً



ملاحظة: مكالك بالسؤال انه Random sample بالتالي كل المعطيات من العينة

reject Ho ⇐ $-2 < -1.96$

↳ وكنتي ثقة مقدارها 0.05 انه $\mu \neq 460$

Test for P

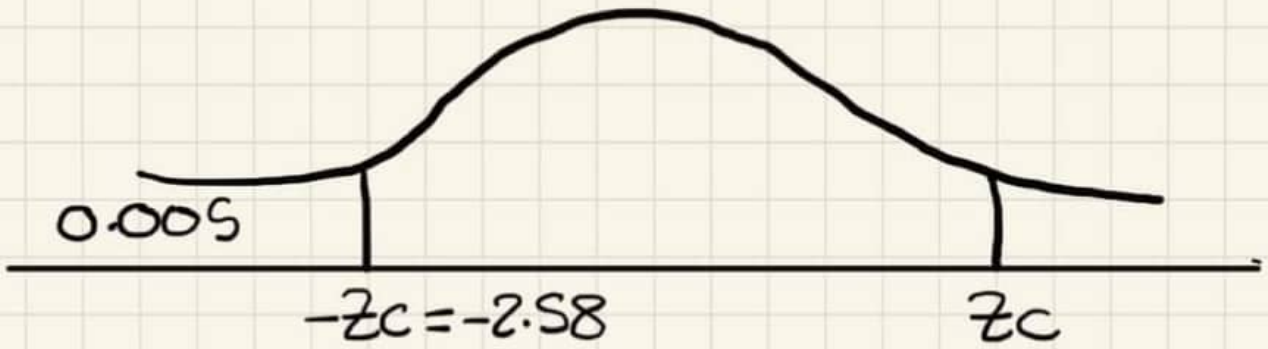
e.g) It was believed in the Arab world that 50% of persons are smokers. During the year 2000, a sample of 1000 persons showed that the no. of smokers is 620. Can you conclude that the proportion of " " different from 50%? use $\alpha = 0.01$

$P = 0.5$ $n = 1000$ $P^0 = 0.5$ $(1-P_0) = 0.5$

$\alpha = 0.01$

$H_a: P \neq 0.5$ $H_0: P = 0.5$

Sol) $Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = 7.59$ two tailed test هنا ≠ ال ho



reject Ho ⇐ $-2.58 < 7.59$

* Test for σ^2 *

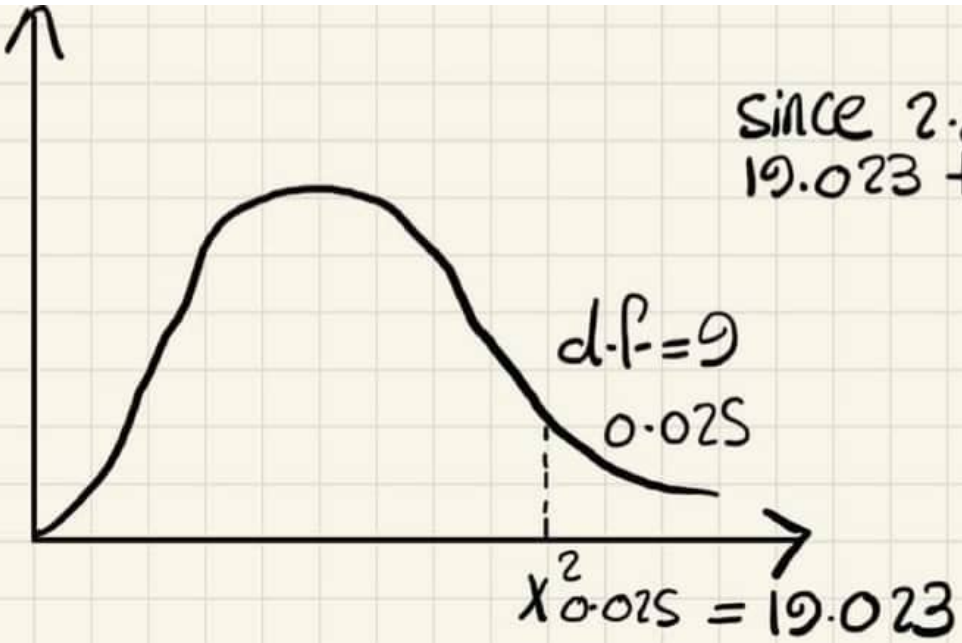
e.g) Quality control engineer wishes to study the weight variation of a new product. A sample of 10 items is taken and provided $\bar{x} = 0.6$ kg and $s = 0.4$ kgs. Assume that the distribution of the weights can be modeled as a normal distribution.

a) test $H_0: \sigma^2 = 0.5$ vs $H_a: \sigma^2 > 0.5$ $\alpha = 0.025$

b) " $H_0: \sigma = 0.74$ vs $H_a: \sigma \neq 0.74$ $\alpha = 0.10$

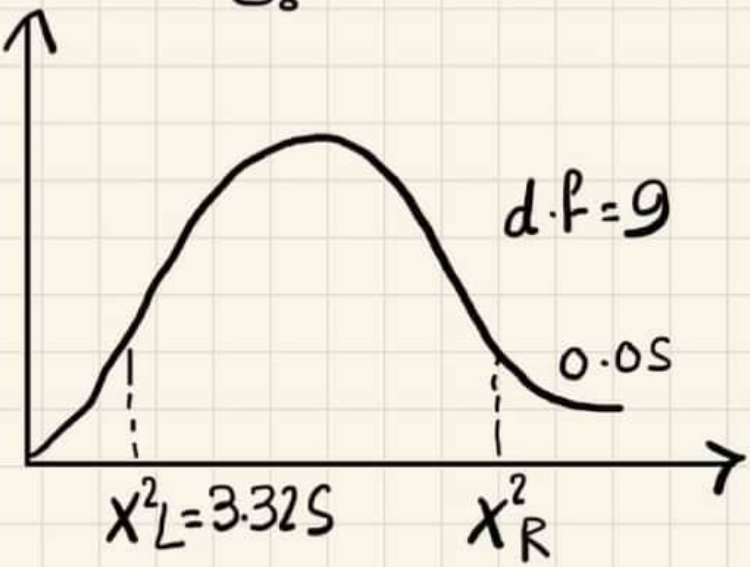
$n = 10$ $\bar{x} = 0.6$ $s = 0.4$

a) $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = 2.88$



since 2.88 is less than 19.023 then accept H_0

b) $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} = 2.63$ $\Rightarrow$ two tailed test



since $2.63 < 3.325$ then reject H_0

* Relationship between C.I and tests:

Let (L, U) be $1-\alpha$ % C.I for an unknown parameter θ. The null hypothesis $H_0: \theta = \theta_0$ is rejected against $H_a: \theta \neq \theta_0$ at significant level α if θ_0 doesn't belong to (L, U)

يعني بشوف القيمة اللي حط H_0 اذا موجودة جوا C.I مقبولة واذا خارجها مرفوضة
بس بشرط يكون two tailed test

eg) A random sample of 8 observations was taken from a normal population. The sample mean and standard deviation are $\bar{x}=70$ and $s=20$. Find a 95% C.I for μ and test at 5% confidence level.

$H_0: \mu = 80$ vs $H_a: \mu \neq 80$

Sol) $n=8$ $\bar{x}=70$ $s=20$ $1-\alpha = 0.95 \rightarrow \alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$$\bar{x} \pm t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} = 70 \pm (2.365) \times \frac{20}{\sqrt{8}} \Rightarrow (53.29, 86.71)$$

بالنسبة لل test كلها 2 tailed و α نقسمها بشوف قيمة μ
اللي عند H_0 وهي 80 ⇐ عمن الفترة ⇐ Accept H_0