Statistics, lecture 9:-

محاضرة ٩ روتين

Expirements

- Determenistic → Boiling water
- Random → tossing a coin → You can not exactly determine what the next outcome is, but You can determine all the possible outcomes

*** The Sample space (Ω):**
The set of all possible outcomes

e.g) Find the sample space for each of the following random experments

i) Tossing a fair coin 1 time

$\Omega = \{H, T\} \rightarrow n(\Omega) = 2^1$

* ملاحظة:
Fair (unbiased) coin: (H,T) كلها وجهين
Biased coin: (T,T)/(H,H) وجه واحد
← مغشوشة

ملاحظة: لمّا نرمي coin مرتين او 2 coins مع وحدة ... نفس النتائج.

ii) Tossing a fair coin 2 times

$\Omega = \{(H,H), (H,T), (T,T), (T,H)\}$

$n(\Omega) = 2^2$

iii) Tossing a fair coin 3 times:-

$\Omega = \{(H,H,H), (H,H,T), (T,H,H), (H,T,H), (T,T,T), (T,T,H), (H,T,T), (T,H,T)\}$

$n(\Omega) = 8 = 2^3$

Note: When tossing a fair coin k-times, we have $n(\Omega) = 2^k$

iv) Trowing a fair die 1 time

$\Omega = \{1, 2, 3, 4, 5, 6\}$
$n(\Omega) = 6^1$

v) Trowing 2 fair dice 1 time

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	- - - -	
2		- - - -	- - - -			
3						
4						
5						
6						(6,6)

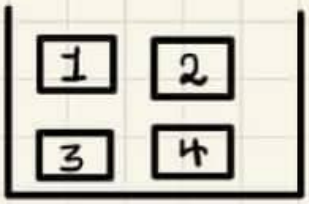
$n(\Omega) = 36 = 6^2$

Note: when throwing a fair die k-times, we have
$n(\Omega) = 6^k$

$n(\Omega) = X^k$

X: no. of Possible results
k: no. of expirements

vii)



2 cards are drawn at-random:

a) With replacement
b) Without replacement

a)

ثاني بطاقة / اول بطاقة	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)
4	(4,1)	(4,2)	(4,3)	(4,4)

$n(\Omega) = 16$

b)

ثاني بطاقة / اول بطاقة	1	2	3	4
1	X	(1,2)	(1,3)	(1,4)
2	(2,1)	X	(2,3)	(2,4)
3	(3,1)	(3,2)	X	(3,4)
4	(4,1)	(4,2)	(4,3)	X

$n(\Omega) = 12$

* The Event:
Any subset of the sample spac

* Types of Events:

1. Simple (Elementary): consists of exactly 1 element of Ω

2. Composite (Combined): consists of more than 1 element of Ω

3. Certain (Sure): Consists of all elements of Ω

4. Impossible (NULL): consists of no elements of Ω.

g) When throwing a fair die 1 time, define:

A: Getting a no. divisible by 5
B: " " prime no.
C: " " no. less than 7
D: " " " more than 6

Find the elements and classify each event above.

Sol) $\Omega = \{1,2,3,4,5,6\}$

A: $\{5\}$ → Simple (elementary)
B: $\{2,3,5\}$ → Composite (compined)
C: $\{1,2,3,4,5,6\}$ → Sure (certain)
D: $\{\} = \emptyset$ (impossible or null)

* The pobability of an event A is defined by:

$$P(A) = \frac{n(A)}{n(\Omega)}$$

e.g) In the previous example, we have:

$$P(A) = \frac{1}{6}, \quad P(B) = \frac{1}{2}, \quad P(C) = 1$$

$$P(D) = 0$$

Note: ① $P(\emptyset) = 0$
② $P(\Omega) = 1$
③ $0 \leq P \leq 1$

****

$P(A) = 0 \Rightarrow$ Impossible
$P(A) = 1 \Rightarrow$ Certain
$P(A) = 0.5 \Rightarrow$ Even chance

خامزة ٩ لقاد مباشر

"فرصة حدوثة مساوية لفرصة عدم حدوثة"

$P(A) = 0.75 \Rightarrow$ Likely

$P(A) = 0.25 \Rightarrow$ Unlikely

"فرصة حدوثة اكبر من عدم حدوثة"
"فرصة حدوثة أقل من عدم حدوثة"

أمّا اذا كانت 0.05 أو أقل تسمى Unusual

↳ High unlikely to occur

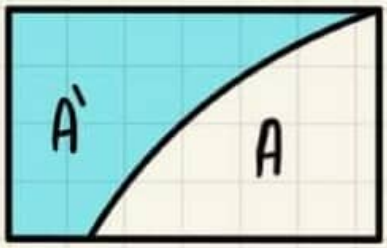
* Likely + more likely توضيح للمصطلحين *

$$P(A) = \frac{1}{6} \qquad P(B) = \frac{1}{2}$$

$P(B) > P(A) \Rightarrow$ B is more likely to occur than A

* Rules of Probability *

$\Omega \rightarrow P(\Omega) = 1$

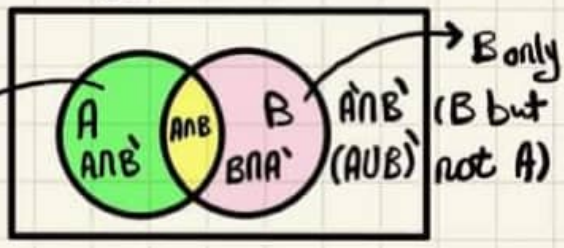


A': complement of A / Not A

تكمل A لتصبح Ω وهي مكونة من عناصر موجودة في Ω وليست موجودة في A.

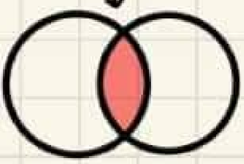
1. $P(A) + P(A') = 1$
or $P(A') = 1 - P(A)$

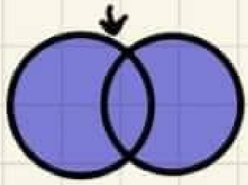
2.



$A \cap B$ → A and B

$A \cup B$ → A or B





2\. i) $P(A \cap B') = P(A) - P(A \cap B)$

ii) $P(B \cap A') = P(B) - P(B \cap A)$

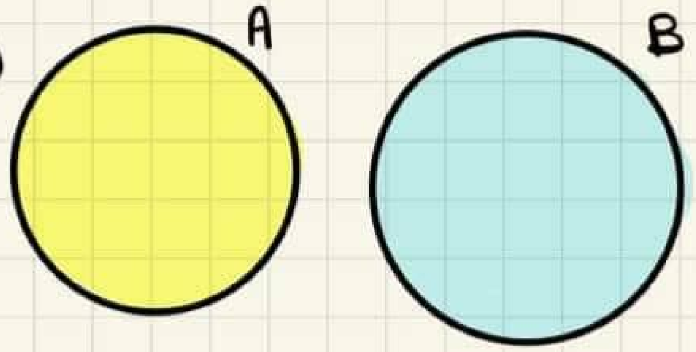
3\. $P(A \cup B) = P(B) + P(A \cap B')$

$P(B) + P(A) - P(A \cap B)$

De` Morgan's laws:

) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$

) 

A and B are said to be mutually exclusive or disjoint.

$P(A \cap B) = 0$ <u>or</u> (ii) $P(A \cup B) = P(A) + P(B)$

واحدة منهم تكفي

6) A and B are said to be independent if they don't influence each other.

Mathmetically,

$P(A \cap B) = P(A) \times P(B)$

7) Conditional probability

$P(A|B)$:B ئه بيلكيلك انه العينة الجديدة

Given B داخل A احتمالية ان كل ... يهو

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Note: If A and B are independent

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)}$$

$= P(A)$ ∴

$P(A|B) = P(A)$

eg) If $P(A) = 0.8$, $P(B) = 0.7$, $P(A \cap B) = 0.6$. Then find:

ملاحظة: الاسئلة مرحلة بناءً كل انا نعلم

P(A),P(B) , P(A∩B) فاذا وحة منهم مجهولة

نجدها قبل البدء بالكل .

i) $P(A') = 1 - P(A) = 1 - 0.8 = 0.2$

ii) $P(B') = 1 - P(B) = 1 - 0.7 = 0.3$

iii) $P(A \cap B') = P(A) - P(A \cap B)$

$= 0.8 - 0.6$

$= 0.2$

iv) $P(A' \cap B) = P(B) - P(A \cap B)$
$= 0.7 - 0.6$
$= 0.1$

v) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.8 + 0.7 - 0.6$
$= 0.9$

vi) $P(A' \cap B') = P(A \cup B)' = 1 - (B \cup A)$
$= 1 - 0.9$
$= 0.1$

i) $P(A' \cup B') = P(A \cap B)' = 1 - (A \cap B)$
$= 1 - 0.6$
$= 0.4$

ii) $P(A \cup B') = P(A) + P(B') - P(A \cap B')$
$= 0.8 + 0.3 - 0.2$
$= 0.9$

x) $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$
$= 0.2 + 0.7 - 0.1$
$= 0.8$

) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.6}{0.7} = \frac{6}{7}$

$P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{0.1}{0.7} = \frac{1}{7}$

$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{0.2}{0.3} = \frac{2}{3}$

xiii) $P(A'|B') = \frac{P(A' \cap B')}{P(B')}$
$= \frac{1 - (A \cup B)}{P(B')} = \frac{0.1}{0.3} = \frac{1}{3}$

xiv) $P(A|B)' = 1 - (A|B)$
$= 1 - \frac{6}{7} = \frac{1}{7}$

Note: $P(A|B)' = P(A'|B)$

Example 1 / Book p169:

1.

~~52~~ 51

~~4~~ kings / 3 kings — 4 Queens

$P(2^{nd} \text{ queen} \mid 1^{st} \text{ king}) = \frac{4}{51} = 0.078$

2. $P(\text{Male} \mid \text{offended by something})$

$\frac{532}{1151} \approx 0.462$

الكيتة عنه فكرة الجدول لاحقاً لكن كتبته
الحل على اي حال

e.g) If $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$. Are A and B mutually exclusive? Independent? neither?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = 0.3 \neq 0$ ∴ A and B are not mutually exclusive

$P(A) \cdot P(B) = 0.3$ A and B are Independent

g) If P(A)=0.7, P(B)=0.6 where A and B are Independent, then find:

i) $P(A\cap B) = P(A)\cdot P(B) = 0.42$
ii) $P(A\cap B') = P(A)\cdot P(B') = 0.28$
iii) $P(A'\cap B) = P(A')\cdot P(B) = 0.18$
iv) $P(A'\cap B') = P(A')\cdot P(B') = 0.12$
v) $P(A|B) = P(A) = 0.7$
vi) $P(A'|B) = P(A') = 0.3$
vii) $P(A|B') = P(A) = 0.7$
viii) $P(A'|B') = P(A') = 0.3$
x) $P(A|B)' = P(A'|B) = P(A') = 0.3$

اذا هكالك Independent مباشرة التقاطع بيتحول لضرب

XXXXX حامرة ١٠ لدكتور

e.g) If A and B are Independent events such that P(A)=2P(B) and P(A∪B)=0.8 then find P(A)

Sol)

$P(B) = x$

$P(A) = 2x$

$P(A\cap B) = 2x^2$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$0.8 = 2x + x - 2x^2$

$2x^2 - 3x + 0.8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4\times 2\times 0.8}}{4}$$

$x = 1.15$ X

$x = 0.347 \rightarrow P(A) = 0.694$

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