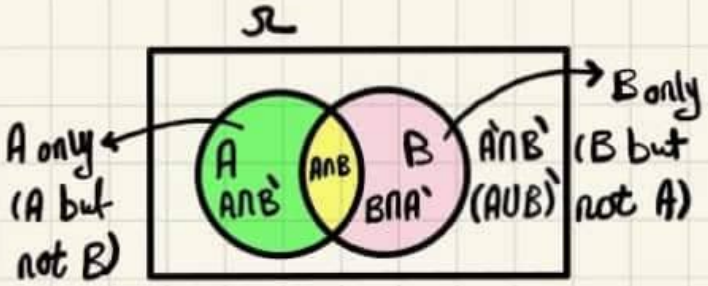
Statistics, lecture 10: محاضرة رقم "١٠" يوتيوب

-Venn-diagram:



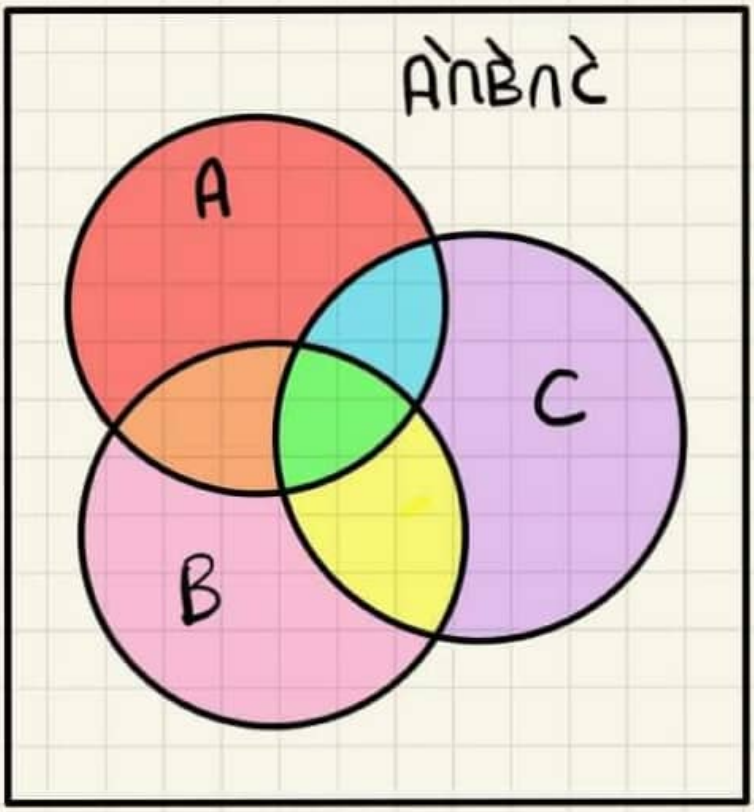
* Probability for an element in both A and B ⇒ intersection "∩"

* Probability for an element in A or B ⇒ Union "U" ⇒ A or B or both ⇒ belongs to at least one set

* Probability for an element in A only (A∩B')

* Probability for an element that belongs to exactly one set ⇒ (A only or B only) ⇒ (A∩B')+(B∩A')

* Probability for an element that belongs at most to one set ⇒ (A only or B only or (A'∩B'))



- A∩B∩C
- A∩B'∩C
- A'∩B'∩C
- A∩B'∩C'
- A'∩B∩C
- A∩B∩C'
- A'∩B∩C'

e.g) A class of 20 students.
10 play football, 8 play basketball
5 " neither.
A student is selected at random, what is the prob. that this student plays:
a) exactly one sport.
b) at least one sport
c) at most one sport.
D) Football given that the student plays basket-ball
e) Exactly 1 sport given that the student plays at most one sport

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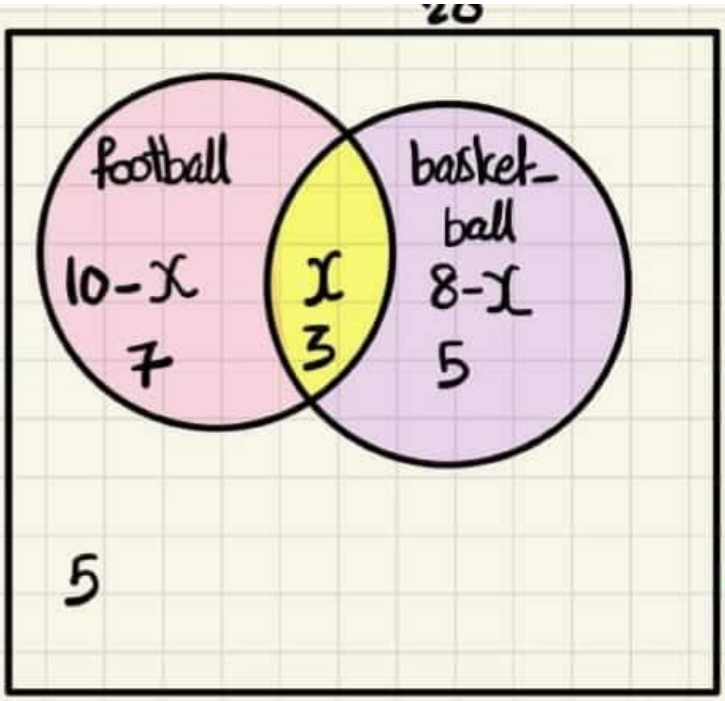

sol) $10-X+X+8-X+5=20$

$X=3$

a) $\frac{7+5}{20}=\frac{12}{20}=0.6$

b) $\frac{5+7+3}{20}=0.75$ "كل الاقل رياضة او اكثر"

c) $\frac{5+7+5}{20}=0.85$ "كل الأكثر رياضة او 2"

D) $P(F|B)=\frac{P(F\cap B)}{P(B)}=\frac{3/20}{8/20}$

$\frac{3}{8}=0.375$

ملاحظة: يمكنه حل الفرع السابع مباشرة بناءً على ان ال new sample space اصبح 8

E) P(Exactly 1 sport | at most 1)

$\frac{7+5}{7+5+5}=\frac{12}{17}\simeq 0.706$

e.g) A class of 30 stududents.
15 play football.
12 play basketball.
6 play volleyball only
7 play football and basketball
6 play " " volleyball
3 play basketball and " only
2 play the 3 sports.

A student is selected at random what is the prob. that this student plays:
a) exactly 1 sport
b) " 2 sports
c) at least 1 sport
d) " " 2 sports
e) at most 1 sport.
f) " " 2 sports.
g) football given that the student plays basketball
h) exactly 1 sport given that the student plays at most 1 sport.

ملاحظة: في هذا النوع من الاسئلة يفضل البدء بالمحطيات من الاخير.

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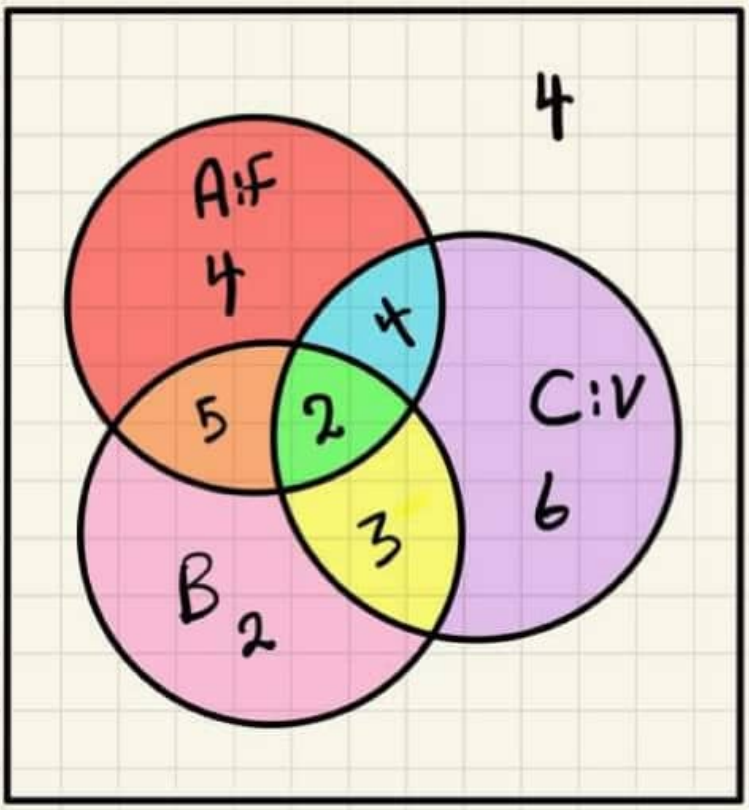


a) $\frac{2+6+4}{30} = 0.4$

b) $\frac{3+4+5}{30} = 0.4$

c) $\frac{26}{30} \simeq 0.87$

d) $\frac{2+3+4+5}{30} = 0.45$

e) $\frac{2+6+4+4}{30} = 0.53$

f) $\frac{28}{30} = 0.93$

g) $P(F|B) = \frac{7}{12} = 0.58$

h) $P(\text{exactly 1} \mid \text{at most 1}) = \frac{12}{16}$

$= 0.75$

Tree diagram

usually we use it when we select more than one item

e.g)

B	W
5	3

5

2 balls are selected at random:

i) without replacement

ii) with replacement "do it ur-self"

Find the prob. of getting:

a) 2 black balls

b) 2 balls of the same colour

c) 2 balls of different colours

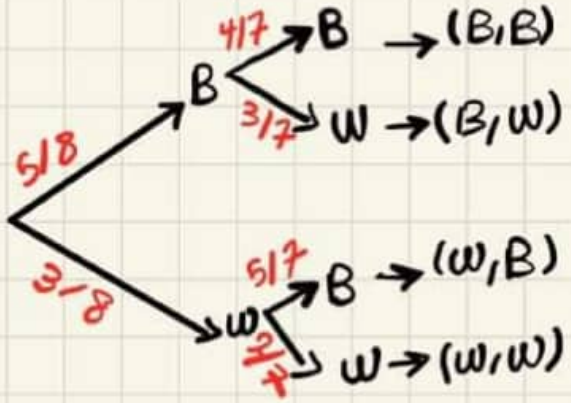
d) exactly 1 black ball

e) At least one black ball

f) At most one black ball

g) the 1st ball is black given that the second one is black

sol)



a) $P(B,B) \Rightarrow \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

b) $P(B,B) + P(W,W) = \frac{20}{56} + \frac{6}{56} = \frac{26}{56}$

c) $P(B,W) + P(W,B) = \frac{15}{56} + \frac{15}{56} = \frac{30}{56}$

D) $P(W,B) + P(B,B) = \frac{30}{56}$

e) $1 - P(W,W) = \frac{50}{56}$

f) $1 - P(B,B) = \frac{36}{56}$

6) $P(1^{st} B \mid 2^{nd} B) = \frac{P(B,B)}{P(B,B)+P(W,B)}$

$6 = \frac{20}{\frac{}{6}}$

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بال Tree diagram انتبه الحالة G عشان ما تتلخبط ... اتشغل المقام اولاً ثم البسط.

n) $P(2^{nd}\ B \mid different) = \dfrac{P(W,B)}{P(B,W)+P(W,B)}$

$= \dfrac{1}{2}$

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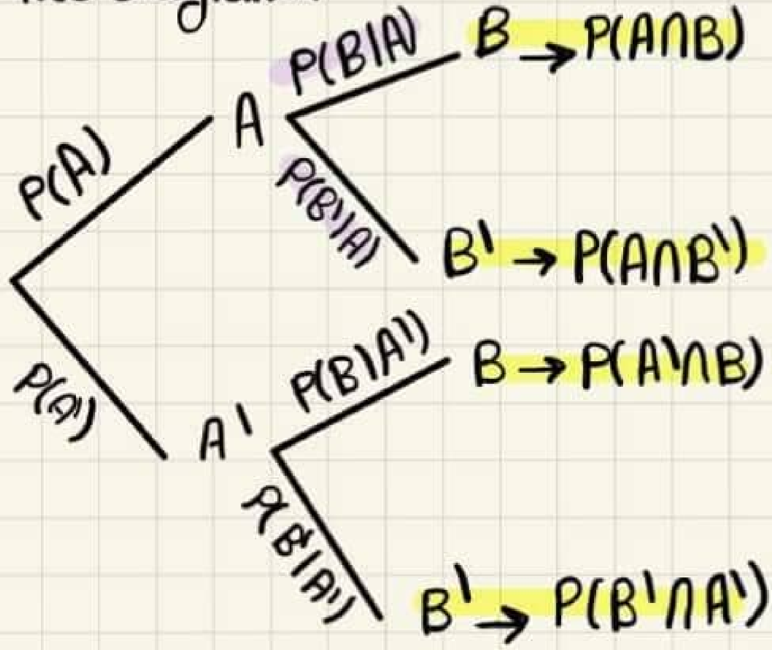
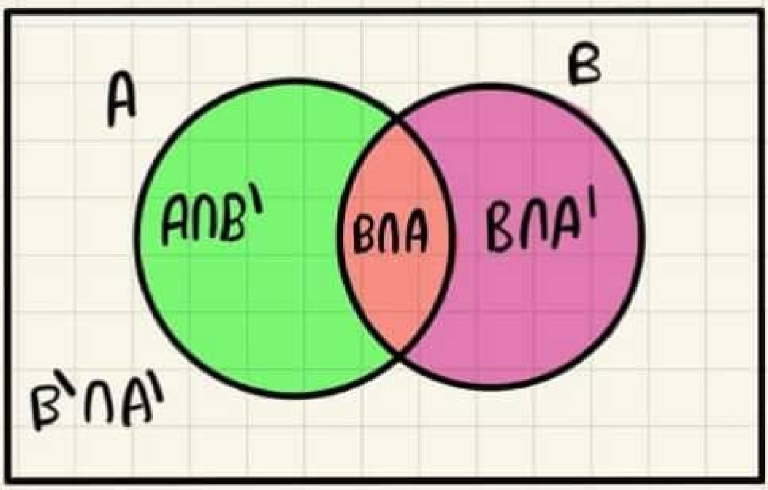
قاصمة ٢ يوتيوب

* Tree diagram:



توضيح: ليش كنّا نضرب لما نطلع الاحتمال ؟

$P(A) \times P(B|A) = \dfrac{P(A) \times P(B\cap A)}{P(A)}$ ✓



ملاحظة:
* الاحتمالات المظللة بالاصفر مجموعهم 1
* الاحتمالات المظللة بالبنفسجي مجموعهم 1
بالنسبة لـ A "على اعتبار ان العينة هي A"

"Total Probability"

$P(B) = P(A\cap B) + P(B\cap A')$

"Bays Rule"

$P(A|B) = \dfrac{P(A\cap B)}{P(B)}$

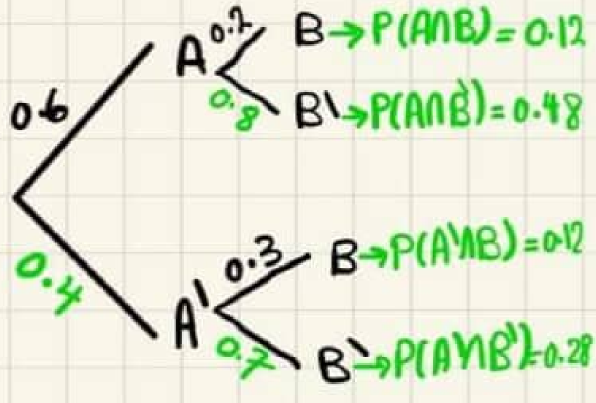
$\dfrac{P(A).P(B|A)}{P(B)}$

$\dfrac{P(A).P(B|A)}{P(A\cap B)+P(B\cap A')}$

$\dfrac{P(A).P(B|A)}{P(A)\cdot P(B|A)+P(A').P(B|A')}$

ملاحظة: اذا عرفت تتعامل مع Tree diagram بشكل جيد فلن تحتاج الصيغتين اعلاه ^^

eg)



a) complete the tree diagram
b) Find P(B)   c) Find P(A|B)

b) $P(B) = P(A\cap B) + P(B\cap A')$

$0.12 + 0.12$

0.24

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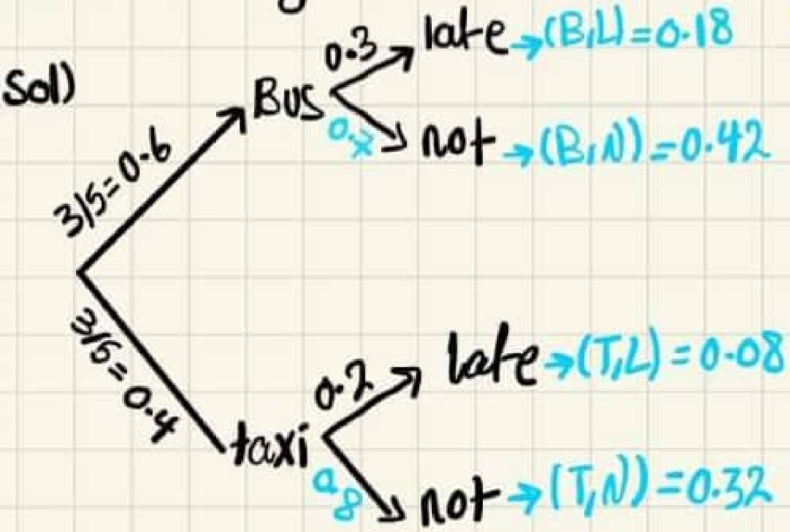
c) $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.12}{0.24} = \frac{1}{2}$

ملاحظة: هنا المثال فيه A و B لكن في الكتاب
ستجد الصيغ عمة اكثر (A وB وC)

e.g) Ahmad goes to school either by bus or by taxi. If he goes by bus, the probability that he will be late is 0.3, but if he goes by taxi then the prob. that he will be late is 0.2. It's knowen that Ahmad goes to school by bus 3 days a week

a) for a given school days, what is the Prob. that
i) Ahmad is late
I) He came by bus if he is late

Sol)



i) $P(L) = 0.18 + 0.08 = 0.26$

ii) $P(B|L) = \frac{P(B\cap L)}{P(L)} = \frac{0.18}{0.26} = \frac{9}{13}$

ملاحظ :

* Tree diagram مفيد عند وجود تشعّب
* افترضنا عدد ايام الدوام 5
* If = Conditional

e.g)

sum		Smoking	Not
10	Male	8	2
10	Female	6	4
20	sum	14	6

a) A student is selected at random, what is the Prob. that the student is:
i) a Female smoker
ii) a Female or a smoker
iii) a Female if the student is a Smoker

b) 2 students are selected at random, what is the Prob. that exactly 1 one of them is a smoker

مهم { and ∩ ; or ∪ }

a)
i) $P(F\cap S) = \frac{6}{20} = 0.3$

ii) $P(F\cup S) = P(F) + P(S) - P(F\cap S)$
$\frac{10}{20} + \frac{14}{20} - \frac{6}{20} = \frac{18}{20}$

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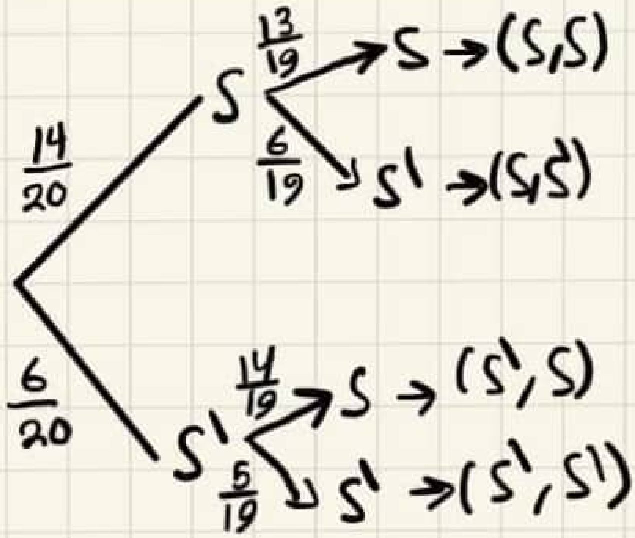
ملاحظة: يُفضل حل الفرع السابق عالقانون
وليس من الجدول مباشرة عشان ما
تلخبط 😊

$$1)\ P(F|S) = \frac{P(F \cap S)}{P(S)} = \frac{3}{7}$$

b)

S	S'
14	6

مش مهتم اذا
ذكر او انثى فقط
مهتم اذا مدخن
او لا .



خلّي هالمعلومة ببالك: اذا ما حدد
with or without replacement
معناها without replacement

$$P(S,S') + P(S' + S) =$$

$$\frac{14}{20} \cdot \frac{6}{19} + \frac{6}{20} \cdot \frac{14}{19}$$

$$\frac{84 + 84}{380} = 0.442$$

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