Chapter 10:Fluids

Lecture 3

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10-7] Bouyancy and Archimedes' principle

In our life we may experience the following:

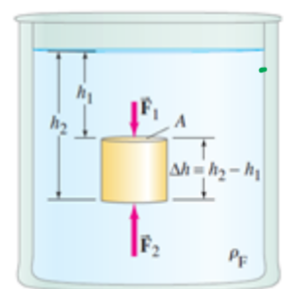
1) In a swimming pool, you feel you have lesser weight than in air (when you are outside the pool).

2) You can easily lift a heavy object when it is in water. But it is more difficult to lift it when it is outside the water.

Objects in fluids experience an upward force due to the fluid. This upward force is called the buoyant force. قوة الطفو

What causes the buoyant force in a fluid?

To understand the physical origin of the buoyant force consider the cylinder shown, which is completely submerged in the liquid.



The pressure due to the liquid at the top surface is

$$P_1 = \rho_f g h_1 \Rightarrow F_1 = P_1 A = \rho_f g h_1 A$$

The pressure due to the liquid at the bottom surface is

$$P_2 = \rho_f g h_2 \Rightarrow F_2 = P_2 A = \rho_f g h_2 A$$

Note $P_2 > P_1 \Rightarrow F_2 > F_1 \Rightarrow$ the liquid acts with a resultant force vertically upwards on the cylinder. This force is called the buoyant force F_B given by: buoyant force

$$+\uparrow \quad F_B = F_2 - F_1 = \rho_f (h_2 - h_1) A g = \rho_f \Delta h A g$$

but $\Delta h A = V$ the volume of displaced fluid. (= volume of cylinder as it is fully submerge under the surface of the fluid)

$$\therefore \; F_B = \rho_f V g = m_f g$$ — weight of displaced fluid.

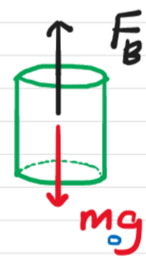
Archimedes' Principle: Buoyant force on an object immersed in a fluid equals the weight of the displaced fluid.

Is the cylinder going to sink or float?

$$+\uparrow \quad F_R = F_B - m_o g$$

$$= \rho_f V g - \rho_o V g$$

(F_R: resultant force acting on the cylinder; m_o: mass of object; ρ_o: density of object (cylinder))



$$F_R = (\rho_f - \rho_o) V g$$

If $\rho_f > \rho_o \Rightarrow F_R \uparrow$ and object floats.

If $\rho_f < \rho_o \Rightarrow F_R \downarrow$ and object sinks.

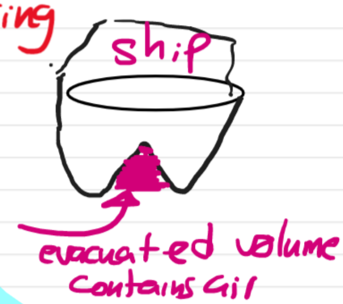
If $\rho_f = \rho_o \Rightarrow$ object hangs in the fluid in equilibrium.

Objects whose densities are greater than the density of the liquid sink.

What causes the buoyant force in fluids?

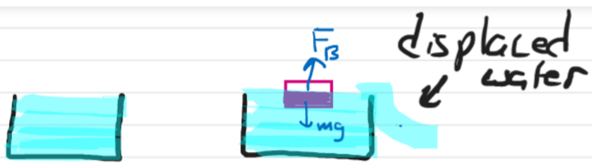
The increase of liquid pressure with increasing depth bleow the liquids surface.

A ship has iron and steel structure, dispite this it floats in the sea. Explain this.



The ship has an evacuated space which is full of air. So, when we divide the total mass of the ship by its total volume the resulting density is less than that of seawater ⇒ the ship floats.

Conceptual Example 10-6]



Two pails of water. Consider two identical pails of water filled to the brim. One pail contains only water, the other has a piece of wood floating in it. Which pail has the greater weight?

RESPONSE Both pails weigh the same. Recall Archimedes' principle: the wood displaces a volume of water with weight equal to the weight of the wood. Some water will overflow the pail, but Archimedes' principle tells us the spilled water has weight equal to the weight of the wood; so the pails have the same weight.

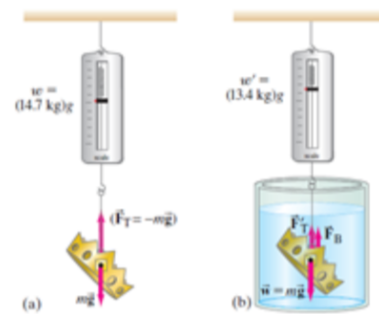
$$V_s \rho_w g = F_B$$

$$= m_o g = \rho_o V g$$

$$V_s \rho_w = V \rho_o$$

Example 10-8]

Is the crown gold? When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold?



In air: weight of object in air

$$W = F_T = mg$$

$$W = mg = \rho_o V g$$

$$14.7g \nearrow W = \rho_o V g \quad -①$$

In water: apparent weight in water (reading of balance in water)

$$m_o g = F_T' + F_B$$

$$m_o g - F_T' = F_B$$

$$W - W' = F_B = \rho_f V g \quad -②$$

$$②/① \Rightarrow \frac{W - W'}{W} = \frac{\rho_f V g}{\rho_o V g}$$

$$\therefore \quad \frac{W - W'}{W} = \frac{\rho_f}{\rho_o} \Rightarrow \rho_o = \left(\frac{W}{W - W'}\right)\rho_f$$

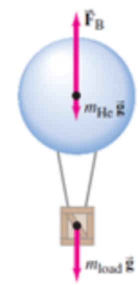
$$\rho_o = \left(\frac{14.7}{14.7 - 13.4}\right)\rho_f \quad (\rho_f = 1000\ kg/m^3)$$

$\therefore \quad \rho_o = 11300\ kg/m^3$, this is the density of <u>lead</u>.

Example 10-10]

Helium balloon. What volume V of helium is needed if a balloon is to lift a load of 180 kg (including the weight of the empty balloon)?

Assume balloon and load in static equilibrium



$$\overset{+}{\uparrow}\; F_B - m_{He}g - m_L g = 0$$

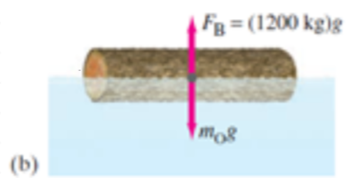
$$\rho_{air} V g - \rho_{He} V g - m_L g = 0$$

$$V = \frac{m_L}{\rho_{air} - \rho_{He}} = \frac{180 \text{ kg}}{1.29 \frac{kg}{m^3} - 0.179 \frac{kg}{m^3}}$$

$$V \cong 162 \text{ m}^3$$

Ignore the buoyant force acting on the box (very small because the volume of the box is small and ρ_{air} is small.

Partial Submersion



(b)

Volume of object = V

volume of object under the surface of the fluid is V_s. NOTE $V_s < V$ — submerged volume of log of wood.

Submerged volume ↓ $V_s < V$ ↑ volume of piece of wood.

Object is floating in equilibrium ⇒

$$F_B = m_o g$$

$$\rho_f V_s g = \rho_o V g$$

$$\therefore \quad \frac{\rho_o}{\rho_f} = \frac{V_s}{V} < 1$$

For partial submersion.

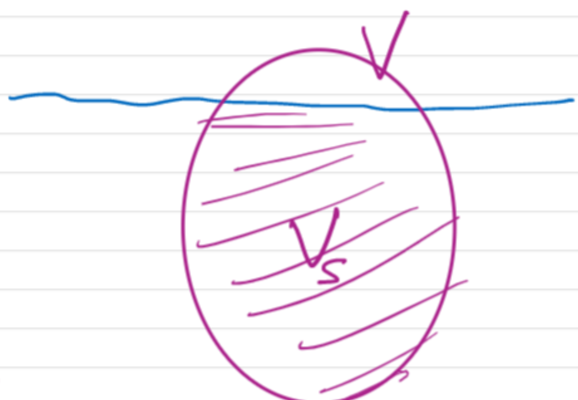
For partial submersion:

$$\frac{\rho_o}{\rho_f} = \frac{V_s}{V}$$

V_s = volume of object under the surface of the fluid = volume of displaced fluid.

V = volume of object.

$$\frac{\rho_{ice}}{\rho_w} = \frac{V_s}{V} \sim 0.9$$



$$V_s = 0.9 V$$

Submerged volume of ice = 90% of the volume of the ice