Chapter 10:Fluids

Lecture 4
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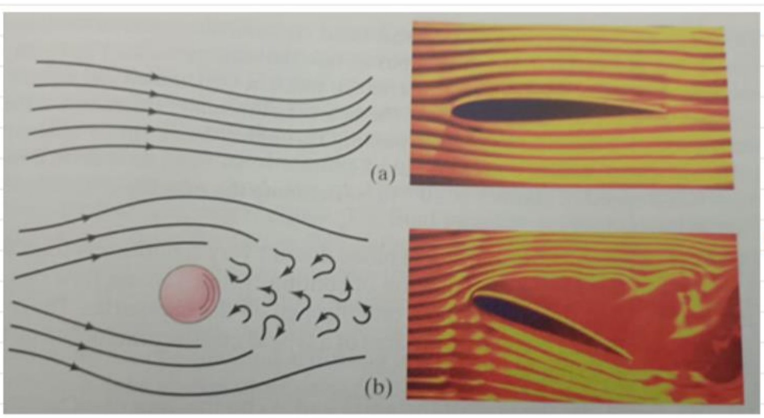
10-8] Fluids in Motion (Dynamics)
Flow rate and equation of continuity

Fluids in motion → dynamics
If fluid is water → hydrodynamics.

Two main types of Fluid flow :



① Laminar (streamline) flow:
Each particle follows a smooth path called streamline.
The paths don't cross one another

② Turbulent flow:
Above certain speed flow becomes turbulent.
Small whirlpool-like circles called eddy currents.
Eddies absorb a great deal of energy.

We shall consider laminar flow ONLY.

Viscosity: internal friction between the layers of moving liquids.
It acts similar to friction between two rough surfaces in contact.

Equation of Continuity

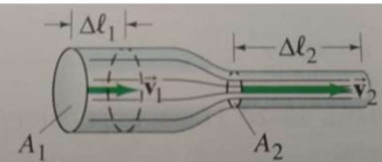


FIGURE 10–19 Fluid flow through a pipe of varying diameter.

\# Assume laminar (streamline) flow.

\# Assume incompressible fluid.

∴ volume of fluid that enters area A1 per unit time MUST equal the volume of fluid that exists area A2 per unit time.

∴ $\dfrac{\Delta V_1}{\Delta t} = \dfrac{\Delta V_2}{\Delta t}$, $\dfrac{\Delta V}{\Delta t}$: volume flow rate, which is the volume of fluid passing a point per unit time. Unit: m^3/s.

But $\Delta V_1 = A_1 \Delta \ell_1$

$\Delta V_2 = A_2 \Delta \ell_2$

Define mass flow rate $\dfrac{\Delta m}{\Delta t} = \rho \dfrac{\overset{\Delta m}{\Delta V}}{\Delta t}$ which is mass of fluid that passes a point per unit time

Unit: kg/s.

$$\rho_1 \frac{\Delta V_1}{\Delta t} = \rho_2 \frac{\Delta V_2}{\Delta t}$$

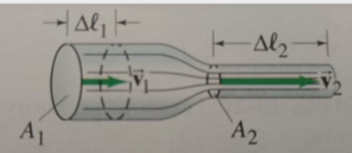


FIGURE 10–19 Fluid flow through a pipe of varying diameter.

but $\Delta V_1 = A_1 \Delta \ell_1$

$\Delta V_2 = A_2 \Delta \ell_2$

$$\Rightarrow \rho_1 A_1 \frac{\Delta \ell_1}{\Delta t} = \rho_2 A_2 \frac{\Delta \ell_2}{\Delta t}$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$ continuity equation.

If fluid is INCOMPRESSIBLE (does NOT change volume under pressure) $\Rightarrow$ ρ is constant i.e $\rho_1 = \rho_2 = \rho$

$$\Rightarrow A_1 v_1 = A_2 v_2$$ $\boxed{\rho \text{ is constant}}$

Small $A \rightarrow$ large v

large $A \rightarrow$ small v.

Partially blocking the hose opening $\Rightarrow$ reduce $A \rightarrow$ larger v and water travels further distances.

As water falls down it speed increases $\Rightarrow$ A decreases.

$\therefore$ Water column narrows as water falls.

A_1, v_1

A_2, v_2

Example 10-12.

blood flow

heart $\rightarrow$ aorta $\rightarrow$ major arteries $\rightarrow$ small arteries (arteriols) $\rightarrow$ capillaries $\rightarrow$ veins

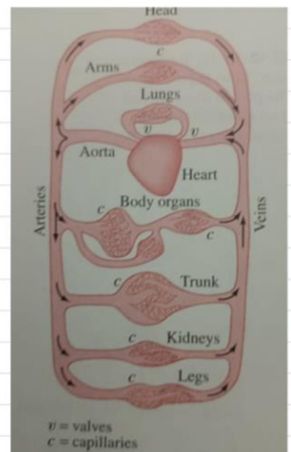


FIGURE 10-20 Human circulatory system.

speed of blood in the aorta

radius of aorta $r_a = 1.2$ cm , $v_a = 40$ cm/s

A typical capillary has radius

$r_c = 4 \times 10^{-4}$ cm

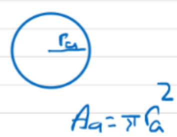
$v_c = 5 \times 10^{-4}$ m/s (speed of blood in capillary)

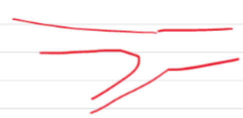
Estimate the number of capillaries in the human body.

$$\underbrace{A_1 v_1}_{\text{for capillaries}} = \underbrace{A_2 v_2}_{\text{for aorta}}$$

$A_1 = N A_c$

N: number of capillaries





$$N A_c v_c = A_a v_a$$

$$\therefore \quad N = \frac{\cancel{\pi} r_a^2 v_a}{\cancel{\pi} r_c^2 v_c} = \left(\frac{r_a}{r_c}\right)^2 \frac{v_a}{v_c}$$

$$N = \left(\frac{1.2\times10^{-2}}{4\times10^{-4}}\right)^2 \left(\frac{0.4}{5\times10^{-4}}\right)$$

note changed units to m and m/s.

$$\therefore \quad N \simeq 7\times10^{9}.$$

10-9] Bernoulli's Equation

Assume:

- steady laminar flow.
- Ignore small viscosity
- Incompressible fluid.

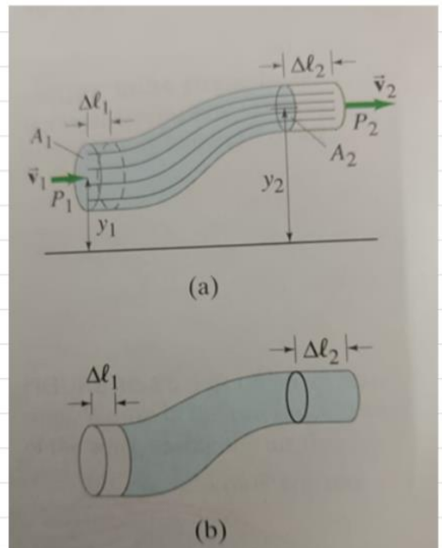


FIGURE 10–22 Fluid flow: for derivation of Bernoulli's equation.

Consider the fluid in blue color. What work is required to move it from position in figure (a) to the new position in figure (b)?

Fluid entering area A_1 moves a distance $\Delta \ell_1$ in time Δt. It pushes the fluid at A_2 and causes it to move a distance $\Delta \ell_2$

in the same time Δt.

Fluid to the left of A_1 exerts pressure P_1 and moves the fluid to its right $\Rightarrow$ It does work W_1:

$$W_1 = F_1 \Delta l_1 = P_1 A_1 \Delta l_1$$

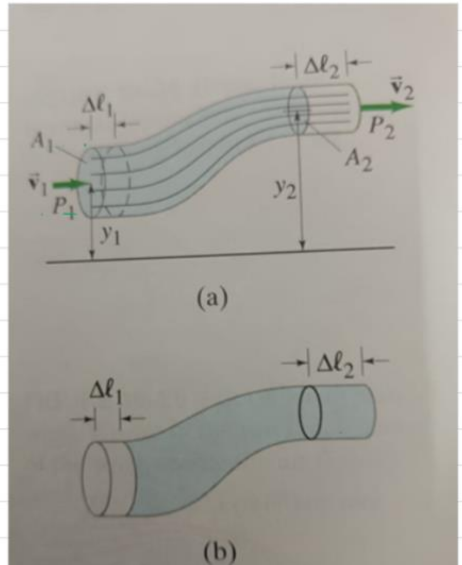


FIGURE 10–22 Fluid flow: for derivation of Bernoulli's equation.

Fluid on the right of A_2 (in fig(a)) opposes the motion of the fluid to its left and exerts a force $F_2 = P_2 A_2$ to the left $\Rightarrow$ it does work W_2

$$W_2 = -F_2 \Delta l_2 = -P_2 A_2 \Delta l_2$$

W_1 and W_2 are works done by the fluid.

Does gravity do any work? YES.

The net process is motion of mass of volume $A_1 \Delta l_1 = A_2 \Delta l_2$ (remember fluid is incompressible) from height y_1 to heigh y_2

$\therefore$ Work done by gravity is

$$W_g = -\Delta U = -mg(y_2 - y_1)$$

mass of moved fluid $m = \rho A_1 \Delta l_1 = \rho A_2 \Delta l_2$

Note: y_1 and y_2 are measured from some level to the center of mass of the tube at points 1 and 2.

$\therefore$ Total work done on the fluid is

$$W_{total} = \Delta K$$

$$W_1 + W_2 + W_g = \Delta K$$

$$P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mg(y_2 - y_1) = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2$$

but $A_1 \Delta l_1 = A_2 \Delta l_2 = V \leftarrow$ volume of fluid moved from point 1 $\rightarrow$ point 2.

$\frac{PV}{\cancel{V}}$ $\quad \frac{J}{m^3}$

$$\therefore\ P_1 V - P_2 V - mgy_2 + mgy_1 = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2$$

$$\therefore\ \underbrace{P_1 V + mgy_1 + \tfrac{1}{2} m v_1^2}_{\text{related to point 1 ONLY}} = \underbrace{P_2 V + mgy_2 + \tfrac{1}{2} m v_2^2}_{\text{related to point 2 ONLY.}}$$

$\frac{N}{m^2}\ m^3 = Nm$

$\therefore$ Each side is A CONSTANT.

For example: If P_1 changes $\Rightarrow y_1$ or v_1 or BOTH MUST change so that Left hand side remains unchanged.

$\frac{N}{m^2} \times m^5 = N.m$

$\vec{F}.\vec{d}$

Divide both sides by $V \Rightarrow$

$\rho g y_1$: potential energy per unit volume

$\frac{1}{2}\rho v_1^2$: KE per unit volume

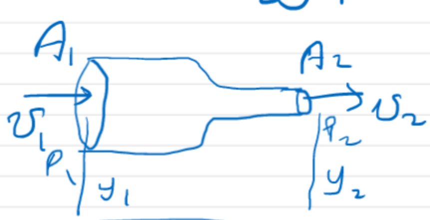
$$P_1 + \overset{m/V}{\rho} g y_1 + \tfrac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2} \rho v_2^2$$

This is Bernoulli's equation.

$\rho = m/V$ density

$\rho g y$: potential energy per unit volume

$\frac{1}{2}\rho v^2$: kinetic energy per unit volume.



$$P_1 + \cancel{\rho g y_1} + \tfrac{1}{2}\rho v_1^2 = P_2 + \cancel{\rho g y_2} + \tfrac{1}{2}\rho v_2^2$$

$y_1 = y_2$

$$v_1 < v_2$$

$$\Rightarrow P_1 > P_2$$

$A_1 v_1 = A_2 v_2$, $v_2 > v_1$