Chapter 23
Light: Geometric Optics

Lecture 3

The University of Jordan/Physics Department
Prof. Mahmoud Jaghoub
أ.د. محمود الجاغوب

Thin Lens Equation

(1) Converging Lens

A : center of the lens.

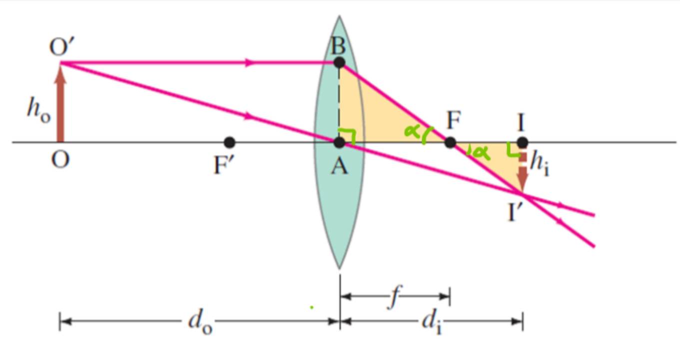
d_o : distance of the object from the center of the lens.

d_i : distance of the image from the center of the lens.

f : focal length.

h_o : height of object

h_i : height of image



Triangles AFB and II′F are similar because the have two equal angles one is 90°, the other is α.

Using the two similar triangles we can write

$$\frac{h_i}{h_o} = \frac{d_i - f}{f} \quad — ①$$

Alo triangles OAO' and IAI' are also similar ⇒

$$\frac{h_i}{h_o} = \frac{d_i}{d_o} \quad — ②$$

⇒ right hand sides of equations ① and ② are equal

$$\therefore \frac{d_i}{d_o} = \frac{d_i - f}{f} = \frac{d_i}{f} - 1$$

$$\div d_i \Rightarrow \frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} \Rightarrow$$

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$$

This is the thin lens equation

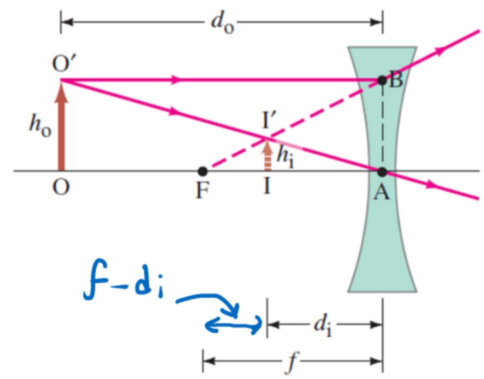
If the object is so far away from the lens ⇒

$d_o \to \infty$ and $\frac{1}{d_o} \to 0$ ⇒

$$\frac{1}{d_i} = \frac{1}{f} \Rightarrow d_i = f$$

This means that the image forms at the position of the focus point.

(2) Diverging Lens



ⓐ IAI' and OAO' are similar triangles

$$\Rightarrow \frac{h_i}{h_o} = \frac{d_i}{d_o} \quad - \text{③}$$

ⓑ IFI' and AFB are similar triangl

$$\Rightarrow \frac{h_i}{h_o} = \frac{f-d_i}{f} \quad \text{④}$$

$$\Rightarrow \frac{d_i}{d_o} = \frac{f-d_i}{f} = 1 - \frac{d_i}{f}$$

$$/d_i \Rightarrow \frac{1}{d_o} = \frac{1}{d_i} - \frac{1}{f}$$

$$\therefore \frac{1}{d_o} - \frac{1}{d_i} = -\frac{1}{f}$$

For a diverging lense if $f \rightarrow -f$ and $d_i \rightarrow -d_i \Rightarrow$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$ the same as for a converging lens.

Therefore, we shall adopt the following sign convensions so that the thin lens equation applies to a converging and a diverging lens :

1) The focal length is positive for converging lenses and negative for diverging lenses.

2) d_o: is positive for a real object and negative for a virtual object.

3) d_i is positive for a real image, and negative for a virtual image.

4) h_i is positive if the image is upright, and h_i is negative if the image is inverted relative to the object.

5) h_o is always taken as upright and positive.

Magnification (m)

It is the ratio of the image height (h_i) to the object height h_o :

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

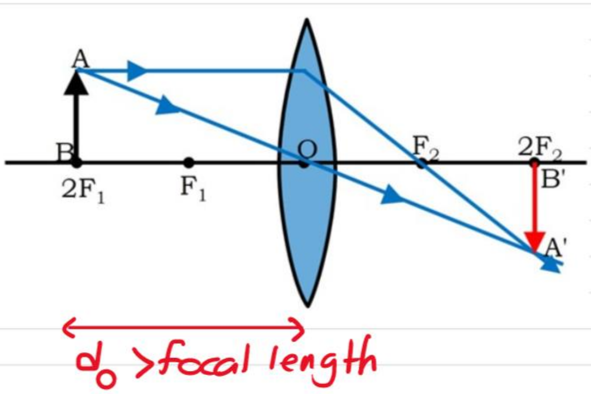
Upright image ⇒ m is positive

Inverted image ⇒ m is negative

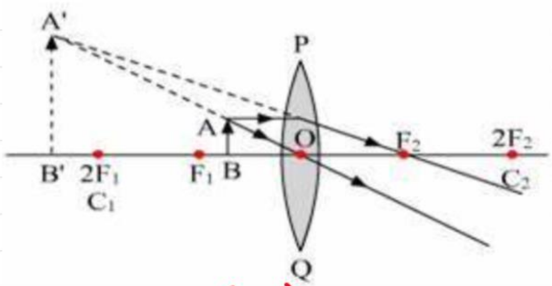
Note:

A diverging lens always produces an upright virtual image for any real object regardless of the value of d_o.

A converging lens produces real inverted images OR virtual upright images depending on the value do.



d_o > focal length ⇒ real inverted image.



d_o < focal length

d_o < focal length

⇒ virtual upright image

EXAMPLE 23–12 **Image formed by converging lens.** What is (*a*) the position, and (*b*) the size, of the image of a 7.6-cm-high leaf placed 1.00 m from a +50.0-mm-focal-length camera lens?

a) real object ⇒ d_o is positive, $d_o = 100$ cm

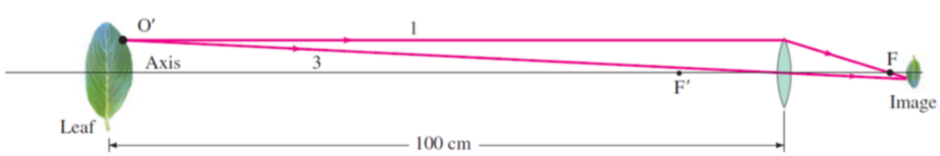
$h_o = 7.6$ cm

Convex camera lens ⇒ $f = 5$ cm

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{5} - \frac{1}{100}$$

$$\frac{1}{d_i} = \frac{20-1}{100} = \frac{19}{100} \Rightarrow d_i = \frac{100}{19} = 5.26 \text{ cm}$$

$d_i > 0 \Rightarrow$ (real image)



b) magnification $m = -\frac{d_i}{d_o} = -\frac{5.26}{100}$

$\therefore m = -0.0526$

↑ - sign means that image is inverted

$|m| < 1 \Rightarrow$ image is reduced.

$m = \frac{h_i}{h_o} \Rightarrow h_i = m h_o = (-0.0526)(7.6)$

$\Rightarrow h_i = -4$ cm

↑ image is inverted.

Note $|h_i| < h_o \Rightarrow$ image is reduced.

EXAMPLE 23–13 **Object close to converging lens.** An object is placed 10 cm from a 15-cm-focal-length converging lens. Determine the image position and size (*a*) analytically, and (*b*) using a ray diagram.

Note: $d_o < f$

a) $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15} - \frac{1}{10}$

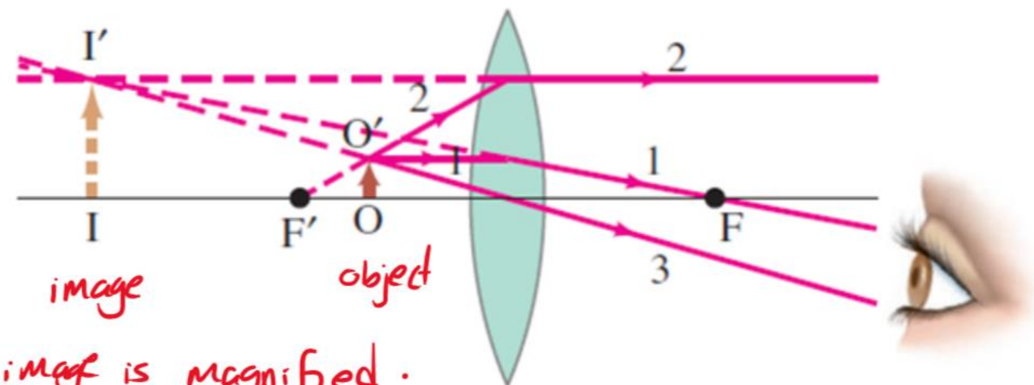
$\therefore \frac{1}{d_i} = \frac{10-15}{150} = \frac{-5}{150} \Rightarrow d_i = -30$ cm.

d_o is negative i.e the image is virtual

(image is on the same side as the object)

b) $m = -\frac{d_i}{d_o} = -\frac{(-30)}{10} = 3$

m is positive ⇒ image is upright and the image is magnified. The lens in this example is used as a magnifying lens.



image

object

image is magnified.

and virtual

EXAMPLE 23–14 **Diverging lens.** Where must a small insect be placed if a 25-cm-focal-length diverging lens is to form a virtual image 20 cm from the lens, on the same side as the object?

diverging lens ⇒ $f = -25$ cm

virtual image ⇒ $d_i = -20$ cm

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{-25} - \frac{1}{-20}$$

$$\frac{1}{d_o} = \frac{1}{-25} + \frac{1}{20} = \frac{-20+25}{25\times 20} = \frac{5}{25\times 20}$$

$$\therefore \quad d_o = \frac{25\times 20}{5} = 100 \text{ cm}$$

∴ Object is 100 cm in front of the lens.

Extra: $m = -\frac{d_i}{d_o} = -\frac{(-20)}{100} = \frac{1}{5}$

⇒ image is upright (since m is positive) and reduce since $m < 1$.

image is virtual and reduced.