

predict  $\sin \theta_2 > 1$ . This occurs if  $\theta_1$  exceeds the critical angle  $\theta_C$  given by

$$\sin \theta_C = \frac{n_2}{n_1}. \quad (23-6)$$

A lens uses refraction to produce a real or virtual image. Parallel rays of light are focused to a point, the **focal point**, by a **converging** lens. The distance of the focal point from the lens is the **focal length**  $f$  of the lens. It is the same on both sides of the lens.

After parallel rays pass through a **diverging** lens, they appear to diverge from a point in front of the lens, which is its focal point; and the corresponding focal length is considered negative.

The **power**  $P$  of a lens, which is  $P = 1/f$  (Eq. 23-7), is given in diopters, which are units of inverse meters ( $\text{m}^{-1}$ ).

For a given object, the position and size of the image formed by a lens can be found approximately by ray tracing. Algebraically, the relation between image and object distances,  $d_i$  and  $d_o$ ,

and the focal length  $f$ , is given by the **thin lens equation**:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \quad (23-8)$$

The ratio of image height to object height, which equals the **magnification**  $m$  for a lens, is

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \quad (23-9)$$

When using the various equations of geometric optics, you must remember the **sign conventions** for all quantities involved: carefully review them (pages 655 and 665) when doing Problems.

[\*When two (or more) thin lenses are used in combination to produce an image, the thin lens equation can be used for each lens in sequence. The image produced by the first lens acts as the object for the second lens.]

[\*The **lensmaker's equation** relates the radii of curvature of the lens surfaces and the lens' index of refraction to the focal length of the lens.]

## Questions

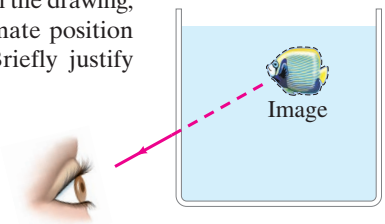
1. Archimedes is said to have burned the whole Roman fleet in the harbor of Syracuse, Italy, by focusing the rays of the Sun with a huge spherical mirror. Is this<sup>†</sup> reasonable?
2. What is the focal length of a plane mirror? What is the magnification of a plane mirror?
3. Although a plane mirror appears to reverse left and right, it doesn't reverse up and down. Discuss why this happens, noting that front to back is also reversed. Also discuss what happens if, while standing, you look up vertically at a horizontal mirror on the ceiling.
4. An object is placed along the principal axis of a spherical mirror. The magnification of the object is  $-2.0$ . Is the image real or virtual, inverted or upright? Is the mirror concave or convex? On which side of the mirror is the image located?
5. If a concave mirror produces a real image, is the image necessarily inverted? Explain.
6. How might you determine the speed of light in a solid, rectangular, transparent object?
7. When you look at the Moon's reflection from a ripply sea, it appears elongated (Fig. 23-47). Explain.



**FIGURE 23-47**  
Question 7.

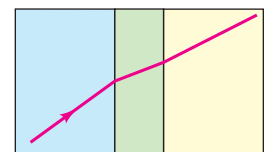
8. What is the angle of refraction when a light ray is incident perpendicular to the boundary between two transparent materials?

9. When you look down into a swimming pool or a lake, are you likely to overestimate or underestimate its depth? Explain. How does the apparent depth vary with the viewing angle? (Use ray diagrams.)
10. Draw a ray diagram to show why a stick or straw looks bent when part of it is under water (Fig. 23-23).
11. When a wide beam of parallel light enters water at an angle, the beam broadens. Explain.
12. You look into an aquarium and view a fish inside. One ray of light from the fish is shown emerging from the tank in Fig. 23-48. The apparent position of the fish is also shown (dashed ray). In the drawing, indicate the approximate position of the actual fish. Briefly justify your answer.



**FIGURE 23-48**  
Question 12.

13. How can you "see" a round drop of water on a table even though the water is transparent and colorless?
14. A ray of light is refracted through three different materials (Fig. 23-49). Which material has (a) the largest index of refraction, (b) the smallest?



**FIGURE 23-49**  
Question 14.

15. A child looks into a pool to see how deep it is. She then drops a small toy into the pool to help decide how deep the pool is. After this careful investigation, she decides it is safe to jump in—only to discover the water is over her head. What went wrong with her interpretation of her experiment?
16. Can a light ray traveling in air be totally reflected when it strikes a smooth water surface if the incident angle is chosen correctly? Explain.

<sup>†</sup>Students at MIT did a feasibility study. See [www.mit.edu/2.009/www/experiments/deathray/10\\_ArchimedesResult.html](http://www.mit.edu/2.009/www/experiments/deathray/10_ArchimedesResult.html).

17. What type of mirror is shown in Fig. 23–50? Explain.



**FIGURE 23–50**  
Question 17 and  
Problem 15.

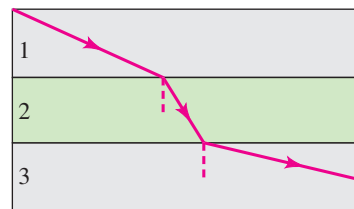
18. Light rays from stars (including our Sun) always bend toward the vertical direction as they pass through the Earth's atmosphere. (a) Why does this make sense? (b) What can you conclude about the apparent positions of stars as viewed from Earth? Draw a circle for Earth, a dot for you, and 3 or 4 stars at different angles.
19. Where must the film be placed if a camera lens is to make a sharp image of an object far away? Explain.
20. A photographer moves closer to his subject and then refocuses. Does the camera lens move farther away from or closer to the camera film or sensor? Explain.
21. Can a diverging lens form a real image under any circumstances? Explain.
22. Light rays are said to be “reversible.” Is this consistent with the thin lens equation? Explain.
23. Can real images be projected on a screen? Can virtual images? Can either be photographed? Discuss carefully.

24. A thin converging lens is moved closer to a nearby object. Does the real image formed change (a) in position, (b) in size? If yes, describe how.
25. If a glass converging lens is placed in water, its focal length in water will be (a) longer, (b) shorter, or (c) the same as in air. Explain.
26. Compare the mirror equation with the thin lens equation. Discuss similarities and differences, especially the sign conventions for the quantities involved.
27. A lens is made of a material with an index of refraction  $n = 1.25$ . In air, it is a converging lens. Will it still be a converging lens if placed in water? Explain, using a ray diagram.
28. (a) Does the focal length of a lens depend on the fluid in which it is immersed? (b) What about the focal length of a spherical mirror? Explain.
29. An underwater lens consists of a carefully shaped thin-walled plastic container filled with air. What shape should it have in order to be (a) converging, (b) diverging? Use ray diagrams to support your answer.
30. The thicker a double convex lens is in the center as compared to its edges, the shorter its focal length for a given lens diameter. Explain.
- \*31. A non-symmetrical lens (say, planoconvex) forms an image of a nearby object. Use the lensmaker's equation to explain if the image point changes when the lens is turned around.
- \*32. Example 23–16 shows how to use a converging lens to measure the focal length of a diverging lens. (a) Why can't you measure the focal length of a diverging lens directly? (b) It is said that for this to work, the converging lens must be stronger than the diverging lens. What is meant by “stronger,” and why is this statement true?

## MisConceptual Questions

1. Suppose you are standing about 3 m in front of a mirror. You can see yourself just from the top of your head to your waist, where the bottom of the mirror cuts off the rest of your image. If you walk one step closer to the mirror (a) you will not be able to see any more of your image. (b) you will be able to see more of your image, below your waist. (c) you will see less of your image, with the cutoff rising to be above your waist.
2. When the reflection of an object is seen in a flat mirror, the image is (a) real and upright. (b) real and inverted. (c) virtual and upright. (d) virtual and inverted.
3. You want to create a spotlight that will shine a bright beam of light with all of the light rays parallel to each other. You have a large concave spherical mirror and a small lightbulb. Where should you place the lightbulb? (a) At the focal point of the mirror. (b) At the radius of curvature of the mirror. (c) At any point, because all rays bouncing off the mirror will be parallel. (d) None of the above; you can't make parallel rays with a concave mirror.

4. When you look at a fish in a still stream from the bank, the fish appears shallower than it really is due to refraction. From directly above, it appears (a) deeper than it really is. (b) at its actual depth. (c) shallower than its real depth. (d) It depends on your height above the water.
5. Parallel light rays cross interfaces from medium 1 into medium 2 and then into medium 3 as shown in Fig. 23–51. What can we say about the relative sizes of the indices of refraction of these media? (a)  $n_1 > n_2 > n_3$ . (b)  $n_3 > n_2 > n_1$ . (c)  $n_2 > n_3 > n_1$ . (d)  $n_1 > n_3 > n_2$ . (e)  $n_2 > n_1 > n_3$ . (f) None of the above.



**FIGURE 23–51**  
MisConceptual  
Question 5.

6. To shoot a swimming fish with an intense light beam from a laser gun, you should aim (a) directly at the image. (b) slightly above the image. (c) slightly below the image.

7. When moonlight strikes the surface of a calm lake, what happens to this light?  
 (a) All of it reflects from the water surface back to the air.  
 (b) Some of it reflects back to the air; some enters the water.  
 (c) All of it enters the water.  
 (d) All of it disappears via absorption by water molecules.
8. If you shine a light through an optical fiber, why does it come out the end but not out the sides?  
 (a) It does come out the sides, but this effect is not obvious because the sides are so much longer than the ends.  
 (b) The sides are mirrored, so the light reflects.  
 (c) Total internal reflection makes the light reflect from the sides.  
 (d) The light flows along the length of the fiber, never touching the sides.
9. A converging lens, such as a typical magnifying glass,  
 (a) always produces a magnified image (taller than object).  
 (b) always produces an image smaller than the object.  
 (c) always produces an upright image.  
 (d) always produces an inverted image (upside down).  
 (e) None of these statements are true.
10. Virtual images can be formed by  
 (a) only mirrors.  
 (b) only lenses.  
 (c) only plane mirrors.  
 (d) only curved mirrors or lenses.  
 (e) plane and curved mirrors, and lenses.
11. A lens can be characterized by its *power*, which  
 (a) is the same as the magnification.  
 (b) tells how much light the lens can focus.  
 (c) depends on where the object is located.  
 (d) is the reciprocal of the focal length.
12. You cover half of a lens that is forming an image on a screen. Compare what happens when you cover the top half of the lens versus the bottom half.  
 (a) When you cover the top half of the lens, the top half of the image disappears; when you cover the bottom half of the lens, the bottom half of the image disappears.  
 (b) When you cover the top half of the lens, the bottom half of the image disappears; when you cover the bottom half of the lens, the top half of the image disappears.  
 (c) The image becomes half as bright in both cases.  
 (d) Nothing happens in either case.  
 (e) The image disappears in both cases.
13. Which of the following can form an image?  
 (a) A plane mirror.  
 (b) A curved mirror.  
 (c) A lens curved on both sides.  
 (d) A lens curved on only one side.  
 (e) All of the above.
14. As an object moves from just outside the focal point of a converging lens to just inside it, the image goes from \_\_\_\_\_ and \_\_\_\_\_ to \_\_\_\_\_ and \_\_\_\_\_.  
 (a) large; inverted; large; upright.  
 (b) large; upright; large; inverted.  
 (c) small; inverted; small; upright.  
 (d) small; upright; small; inverted.

For assigned homework and other learning materials, go to the MasteringPhysics website.

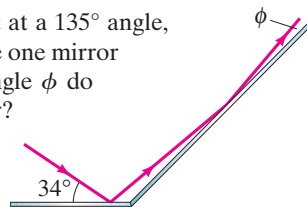


## Problems

### 23–2 Reflection; Plane Mirrors

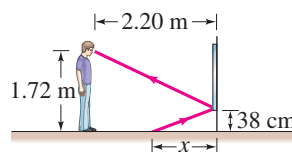
1. (I) When you look at yourself in a 60-cm-tall plane mirror, you see the same amount of your body whether you are close to the mirror or far away. (Try it and see.) Use ray diagrams to show why this should be true.
2. (I) Suppose that you want to take a photograph of yourself as you look at your image in a mirror 3.1 m away. For what distance should the camera lens be focused?
3. (II) Two plane mirrors meet at a  $135^\circ$  angle, Fig. 23–52. If light rays strike one mirror at  $34^\circ$  as shown, at what angle  $\phi$  do they leave the second mirror?

FIGURE 23–52  
Problem 3.



4. (II) A person whose eyes are 1.72 m above the floor stands 2.20 m in front of a vertical plane mirror whose bottom edge is 38 cm above the floor, Fig. 23–53. What is the horizontal distance  $x$  to the base of the wall supporting the mirror of the nearest point on the floor that can be seen reflected in the mirror?

FIGURE 23–53  
Problem 4.



5. (II) Stand up two plane mirrors so they form a  $90.0^\circ$  angle as in Fig. 23–54. When you look into this double mirror, you see yourself as others see you, instead of reversed as in a single mirror. Make a ray diagram to show how this occurs.

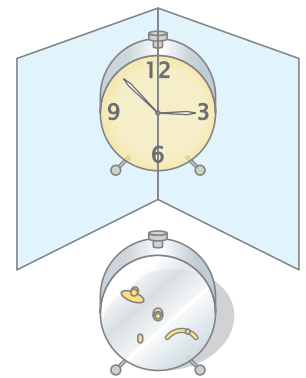


FIGURE 23–54  
Problem 5.

6. (II) Two plane mirrors, nearly parallel, are facing each other 2.3 m apart as in Fig. 23–55. You stand 1.6 m away from one of these mirrors and look into it. You will see multiple images of yourself. (a) How far away from you are the first three images of yourself in the mirror in front of you? (b) Are these first three images facing you or away from you?

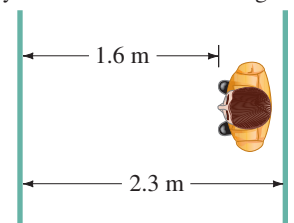


FIGURE 23–55  
Problem 6.

### 23–3 Spherical Mirrors

7. (I) A solar cooker, really a concave mirror pointed at the Sun, focuses the Sun's rays 18.8 cm in front of the mirror. What is the radius of the spherical surface from which the mirror was made?
8. (I) How far from a concave mirror (radius 21.0 cm) must an object be placed if its image is to be at infinity?
9. (II) A small candle is 38 cm from a concave mirror having a radius of curvature of 24 cm. (a) What is the focal length of the mirror? (b) Where will the image of the candle be located? (c) Will the image be upright or inverted?
10. (II) An object 3.0 mm high is placed 16 cm from a convex mirror of radius of curvature 16 cm. (a) Show by ray tracing that the image is virtual, and estimate the image distance. (b) Show that the (negative) image distance can be computed from Eq. 23–2 using a focal length of  $-8.0$  cm. (c) Compute the image size, using Eq. 23–3.
11. (II) A dentist wants a small mirror that, when 2.00 cm from a tooth, will produce a  $4.0\times$  upright image. What kind of mirror must be used and what must its radius of curvature be?
12. (II) You are standing 3.4 m from a convex security mirror in a store. You estimate the height of your image to be half of your actual height. Estimate the radius of curvature of the mirror.
13. (II) The image of a distant tree is virtual and very small when viewed in a curved mirror. The image appears to be 19.0 cm behind the mirror. What kind of mirror is it, and what is its radius of curvature?
14. (II) A mirror at an amusement park shows an upright image of any person who stands 1.9 m in front of it. If the image is three times the person's height, what is the radius of curvature of the mirror? (See Fig. 23–50.)
15. (II) In Example 23–4, show that if the object is moved 10.0 cm farther from the concave mirror, the object's image size will equal the object's actual size. Stated as a multiple of the focal length, what is the object distance for this "actual-sized image" situation?
16. (II) You look at yourself in a shiny 8.8-cm-diameter Christmas tree ball. If your face is 25.0 cm away from the ball's front surface, where is your image? Is it real or virtual? Is it upright or inverted?
17. (II) Some rearview mirrors produce images of cars to your rear that are smaller than they would be if the mirror were flat. Are the mirrors concave or convex? What is a mirror's radius of curvature if cars 16.0 m away appear 0.33 their normal size?
18. (II) When walking toward a concave mirror you notice that the image flips at a distance of 0.50 m. What is the radius of curvature of the mirror?
19. (II) (a) Where should an object be placed in front of a concave mirror so that it produces an image at the same location as the object? (b) Is the image real or virtual? (c) Is the image inverted or upright? (d) What is the magnification of the image?

20. (II) A shaving or makeup mirror is designed to magnify your face by a factor of 1.40 when your face is placed 20.0 cm in front of it. (a) What type of mirror is it? (b) Describe the type of image that it makes of your face. (c) Calculate the required radius of curvature for the mirror.
21. (II) Use two techniques, (a) a ray diagram, and (b) the mirror equation, to show that the magnitude of the magnification of a concave mirror is less than 1 if the object is beyond the center of curvature  $C$  ( $d_o > r$ ), and is greater than 1 if the object is within  $C$  ( $d_o < r$ ).
22. (III) Show, using a ray diagram, that the magnification  $m$  of a convex mirror is  $m = -d_i/d_o$ , just as for a concave mirror. [Hint: Consider a ray from the top of the object that reflects at the center of the mirror.]
23. (III) An object is placed a distance  $r$  in front of a wall, where  $r$  exactly equals the radius of curvature of a certain concave mirror. At what distance from the wall should this mirror be placed so that a real image of the object is formed on the wall? What is the magnification of the image?

### 23–4 Index of Refraction

24. (I) The speed of light in ice is  $2.29 \times 10^8$  m/s. What is the index of refraction of ice?
- ✓ 25. (I) What is the speed of light in (a) ethyl alcohol, (b) lucite, (c) crown glass?
- ✓ 26. (II) The speed of light in a certain substance is 82% of its value in water. What is the index of refraction of that substance?

### 23–5 Refraction; Snell's Law

27. (I) A flashlight beam strikes the surface of a pane of glass ( $n = 1.56$ ) at a  $67^\circ$  angle to the normal. What is the angle of refraction?
- ✓ 28. (I) A diver shines a flashlight upward from beneath the water at a  $35.2^\circ$  angle to the vertical. At what angle does the light leave the water?
29. (I) A light beam coming from an underwater spotlight exits the water at an angle of  $56.0^\circ$ . At what angle of incidence did it hit the air–water interface from below the surface?
30. (I) Rays of the Sun are seen to make a  $36.0^\circ$  angle to the vertical beneath the water. At what angle above the horizon is the Sun?
- ✓ 31. (II) An aquarium filled with water has flat glass sides whose index of refraction is 1.54. A beam of light from outside the aquarium strikes the glass at a  $43.5^\circ$  angle to the perpendicular (Fig. 23–56). What is the angle of this light ray when it enters (a) the glass, and then (b) the water? (c) What would be the refracted angle if the ray entered the water directly?

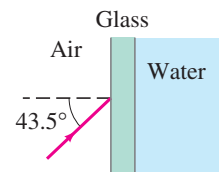


FIGURE 23–56  
Problem 31.

32. (II) In searching the bottom of a pool at night, a watchman shines a narrow beam of light from his flashlight, 1.3 m above the water level, onto the surface of the water at a point 2.5 m from his foot at the edge of the pool (Fig. 23–57). Where does the spot of light hit the bottom of the 2.1-m-deep pool? Measure from the bottom of the wall beneath his foot.

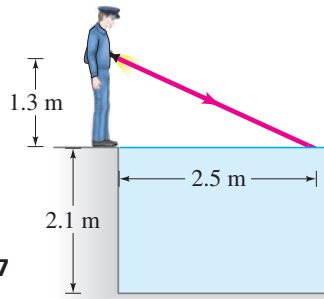


FIGURE 23–57 Problem 32.

### 23–6 Total Internal Reflection

33. (I) What is the critical angle for the interface between water and crown glass? To be internally reflected, the light must start in which material?
34. (I) The critical angle for a certain liquid–air surface is  $47.2^\circ$ . What is the index of refraction of the liquid?
35. (II) A beam of light is emitted in a pool of water from a depth of 82.0 cm. Where must it strike the air–water interface, relative to the spot directly above it, in order that the light does *not* exit the water?
36. (II) A beam of light is emitted 8.0 cm beneath the surface of a liquid and strikes the air surface 7.6 cm from the point directly above the source. If total internal reflection occurs, what can you say about the index of refraction of the liquid?
37. (III) (a) What is the minimum index of refraction for a glass or plastic prism to be used in binoculars (Fig. 23–28) so that total internal reflection occurs at  $45^\circ$ ? (b) Will binoculars work if their prisms (assume  $n = 1.58$ ) are immersed in water? (c) What minimum  $n$  is needed if the prisms are immersed in water?
38. (III) A beam of light enters the end of an optic fiber as shown in Fig. 23–58. (a) Show that we can guarantee total internal reflection at the side surface of the material (at point A), if the index of refraction is greater than about 1.42. In other words, regardless of the angle  $\alpha$ , the light beam reflects back into the material at point A, assuming air outside. (b) What if the fiber were immersed in water?

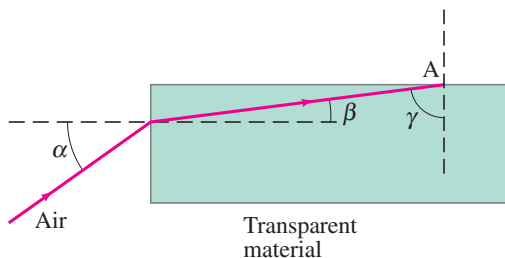


FIGURE 23–58 Problem 38.

### 23–7 and 23–8 Thin Lenses

39. (I) A sharp image is located 391 mm behind a 215-mm-focal-length converging lens. Find the object distance (a) using a ray diagram, (b) by calculation.
40. (I) Sunlight is observed to focus at a point 16.5 cm behind a lens. (a) What kind of lens is it? (b) What is its power in diopters?

41. (I) (a) What is the power of a 32.5-cm-focal-length lens? (b) What is the focal length of a  $-6.75\text{-D}$  lens? Are these lenses converging or diverging?
42. (II) A certain lens focuses light from an object 1.55 m away as an image 48.3 cm on the other side of the lens. What type of lens is it and what is its focal length? Is the image real or virtual?
43. (II) A 105-mm-focal-length lens is used to focus an image on the sensor of a camera. The maximum distance allowed between the lens and the sensor plane is 132 mm. (a) How far in front of the sensor should the lens (assumed thin) be positioned if the object to be photographed is 10.0 m away? (b) 3.0 m away? (c) 1.0 m away? (d) What is the closest object this lens could photograph sharply?
44. (II) Use ray diagrams to show that a real image formed by a thin lens is always inverted, whereas a virtual image is always upright if the object is real.
45. (II) A stamp collector uses a converging lens with focal length 28 cm to view a stamp 16 cm in front of the lens. (a) Where is the image located? (b) What is the magnification?
46. (II) A  $-7.00\text{-D}$  lens is held 12.5 cm from an ant 1.00 mm high. Describe the position, type, and height of the image.
47. (II) An object is located 1.50 m from a 6.5-D lens. By how much does the image move if the object is moved (a) 0.90 m closer to the lens, and (b) 0.90 m farther from the lens?
48. (II) (a) How far from a 50.0-mm-focal-length lens must an object be placed if its image is to be magnified  $2.50\times$  and be real? (b) What if the image is to be virtual and magnified  $2.50\times$ ?
49. (II) Repeat Problem 48 for a  $-50.0\text{-mm-focal-length lens}$ . [Hint: Consider objects real or virtual (formed by some other piece of optics).]
50. (II) How far from a converging lens with a focal length of 32 cm should an object be placed to produce a real image which is the same size as the object?
51. (II) (a) A 2.40-cm-high insect is 1.30 m from a 135-mm-focal-length lens. Where is the image, how high is it, and what type is it? (b) What if  $f = -135\text{ mm}$ ?
52. (III) A bright object and a viewing screen are separated by a distance of 86.0 cm. At what location(s) between the object and the screen should a lens of focal length 16.0 cm be placed in order to produce a sharp image on the screen? [Hint: First draw a diagram.]
53. (III) How far apart are an object and an image formed by an 85-cm-focal-length converging lens if the image is  $3.25\times$  larger than the object and is real?
54. (III) In a film projector, the film acts as the object whose image is projected on a screen (Fig. 23–59). If a 105-mm-focal-length lens is to project an image on a screen 25.5 m away, how far from the lens should the film be? If the film is 24 mm wide, how wide will the picture be on the screen?

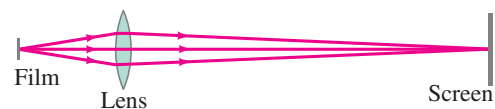


FIGURE 23–59 Film projector, Problem 54.

### \*23–9 Lens Combinations

- \*55. (II) A diverging lens with  $f = -36.5$  cm is placed 14.0 cm behind a converging lens with  $f = 20.0$  cm. Where will an object at infinity be focused?
- \*56. (II) Two 25.0-cm-focal-length converging lenses are placed 16.5 cm apart. An object is placed 35.0 cm in front of one lens. Where will the final image formed by the second lens be located? What is the total magnification?
- \*57. (II) A 38.0-cm-focal-length converging lens is 28.0 cm behind a diverging lens. Parallel light strikes the diverging lens. After passing through the converging lens, the light is again parallel. What is the focal length of the diverging lens? [Hint: First draw a ray diagram.]
- \*58. (II) A lighted candle is placed 36 cm in front of a converging lens of focal length  $f_1 = 13$  cm, which in turn is 56 cm in front of another converging lens of focal length  $f_2 = 16$  cm (see Fig. 23–60). (a) Draw a ray diagram and estimate the location and the relative size of the final image. (b) Calculate the position and relative size of the final image.

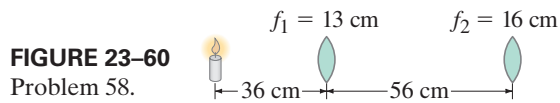


FIGURE 23–60  
Problem 58.

### \*23–10 Lensmaker's Equation

- \*59. (I) A double concave lens has surface radii of 33.4 cm and 28.8 cm. What is the focal length if  $n = 1.52$ ?
- \*60. (I) Both surfaces of a double convex lens have radii of 34.1 cm. If the focal length is 28.9 cm, what is the index of refraction of the lens material?
- \*61. (I) A planoconvex lens (Fig. 23–31a) with  $n = 1.55$  is to have a focal length of 16.3 cm. What is the radius of curvature of the convex surface?
- \*62. (II) A prescription for an eyeglass lens calls for +3.50 diopters. The lensmaker grinds the lens from a “blank” with  $n = 1.56$  and convex front surface of radius of curvature of 30.0 cm. What should be the radius of curvature of the other surface?
- \*63. (III) An object is placed 96.5 cm from a glass lens ( $n = 1.52$ ) with one concave surface of radius 22.0 cm and one convex surface of radius 18.5 cm. Where is the final image? What is the magnification?

## General Problems

64. Sunlight is reflected off the Moon. How long does it take that light to reach us from the Moon?
65. You hold a small flat mirror 0.50 m in front of you and can see your reflection twice in that mirror because there is a full-length mirror 1.0 m behind you (Fig. 23–61). Determine the distance of each image from you.

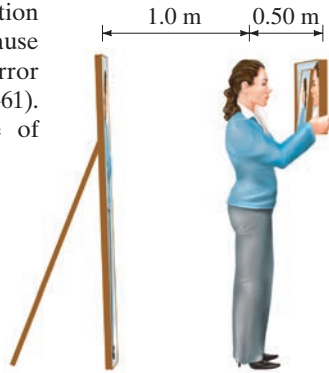


FIGURE 23–61  
Problem 65.

66. We wish to determine the depth of a swimming pool filled with water by measuring the width ( $x = 6.50$  m) and then noting that the far bottom edge of the pool is just visible at an angle of  $13.0^\circ$  above the horizontal as shown in Fig. 23–62. Calculate the depth of the pool.

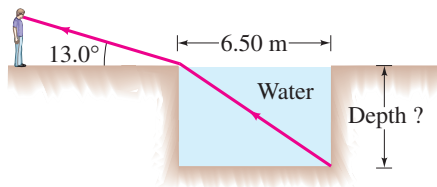


FIGURE 23–62  
Problem 66.

67. The critical angle of a certain piece of plastic in air is  $\theta_C = 37.8^\circ$ . What is the critical angle of the same plastic if it is immersed in water?
68. A pulse of light takes 2.63 ns (see Table 1–4) to travel 0.500 m in a certain material. Determine the material's index of refraction, and identify this material.
69. When an object is placed 60.0 cm from a certain converging lens, it forms a real image. When the object is moved to 40.0 cm from the lens, the image moves 10.0 cm farther from the lens. Find the focal length of this lens.
70. A 4.5-cm-tall object is placed 32 cm in front of a spherical mirror. It is desired to produce a virtual image that is upright and 3.5 cm tall. (a) What type of mirror should be used? (b) Where is the image located? (c) What is the focal length of the mirror? (d) What is the radius of curvature of the mirror?
71. Light is emitted from an ordinary lightbulb filament in wave-train bursts of about  $10^{-8}$  s in duration. What is the length in space of such wave trains?
72. If the apex angle of a prism is  $\phi = 75^\circ$  (see Fig. 23–63), what is the minimum incident angle for a ray if it is to emerge from the opposite side (i.e., not be totally internally reflected), given  $n = 1.58$ ?

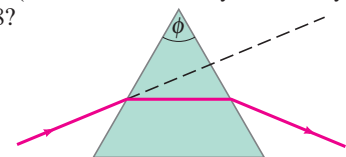
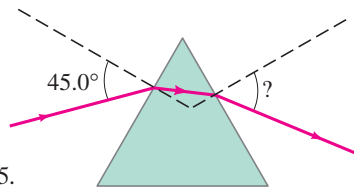


FIGURE 23–63  
Problem 72.

73. An object is placed 18 cm from a certain mirror. The image is half the height of the object, inverted, and real. How far is the image from the mirror, and what is the radius of curvature of the mirror?
74. Light is incident on an equilateral glass prism at a  $45.0^\circ$  angle to one face, Fig. 23–64. Calculate the angle at which light emerges from the opposite face. Assume that  $n = 1.54$ .



**FIGURE 23–64**  
Problems 74 and 75.

75. Suppose a ray strikes the left face of the prism in Fig. 23–64 at  $45.0^\circ$  as shown, but is totally internally reflected at the opposite side. If the apex angle (at the top) is  $\theta = 65.0^\circ$ , what can you say about the index of refraction of the prism?
76. (a) An object 37.5 cm in front of a certain lens is imaged 8.20 cm in front of that lens (on the same side as the object). What type of lens is this, and what is its focal length? Is the image real or virtual? (b) If the image were located, instead, 44.5 cm in front of the lens, what type of lens would it be and what focal length would it have?
77. How large is the image of the Sun on a camera sensor with (a) a 35-mm-focal-length lens, (b) a 50-mm-focal-length lens, and (c) a 105-mm-focal-length lens? The Sun has diameter  $1.4 \times 10^6$  km, and it is  $1.5 \times 10^8$  km away.
78. Figure 23–65 is a photograph of an eyeball with the image of a boy in a doorway. (a) Is the eye here acting as a lens or as a mirror? (b) Is the eye being viewed right side up or is the camera taking this photo upside down? (c) Explain, based on all possible images made by a convex mirror or lens.



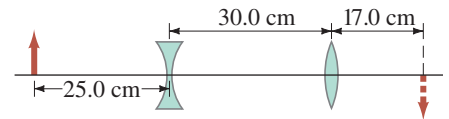
**FIGURE 23–65**  
Problem 78.

79. Which of the two lenses shown in Fig. 23–66 is converging, and which is diverging? Explain using ray diagrams and show how each image is formed.



**FIGURE 23–66** Problem 79.

- \*80. (a) Show that if two thin lenses of focal lengths  $f_1$  and  $f_2$  are placed in contact with each other, the focal length of the combination is given by  $f_T = f_1 f_2 / (f_1 + f_2)$ . (b) Show that the power  $P$  of the combination of two lenses is the sum of their separate powers,  $P = P_1 + P_2$ .
- \*81. Two converging lenses are placed 30.0 cm apart. The focal length of the lens on the right is 20.0 cm, and the focal length of the lens on the left is 15.0 cm. An object is placed to the left of the 15.0-cm-focal-length lens. A final image from both lenses is inverted and located halfway between the two lenses. How far to the left of the 15.0-cm-focal-length lens is the original object?
- \*82. An object is placed 30.0 cm from a +5.0-D lens. A spherical mirror with focal length 25 cm is placed 75 cm behind the lens. Where is the final image? (Note that the mirror reflects light back through the lens.) Be sure to draw a diagram.
- \*83. A small object is 25.0 cm from a diverging lens as shown in Fig. 23–67. A converging lens with a focal length of 12.0 cm is 30.0 cm to the right of the diverging lens. The two-lens system forms a real inverted image 17.0 cm to the right of the converging lens. What is the focal length of the diverging lens?



**FIGURE 23–67**  
Problem 83.

## Search and Learn

- Both a converging lens and a concave mirror can produce virtual images that are larger than the object. Concave mirrors can be used as makeup mirrors, but converging lenses cannot. (a) Draw ray diagrams to explain why not. (b) If a concave mirror has the same focal length as a converging lens, and an object is placed first at a distance of  $\frac{1}{2}f$  from the lens and then at a distance of  $\frac{1}{2}f$  from the mirror, how will the magnification of the object compare in the two cases?
- (a) Did the person we see in Fig. 23–68 shoot the picture we are looking at? We see her in three different mirrors. Describe (b) what type of mirror each is, and (c) her position relative to the focal point and center of curvature.



FIGURE 23–68 Search and Learn 2.

- Justify the second part of sign convention 3, page 665, starting “Equivalently.” Use ray diagrams for all possible situations. Cite Figures already in the text and draw any others needed.
- Make a table showing the sign conventions for mirrors and lenses themselves and for the image and object heights and distances for each.
- Figure 23–69 shows a converging lens held above three equal-sized letters A. In (a) the lens is 5 cm from the paper, and in (b) the lens is 15 cm from the paper. Estimate the focal length of the lens. What is the image position for each case?

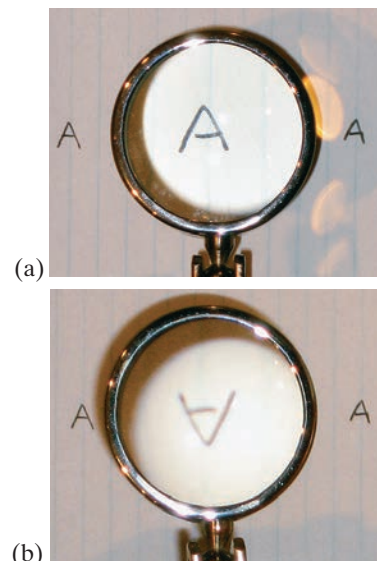


FIGURE 23–69 Search and Learn 5. (b)

## ANSWERS TO EXERCISES

- A:** No.  
**B:** (b).  
**C:** Toward.  
**D:** None.

- E:** No total internal reflection,  $\theta_C > 45^\circ$ .  
**F:** (c).  
**G:** Closer to it.