Chapter 3:Kinematics in Two Dimensions: Vectors

Lecture 2
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3-3] Subtraction of vectors.

First need to introduce two important points:

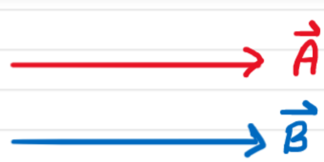
1) Equality of two vectors.

If $\vec{A} = \vec{B} \Rightarrow$

(i) $|\vec{A}| = |\vec{B}|$ i.e the have equal magnitudes

(ii) $\vec{A}$ is parallel to $\vec{B}$ i.e they are in the same direction.

NOTE that both conditions must be satisfied.



Since vectors $\vec{A}$ and $\vec{B}$ have the same length $\Rightarrow |\vec{A}| = |\vec{B}|$

2) Negative of a vector

If $\vec{A} = -\vec{B} \Rightarrow$

(i) $|\vec{A}| = |\vec{B}|$

(ii) $\vec{A}$ is antiparallel to $\vec{B}$ i.e they are in opposite directions.

Now can find $\vec{A} - \vec{B}$ as follows: we turn subtraction into addition:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

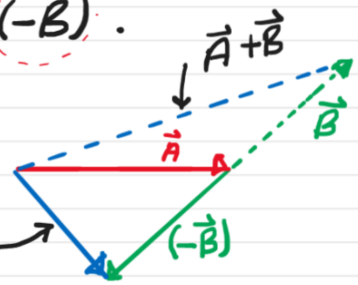
so we add $\vec{A}$ to vector $(-\vec{B})$.

Suppose

Find $\vec{A}-\vec{B}$.   add $-\vec{B}$ to vector $\vec{A}$.

$\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$.

$\vec{A}-\vec{B} =$
$\vec{A}+(-\vec{B})$



Remember:

$\vec{A}+\vec{B} = \vec{B}+\vec{A}$ commutative

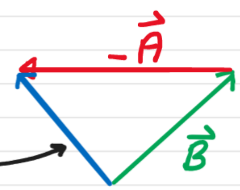
$\vec{A}-\vec{B} \neq \vec{B}-\vec{A}$

Now find $\vec{B}-\vec{A} = \vec{B}+(-\vec{A})$

subtraction is NOT commutative.

$|\vec{A}-\vec{B}| = |\vec{B}-\vec{A}|$ but antiparallel.

$\vec{B}-\vec{A}$
$= \vec{B}+(-\vec{A})$



NOTE: $\vec{A}-\vec{B} \neq \vec{B}-\vec{A}$

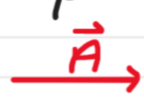
instead $\vec{A}-\vec{B} = -(\vec{B}-\vec{A})$

i.e $\vec{A}-\vec{B}$ is the negative of $\vec{B}-\vec{A}$.

Multiplication of a vector by a scalar.

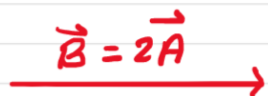
Let $|\vec{A}| = 1m$ along the positive x-direction.

i) sketch $\vec{A}$.



ii) sketch $\vec{B} = 2\vec{A}$

this means $|\vec{B}| = 2|\vec{A}|$ and $\vec{B}$ is parallel to $\vec{A}$.
$= 2m$



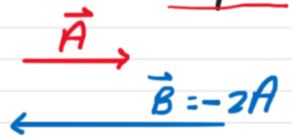
iii) $\vec{C} = \frac{1}{2}\vec{A}$

$\therefore |\vec{C}| = \frac{1}{2}|\vec{A}|$ and $\vec{C}$ is parallel to $\vec{A}$.

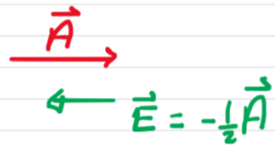
$= \frac{1}{2}$ m $\longrightarrow \vec{C} = \frac{1}{2}\vec{A}$

iv) $\vec{D} = -2\vec{A}$ But $\vec{D}$ is antiparallel (opposite) to $\vec{A}$

$\therefore |\vec{D}| = 2|\vec{A}| = 2m$



v) $\vec{E} = -\frac{1}{2}\vec{A}$



Newton's 2^{nd} Law

$\vec{F} = m\vec{a}$

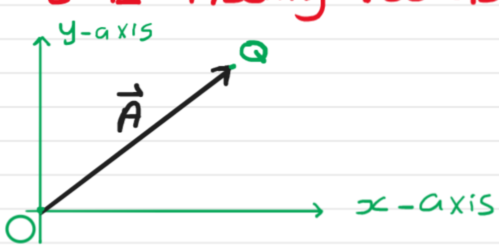
$\vec{a} = \frac{1}{m}\vec{F}$

↑ mass > 0

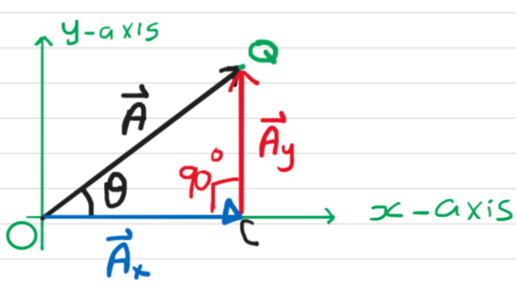
$\vec{a}$ ALWAYS // to $\vec{F}$

Parallel

3-4 Adding Vectors by Components



Suppose a car moved from the origin to point Q, a displacement $\vec{A}$. Is there an alternative route by moving along the positive x and y axes? The answer is YES.



Start from the origin:

(i) move a displacement $\vec{A}_x$ along the positive x-axis.

(ii) turn and make a displacement $\vec{A}_y$ along the positive y-axis

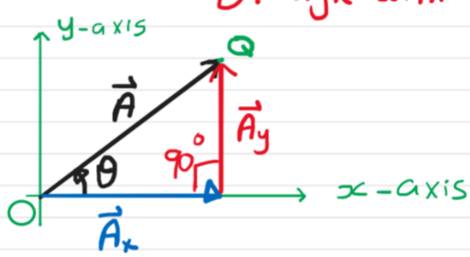
$\vec{A}$ is the resultant displacement of $\vec{A}_x$ and $\vec{A}_y$.

$$\therefore \quad \vec{A} = \vec{A}_x + \vec{A}_y$$

$|\vec{A}_x| \equiv A_x$ is called the x-component of $\vec{A}$.

$|\vec{A}_y| \equiv A_y$ is called the y-component of $\vec{A}$.

θ: angle with +ve x-axis in an anticlockwise direction.



A, A_x and A_y form a right-angle triangle $\Rightarrow$

$A^2 = A_x^2 + A_y^2$ (Pythagoras' theorem)

Also $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}} \Rightarrow \cos\theta = \dfrac{A_x}{A} \Rightarrow A_x = A\cos\theta \quad -①$

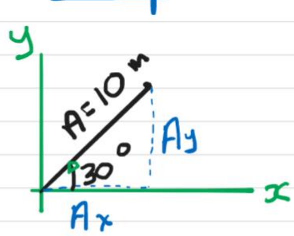
$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}} \Rightarrow \sin\theta = \dfrac{A_y}{A} \Rightarrow A_y = A\sin\theta \quad -②$

$② / ① \Rightarrow \tan\theta = \dfrac{A_y \leftarrow \text{opposite}}{\text{adjacent} \rightarrow A_x} \Rightarrow \theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)$ ← tan inverse.

Also $①^2 + ②^2 \Rightarrow A_x^2 + A_y^2 = A^2(\underbrace{\sin^2\theta + \cos^2\theta}_{=1}) = A^2$ as before.

If we know A_x and A_y $\Rightarrow$ can calculate A and θ and vice versa. $(A, \theta \Rightarrow A_x, A_y)$.

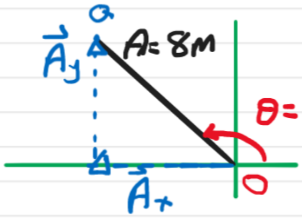
Examples



$A_x = A\cos 30^\circ = 10\left(\frac{\sqrt{3}}{2}\right) = 5\sqrt{3}$ m.

$A_y = A\sin 30^\circ = 10\left(\frac{1}{2}\right) = 5$ m.

Example



along -ve x-direction

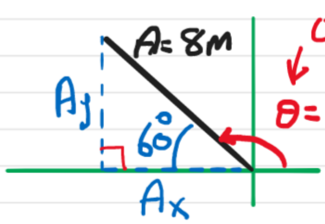
$A_x = 8\cos 120^\circ = 8(-\tfrac{1}{2}) = -4\,m$

$A_y = 8\sin 120^\circ = 8(\tfrac{\sqrt{3}}{2}) = +4\sqrt{3}\ m.$

along positive y-direction.

Second quadrant	First quadrant
$\sin\theta + \to A_y +$	$\sin\theta + \to A_y +$
$\cos\theta - \to A_x -$	$\cos\theta + \to A_x +$
$\sin\theta - \to A_y -$	$\sin\theta - \to A_y -$
$\cos\theta - \to A_x -$	$\cos\theta + \to A_x +$
Third quadrant	Fourth quadrant

Alternatively: use the acute angle of 60°.



since $\vec{A}$ lies in 2^{nd} quadrant.

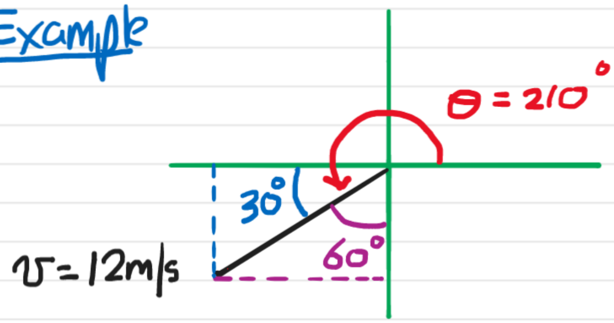
$A_x = -8\cos 60^\circ = -8(\tfrac{1}{2}) = -4\ m$

$A_y = 8\sin 60^\circ = 8(\tfrac{\sqrt{3}}{2}) = 4\sqrt{3}\ m$

NOTE: When we use θ with positive x-axis in an anticlockwise direction $\Rightarrow$ the sign of each component comes out of the angle automatically.

When we use the acute angle $\Rightarrow$ we must insert the sign of each component by hand depending on the quarant where the vector is.

<u>Example</u>



Three possible ways to find v_x and v_y.

(1) $v_x = 12 \cos 210^\circ$
$= 12\left(-\frac{\sqrt{3}}{2}\right) = -6\sqrt{3}$ m/s

$v_y = 12 \sin 210^\circ$
$= 12\left(\frac{-1}{2}\right) = -6$ m/s

(3) take right-angle triangle with 60° acute angle

$v_x = -12 \sin 60^\circ = -12\left(\frac{\sqrt{3}}{2}\right) = -6\sqrt{3}$ m

$v_y = -12 \cos 60^\circ = -12\left(\frac{1}{2}\right) = -6$ m

② take right-angle triangle with 30° acute angle.

$$v_x = -12\cos 30^\circ = -12\left(\frac{\sqrt{3}}{2}\right) = -6\sqrt{3} \text{ m/s}$$

$$v_y = -12\sin 30^\circ = -12\left(\frac{1}{2}\right) = -6 \text{ m/s}$$

③ take right-angle triangle with 60° acute angle

$$v_x = -12\sin 60^\circ = -12\left(\frac{\sqrt{3}}{2}\right) = -6\sqrt{3} \text{ m}$$

$$v_y = -12\cos 60^\circ = -12\left(\frac{1}{2}\right) = -6 \text{ m}$$