Chapter 30
Nuclear Physics and Radioactivity

Lecture 2
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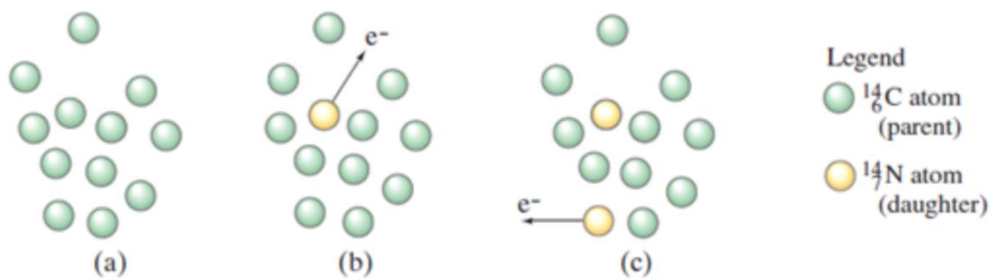
30-8] Half-Life and Rate of Decay.



The number of decays that occur in the short time interval Δt is designated ΔN because each decay that occurs corresponds to a decrease by one in the number N of parent nuclei present. That is, radioactive decay is a "one-shot" process, Fig. 30–9. Once a particular parent nucleus decays into its daughter, it cannot do it again.

$^{14}_{6}C_8$ is an unstable nucleus $\Rightarrow$ it decays by emitting β^- particles (e^-) and turns into $^{14}_{7}N_7$ which is stable.

$n \rightarrow e^- + \bar{\nu}_e + p$

$$\underbrace{^{14}_{6}C_8}_{\text{parent nucleus}} \longrightarrow \underbrace{^{14}_{7}N_7}_{\text{daughter nucleus}} + \underbrace{e^-}_{\text{electron }(\beta^-)} + \underbrace{\bar{\nu}_e}_{\text{antineutrino}}$$

$A = N + Z$

As time passes, the number of $^{14}_{6}C$ radioactive nuclei decreases, while the number of daughter stable $^{14}_{7}N$ nuclei increases.

How does the number of $^{14}_{6}C$ nuclei change with time?

This is given by the radioactive decay law.

$$N = N_0 \, e^{-\lambda t}$$

$\log_{10}$, $\log_e$ ← Ln

natural logarithm.

N : remaining number of radioactive nuclei at time t.

$\log_{10} 100 = 2$

$\Rightarrow 10^2 = 100$

N_0 : initial number of radioactive nuclei at time $t_0 = 0$.

λ : decay constant. Has units of $\frac{1}{time}$

for example s^{-1}, min^{-1}, etc.

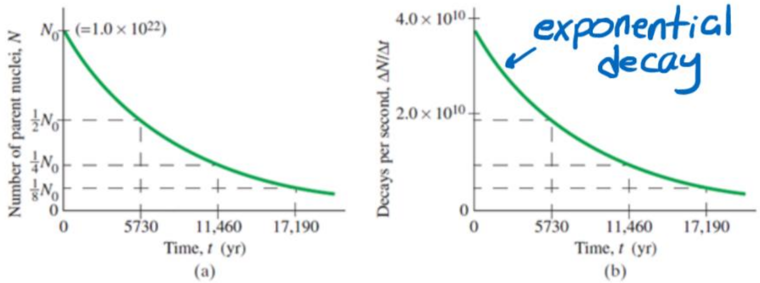


FIGURE 30–10 (a) The number N of parent nuclei in a given sample of $^{14}_{6}C$ decreases exponentially. We assume a sample that has $N_0 = 1.00 \times 10^{22}$ nuclei. (b) The number of decays per second also decreases exponentially. The half-life of $^{14}_{6}C$ is 5730 yr, which means that the number of parent nuclei, N, and the rate of decay, $\Delta N/\Delta t$, decrease by half every 5730 yr.

$$N_0 \longrightarrow \frac{N_0}{2} \longrightarrow \frac{N_0}{4} \longrightarrow \frac{N_0}{8}$$

$$\underline{\quad t_{1/2} \quad} \quad \underline{\quad t_{1/2} \quad} \quad \underline{\quad t_{1/2} \quad}$$

$$1000 \xrightarrow[t_{1/2}]{} 500 \xrightarrow[t_{1/2}]{} 250$$

^{14}C has a half-life of 5730 yr.

half-life ($t_{1/2}$): time needed for the number of radioactive nuclei to become half the original number.

$$N_0 \xrightarrow{t_{1/2}} \frac{N_0}{2} \xrightarrow{t_{1/2}} \frac{N_0}{4} \xrightarrow{t_{1/2}} \frac{N_0}{8} \xrightarrow{t_{1/2}} \frac{N_0}{16} \xrightarrow{t_{1/2}} \frac{N_0}{32} \; \dots$$

time →

	0	$t_{1/2}$	$2t_{1/2}$	$3t_{1/2}$	$4t_{1/2}$	$5t_{1/2}$	
for ^{14}C		5730	11,460	17,190	22,920	28,650	....

Calculate the half-life ($t_{1/2}$) in terms of the decay constant.

remaining number $\rightarrow N = N_0 \, e^{-\lambda t}$, after $t_{1/2}$ we have $N \rightarrow \frac{N_0}{2}$

$$\therefore \frac{\cancel{N_0}}{2} = \cancel{N_0} \, e^{-\lambda t_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}} \qquad \ln e^{x} = x$$

$$\ln \frac{1}{2} = \ln e^{-\lambda t_{1/2}} = -\lambda t_{1/2}$$

natural logarithm

$$\therefore t_{1/2} = -\frac{\ln(\frac{1}{2})}{\lambda} = \frac{\ln 2}{\lambda} \simeq \frac{0.693}{\lambda}$$

$$-\ln(\tfrac{1}{2}) = \ln(\tfrac{1}{2})^{-1} = \ln 2$$

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$$

Each radioactive nuclide has its own λ.

large $\lambda \Rightarrow$ small $t_{1/2} \Rightarrow$ fast decay.

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

small $\lambda \Rightarrow$ large $t_{1/2} \Rightarrow$ slow decay.

For ^{238}U : $t_{1/2} \simeq 4.5 \times 10^{9}$ yr $\Rightarrow \lambda = \frac{\ln 2}{4.5} \times 10^{-9}$ yr^{-1}

very small $\lambda \Rightarrow$ very large $t_{1/2} \Rightarrow$ slow decay.

For Iodine-131 (^{131}I) (used to treat thyroid gland cancer and also treatment of an overactiv thyroid gland).

$t_{1/2} \simeq 8$ days

$t_{1/2} \simeq 8 \text{ days} \Rightarrow \lambda = \frac{\ln 2}{8} \simeq 0.087 \text{ day}^{-1}$

large λ (compared to ^{235}U) $\Rightarrow$ small $t_{1/2}$ $\Rightarrow$ fast decay.

Activity (A) : number of decays per second. [rate of decay]

$$N = N_0 e^{-\lambda t}$$

$$A = -\frac{dN}{dt} = -[N_0(-\lambda) e^{-\lambda t}]$$

$$\therefore \quad A = \lambda \underbrace{N_0 e^{-\lambda t}}$$

$$A = \lambda N$$

$$A = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$$

initial activity

activity at time t.

[The book uses R instead of A]

Mean life (τ): average life time of all the radioactive nuclei of a given radioactive element.

(Tau) $\tau = \frac{1}{\lambda} = \frac{1}{\left(\frac{\ln 2}{t_{1/2}}\right)} = \frac{t_{1/2}}{\ln 2}$

30-9] Calculations Involving Decay Rate (activity) and Half-life.

Example 30-9] Sample activity. The isotope $^{14}_{6}C$ has a half-life of 5730 yr. If a sample contains 1.0×10^{22} carbon-14 nuclei, what is the activity of the sample?

$\underbrace{^{14}_{6}C,\ ^{13}_{6}C,\ ^{12}_{6}C}_{\text{Isotopes}}$.

$A = \lambda N \Rightarrow$ need to calculate λ.

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{\underbrace{5730}_{yr}\times\underbrace{365\times24\times60\times60}_{s/yr}} \simeq 3.84\times10^{-12}\ s^{-1}$$

$\therefore\ A = (3.84\times10^{-12}\ s^{-1})(1.0\times10^{22})$

$A = 3.84\times10^{10}$ decays/s. ← unit of activity

Can say $A = 3.84\times10^{10}\ s^{-1}$.

The SI unit of activity is the Becquerel (Bq)

1 Bq = 1 decay/s or simply s^{-1}

Another more common unit of activity is the curie (Ci)

1 Ci = 3.7×10^{10} decays/s

$1\,Ci = 3.7 \times 10^{10}\ Bq$.

micro
$\mu \equiv 10^{-6}$

Example 30-11] A laboratory has 1.49 μg of pure $^{13}_{7}N$, which has a half-life of 10.0 min.

$m \equiv 10^{-3}$
$\mu \equiv 10^{-6}$
$n \equiv 10^{-9}$
$p = 10^{-12}$

a) How many nuclei are present initially?

Remember: $\underbrace{\text{mass of one mole of a substance}}^{\text{molar mass}}$ contains Avogadro's number of the particles or molecules of that substance.
(Avogadro's number $N_A = 6.02 \times 10^{23}$)

Molar mass equals the mass number in units of grams.

Nuclide	molar mass (grams/mole)
mass number ^{12}C	12
^{13}N	13
^{131}I	131

13 grams of ^{13}N $\longrightarrow$ 1 mole

1.49×10^{-6} grams of ^{13}N $\longrightarrow$ x mole

$x \to y$
$z \to a$

$ax = zy$

$a = \frac{zy}{x}$

$\therefore$ number of moles of ^{13}N is

$$x = \frac{1.49 \times 10^{-6} \cancel{\text{grams}} \times 1\text{mole}}{13 \cancel{\text{grams}}} \simeq \underline{1.146 \times 10^{-7} \text{mole}}$$

$\therefore$ Number of nuclei of ^{14}N is $x \times N_A \simeq 6.9 \times 10^{16}$ nuclei

b) What is the initial activity (initial decay rate)?

$$A = A_0 e^{-\lambda t}$$
$$= \lambda N_0 e^{-\lambda t}$$

$e^{-0} = 1$

$\therefore \quad A_0 = \lambda N_0$ ← 6.9×10^{16}

need to calculate λ

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{10 \times 60} \simeq 0.001155 \ s^{-1}$$

$\therefore A_0 \simeq 7.97 \times 10^{13}$ Bq ← 1 decay/s

$m = 1.49 \times 10^{-6}$ grams

Find out the activity of 1.49μgram of ^{238}U.

c) What is the activity after 1h ?

$$A = A_0 e^{-\lambda t} = 7.97 \times 10^{13} e^{-\lambda t}$$

$$\lambda t = \frac{\ln 2}{t_{1/2}} \times t = \frac{\ln 2}{10 \cancel{\text{min}}} \times \overbrace{60 \cancel{\text{min}}}^{1h}$$

$$\lambda t = 6 \ln 2$$

$$\therefore \; A = \overbrace{7.97\times10^{13}}^{A_o} \times e^{-6\ln 2}$$

$$\simeq 1.25\times10^{12}\ Bq = 1.25\times10^{12}\ \cancel{Bq} \times \frac{1\,Ci}{3.7\times10^{10}\ \cancel{Bq}}$$

$$\therefore \; A \simeq 33.8\ Ci$$

Alternatively:

$$1h = 60\ min = 6\times10 = 6\ t_{1/2}$$

$$N_o \xrightarrow{t_{1/2}} \frac{N_o}{2} \xrightarrow{t_{1/2}} \frac{N_o}{4} \xrightarrow{t_{1/2}} \frac{N_o}{8} \xrightarrow{t_{1/2}} \frac{N_o}{16} \to \frac{N_o}{32} \to \frac{N_o}{64}$$

So, the activity will decrease to:

$$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

i.e. $A \to A_o \times \frac{1}{64} = \frac{7.97\times10^{13}}{64} \simeq 1.25\times10^{12}\ Bq$

as before.

$$N = N_o e^{-\lambda t}$$

$$A = -\frac{d}{dt}N = \lambda N$$

d) After approximately how long will the activity drop to less than 1 decay/s?

$$A = A_o\, e^{-\lambda t}$$

$$1 = 7.97\times10^{13}\, e^{-\frac{\ln 2}{600}\times t}$$

$$\ln\left(\frac{1}{7.97\times10^{13}}\right) = -\frac{\ln 2}{600}\, t$$

$$\therefore t \sim 2.7901\times10^{4}\ s$$

$$\sim 7.75\ hr$$

$\therefore$ after approximately $2.7901\times10^{4}\ s$