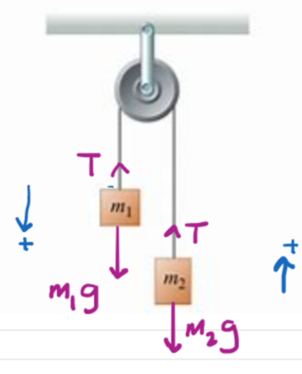
Solutions to Problems Sets of Chapter 4

The University of Jordan/Physics Department
Prof. Mahmoud Jaghoub
أ.د. محمود الجاغوب

Example: The figure shows an Atwood's Machine. If $m_1 = 4k$, $m_2 = 2k$ find the acceleration of the system and the tension in the string.

Assume system started from rest. (masses were initially at rest).

- sketch free-body diagram for each mass.
- since $m_1 > m_2 \Rightarrow m_1$ moves down while m_2 moves up.



for m_1: $\downarrow +$

$$m_1 g - T = m_1 a \quad -① $$

for m_2: $\uparrow +$

$$T - m_2 g = m_2 a \quad -② $$

① + ② $\Rightarrow$ $m_1 g - m_2 g = (m_1 + m_2) a$

$$\therefore \quad a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g = \frac{2}{6} g = \frac{1}{3} g \ m/s^2$$

Substitute for a in ① $\Rightarrow$

$$T = m_2 g + m_2 \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

$$= \frac{m_2(m_1 + m_2) + m_2(m_1 - m_2)}{m_1 + m_2} g$$

$$= \frac{2 m_1 m_2}{m_1 + m_2} g = 2 \frac{8}{6} g = \frac{8}{3} g \ \text{Newton.}$$

Example

From GianColi textbook

$\overset{+}{\uparrow}$ $2F_T - mg = ma$

$2F_T = m(g+a)$

$\therefore F_T = \frac{m}{2}(g+a)$

If $a = 0$ $\Rightarrow$ constant velocity

$F_T = \frac{1}{2}mg$

Special case

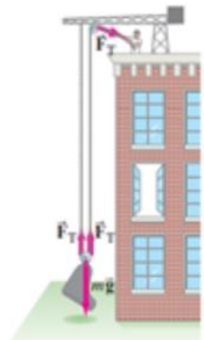


FIGURE 4–24 Example 4–14.

CONCEPTUAL EXAMPLE 4–14 **The advantage of a pulley.** A mover is trying to lift a piano (slowly) up to a second-story apartment (Fig. 4–24). He is using a rope looped over two pulleys as shown. What force must he exert on the rope to slowly lift the piano's 1600-N weight?

RESPONSE The magnitude of the tension force F_T within the rope is the same at any point along the rope if we assume we can ignore its mass. First notice the forces acting on the lower pulley at the piano. The weight of the piano ($= mg$) pulls down on the pulley. The tension in the rope, looped through this pulley, pulls up *twice*, once on each side of the pulley. Let us apply Newton's second law to the pulley–piano combination (of mass m), choosing the upward direction as positive:

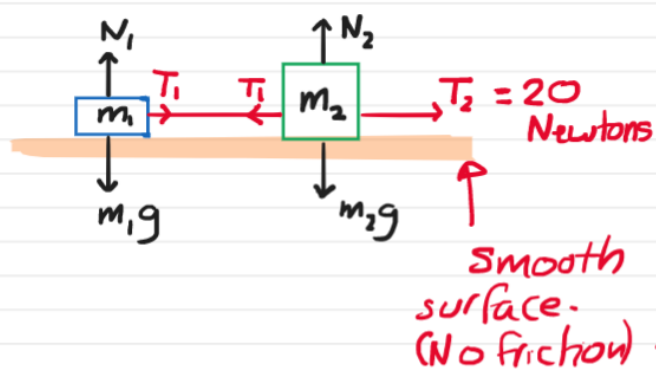
$$2F_T - mg = ma.$$

To move the piano with constant speed (set $a = 0$ in this equation) thus requires a tension in the rope, and hence a pull on the rope, of $F_T = mg/2$. The piano mover can exert a force equal to half the piano's weight.

NOTE We say the pulley has given a **mechanical advantage** of 2, since without the pulley the mover would have to exert twice the force.

Note: we need to exert a force equals half the weight of the object (when $a = 0$).

Example Find a and T_1.
$m_1 = 2\,kg$, $m_2 = 3\,kg$.



For m_1: $\rightarrow +$

$T_1 = m_1 a$ — ①

For m_2: $\rightarrow +$

$T_2 - T_1 = m_2 a$ — ②

① + ② $\Rightarrow$ $T_2 = (m_1 + m_2)a$

$a = \frac{T_2}{m_1+m_2} = \frac{20}{2+3} = 4\ m/s^2$.

Using ① $\Rightarrow$ $T_1 = (2)(4) = 8$ Newton.

$\underline{\text{Also note}}$ $\overset{+}{\uparrow}$ $N_1 - m_1 g = 0 \Rightarrow N_1 = m_1 g$.

$\overset{+}{\uparrow}$ $N_2 - m_2 g = 0 \Rightarrow N_2 = m_2 g$.

Friction

Objects look smooth to the naked eye. But under a microscope objects have irregularities.

As the surfaces which are in contact move against each other the irregularities impede the motion.

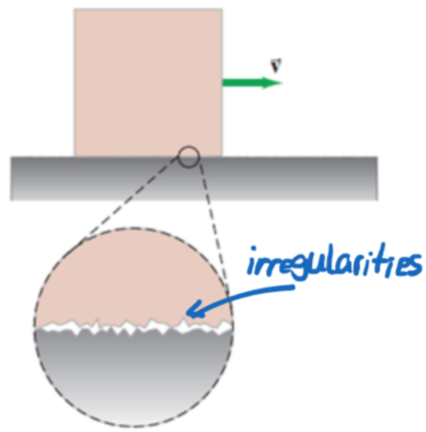
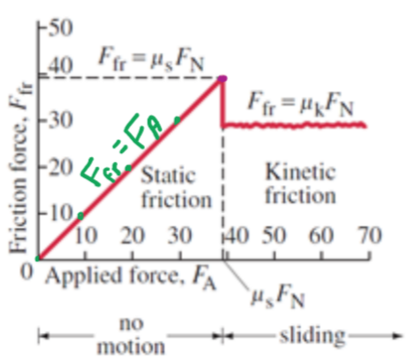
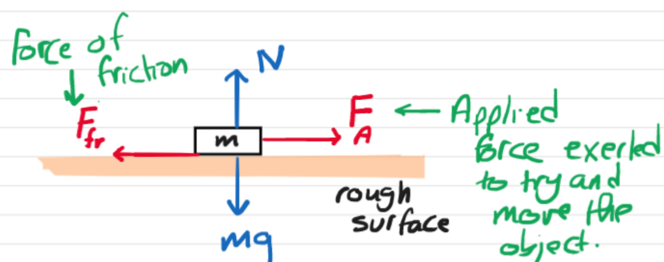


FIGURE 4–26 An object moving to the right on a table. The two surfaces in contact are assumed smooth, but are rough on a microscopic scale.





Mass m is at rest on a rough surface. A force F_A is trying to move the mass but it remains at rest.

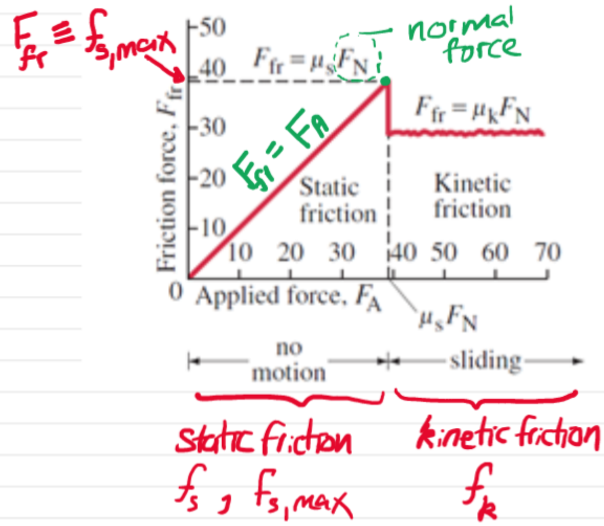
Using $\Sigma F_x = ma_x = m(0) = 0 \Rightarrow$

$\rightarrow + \quad F_A - F_{fr} = 0 \Rightarrow F_A = F_{fr}$.

As long as the mass is stationary $\Rightarrow F_A = F_{fr}$

This is given by the straight line in the figure.

The force of friction that acts while the object is at REST is called static friction and I shall refer to it as f_s .



Whe the object is on the verge of moving (about to move) But still has not moved we have the maximum possible value of static friction called maximum static friction

$f_{s,max} = \mu_s N \leftarrow$ Normal force , $\mu_s = \dfrac{f_{s,max}}{N}$ $\quad \dfrac{\cancel{\text{Newton}}}{\cancel{\text{Newton}}}$

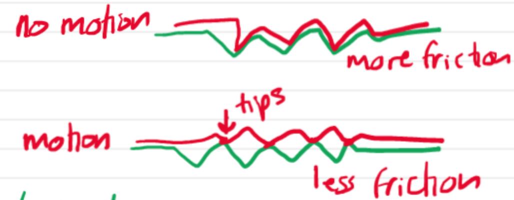
$\uparrow$

coefficient of static friction (dimensionless i.e has no units)

As soon as the object starts moving $\Rightarrow$ we have kinetic friction (f_k)

$f_k = \mu_k N$ ← normal force

↑ coefficient of kinetic friction (has no units)



Note that $f_k < f_{s,max}$. $\Rightarrow \mu_k < \mu_s$

when the object starts moving the contact between the irregularities is mostly at the tips which reduces the surface area of contact and hence reduces the force of friction.

NOTE : $\mu_s > \mu_k$

μ_s and μ_k depend on the nature of the surfaces in contact.

μ_s and μ_k don't depend on the surface area of the two surfaces.

TABLE 4–2 Coefficients of Friction†

Surfaces	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	0.01	0.01

† Values are approximate and intended only as a guide.

f_s , $f_{s,max} = \mu_s N$, $f_k = \mu_k N$

Example: Find the force of friction in each of the following cases: (all objects are at REST)

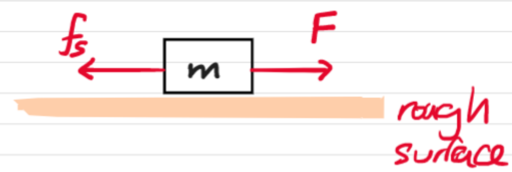
(i) No force is trying to move object along x-axis $\Rightarrow f_s = 0$.

Note: f_s only arises when object tries to move due to an acting force

(ii) static equilibrium

$\rightarrow +$ $F - f_s = 0 \Rightarrow F = f_s$

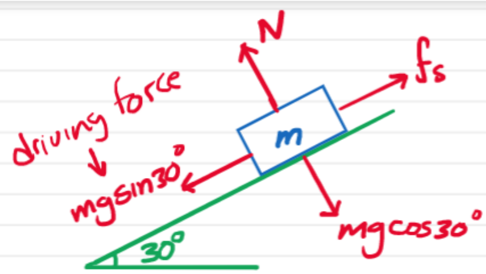
this force tries to move the object



(iii) Static equilibrium

$+\swarrow$ $mg\sin 30^\circ - f_s = 0$

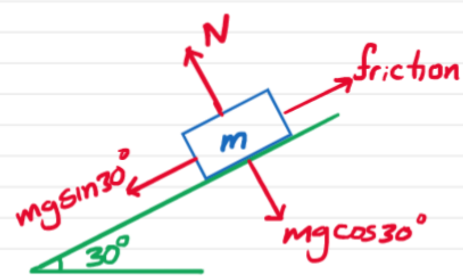
$\therefore f_s = mg\sin 30 = \frac{1}{2}mg$.



(iv) In the figure $m = 4\,kg$, $\mu_s = 0.5$ and $\mu_k = 0.1$.

(a) Is the object going to move?

(b) Find a.



$+\nwarrow$ $N - mg\cos 30^\circ = 0$

$N = mg\sqrt{\frac{3}{2}}$

(a) For the object to slide (move) down the inclined plane $\Rightarrow$

$mg\sin 30^\circ > f_{s,max}$.

$mg\sin 30^\circ = 4 \times 10 \times \frac{1}{2} = 20$ Newtons.

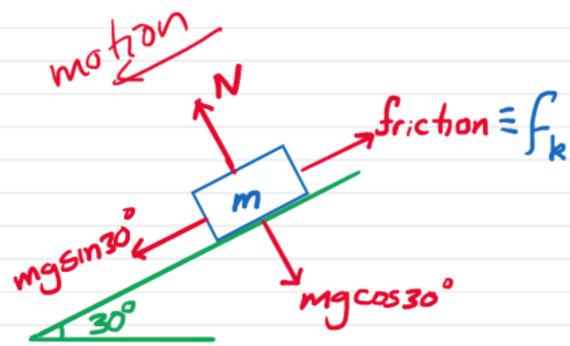
$f_{s,max} = \mu_s N = (0.5)(4 \times 10 \times \frac{\sqrt{3}}{2}) = 10\sqrt{3}$ Newtons.

$\therefore$ $mg\sin 30^\circ > f_{s,max} \Rightarrow$ object moves down the inclined plane.

To find the acceleration apply Newton's 2^{nd} law

$+$

$$mg\sin 30 - \overbrace{\mu_k N}^{f_k = \mu_k N} = ma$$



$$a = \frac{mg\sin 30 - \mu_k N}{m}$$

$$a = \frac{20 - 0.1 \times 4 \times 10 \times \frac{\sqrt{3}}{2}}{4} = \frac{16.53}{4} \sim 4.1 \text{ m/s}^2.$$