Chapter 6:Work and Energy

Lecture 2
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Kinetic Energy

An object of mass m moving with velocity v has a form of energy called Kinetic Energy (K) given by $K = \frac{1}{2} m v^2$ measured in (J)

clearly $K \geq 0$. If $K=0 \Rightarrow$ object is at rest.

Work - kinetic energy theorem

Where does the work done on an object go?

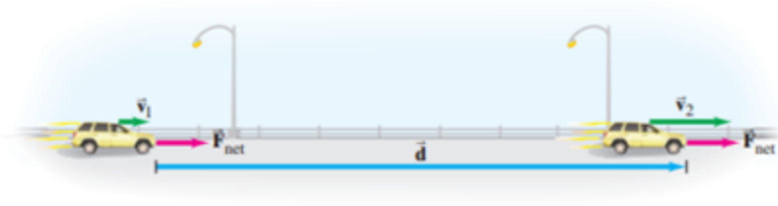


FIGURE 6–7 A constant net force F_{net} accelerates a car from speed v_1 to speed v_2 over a displacement d. The net work done is $W_{net} = F_{net}\, d$.

In the figure, a net force acts on the car moves it a displacement $\vec{d}$ in the direction of $\vec{F}_{net}$ and hence accelerates it from an initial velocity $\vec{v}_i$ to a final velocity $\vec{v}_f$. What is the work done on the car by the force $\vec{F}_{net}$?

$$W_{net} = F_{net}\, d \cos(0) = F_{net}\, d \;.$$

From the equations of motion, we have

$$v_f^2 - v_i^2 = 2ad$$

$$\times \tfrac{1}{2}m \Rightarrow \tfrac{1}{2}m v_f^2 - \tfrac{1}{2}m v_i^2 = \cancel{\tfrac{2}{2}}\, m a d$$

$$= (ma)\,d$$

$$= F_{net}\, d$$

$F_{net} = ma$
Newton's 2nd law

$$\therefore \underbrace{K_f - K_i}_{\Delta K} = W_{net}$$

total (net) work done on the car.

Final kinetic energy

Initial kinetic energy

The total work done on an object equals the change in its kinetic energy.

$$\boxed{W_{net} = \Delta K}$$

$W_{net} = \frac{1}{2}m(v_f^2 - v_i^2)$
work-kinetic energy theorem

When $W_{net} = 0 \Rightarrow \Delta K = 0 \Rightarrow K_f = K_i \Rightarrow$ speed does Not change.
$W_{net} > 0 \Rightarrow v_f > v_i \Rightarrow$ object accelerates, $W_{net} < 0 \Rightarrow v_f < v_i \Rightarrow$ object decelerates.

Example: An object of mass m is projected vertically upwards from the earth's surface with an initial speed of 20 m/s.

(i) Find its maximum height.

Free Fall. Only force that acts on the object is its weight downwards. Displacement $\vec{d}$ is upwards $\Rightarrow \theta = 180°$.

$$W_{net} = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$(\cancel{m}g)(y_{max})\cos(180°) = \frac{1}{2}\cancel{m}(0 - (20)^2)$$

$$\therefore \; -g\,y_{max} = -200 \Rightarrow y_{max} = 20 \text{ m}.$$

② Find the speed of the object when it is at a height of 15 m.

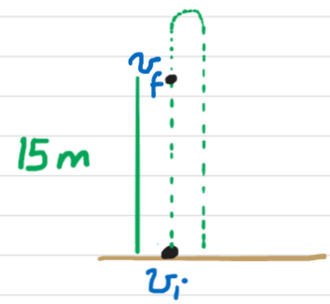
$$W_{net} = \Delta K = K_f - K_i = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2$$

$$(\cancel{m}g)(15)\cos 180^\circ = \tfrac{1}{2}\cancel{m}\left(v_f^2 - (20)^2\right)$$

$$-150 = \tfrac{1}{2} v_f^2 - 200$$

$$v_f^2 = 100$$

$$\therefore\ v_f = 10 \ \text{m/s}.$$



③ Find the speed of the object just before hitting the ground.

$$W_{net} = (mg)(\overset{=0}{d})\cos\theta$$

Return to ground $\Rightarrow \vec{d} = 0 \Rightarrow W_{net} = 0$

$$0 = \tfrac{1}{2} m (v_f^2 - v_i^2) \Rightarrow |v_f| = |v_i| = 20 \text{ m/s} \quad \text{(speeds)}$$

[for velocities $v_f = \pm v_i$]

if $\overset{+}{\uparrow}$ $v_f = -v_i = -20$ m/s, if $\underset{+}{\downarrow}$ $v_f = +20$ m/s.

CONCEPTUAL EXAMPLE 6–5 **Work to stop a car.** A car traveling 60 km/h can brake to a stop in a distance d of 20 m (Fig. 6–10a). If the car is going twice as fast, 120 km/h, what is its stopping distance (Fig. 6–10b)? Assume the maximum braking force is approximately independent of speed.

Stopping distance $\equiv d$

RESPONSE Again we model the car as if it were a particle. Because the net stopping force F is approximately constant, the work needed to stop the car, Fd, is proportional to the distance traveled. We apply the work-energy principle, noting that $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$ are in opposite directions and that the final speed of the car is zero:

$$W_{net} = Fd\cos 180^\circ = -Fd.$$

Then

$$-Fd = \underbrace{\Delta \mathrm{KE}}_{\Delta K} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$
$$= 0 - \tfrac{1}{2}mv_1^2.$$

$v_1 = v_i \qquad v_2 = v_f = 0$

$v_i' = 2v_i$

$Fd = \frac{1}{2}mv_i^2$

$d = \frac{m}{2F}v_i^2$

Thus, since the force and mass are constant, we see that the stopping distance, d, increases with the square of the speed:

$$d \propto v^2.$$

Stopping distance $d \propto v_i^2$

$d \propto v_i^2$

what is d'?

If the car's initial speed is doubled, the stopping distance is $(2)^2 = 4$ times as great, or 80 m.

$d' = \frac{m}{2F}(2v_i)^2 = 4\left(\frac{m}{2F}v_i^2\right) = 4d$

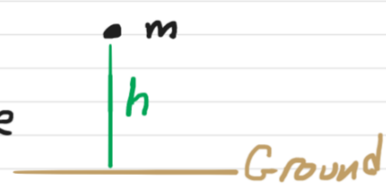
new stopping distance.

at $v_i = 60$ km/h, $d = 20$ m $\Rightarrow$ at $v_i = 120$ km/h, $d' = 4d = 4(20) = 80$ m.

6-4] Potential Energy

Gravitational Potential Energy

Object of mass m is at a height h above the surface of the ground.

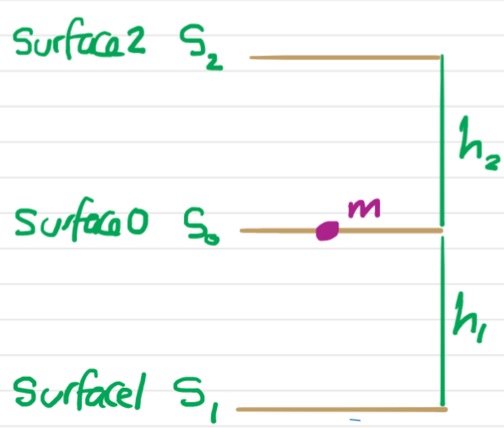


If you release this object it fall towards the ground. This shows that the object possesed energy when it was at height h above the ground. This energy is called the gravitational potential energy which is the energy possesed by the object due to its position above the ground.

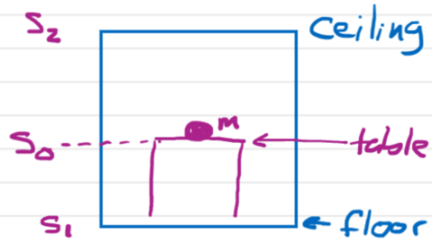
gravitational potential energy $\longrightarrow U = mgh$

Gravitational potential energy is defined with respect to a surface.

$U_2 = -mgh_2$ potential energy of mass m relative to S_2. Negative U_2 means work must be done to rais m to S_2.



$U_1 = mgh_1$ potential energy of mass m relative to S_1. Positive value means if you release m it falls towards S_1.

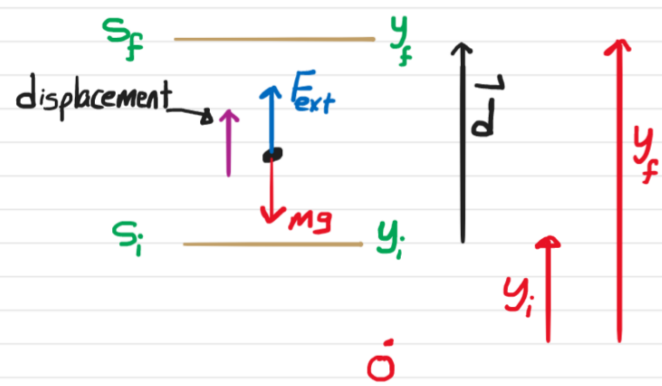


$mgh = mg(0) = 0$

$U_0 = 0$ potential energy of mass m with respect to surface S_0. Note m is on the surface $S_0 \Rightarrow h = 0$.

Unlike kinetic energy K, the potential energy can be positive, negative or zero.

Find the work done by the weight (mg) while an object is moved from surface $S_i \rightarrow S_f$.



$$W_{mg} = (mg)(d)\cos 180^\circ$$
$$= mg(y_f - y_i)(-1)$$

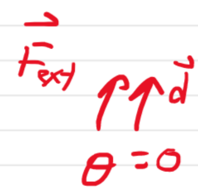
$$W_{mg} = -(mgy_f - mgy_i)$$

$$W_{mg} = -(U_f - U_i) = -\Delta U$$

$$\boxed{W_{mg} = -\Delta U}$$ only depends on the displacement along the y-direction.

If the mass is moved up at <u>constant</u> <u>velocity</u> $(a=0) \Rightarrow F_{ext} = mg$.

Work done by the external force is



$$W_{ext} = \underbrace{(mg)}_{F_{ext}}(d)\cos(0) = mgd = mg(y_f - y_i)$$

$$W_{ext} = U_f - U_i = \Delta U$$

$\therefore$ $$\boxed{W_{ext} = \Delta U}, \quad W_{mg} = -\Delta U$$

$W_{net} = W_{mg} + W_{ext} = 0 \Rightarrow$ object moves at <u>constant speed</u>. upwards.

Conservative and nonconservative forces.

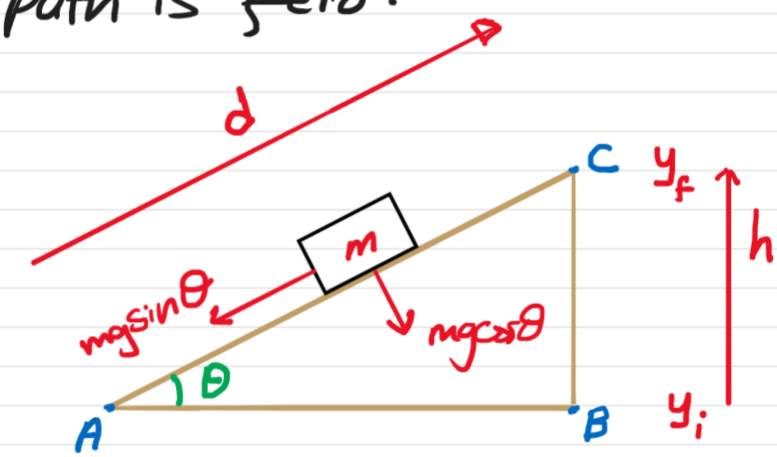
Properties of conservative forces

(The work done by)

(i) The work of a conservative force does NOT depend on the path.

(ii) The work of a conservative force round a closed path is zero.

Find the work done by the force of gravity (mg) when the object of mass m moves from point A to C through two different paths:



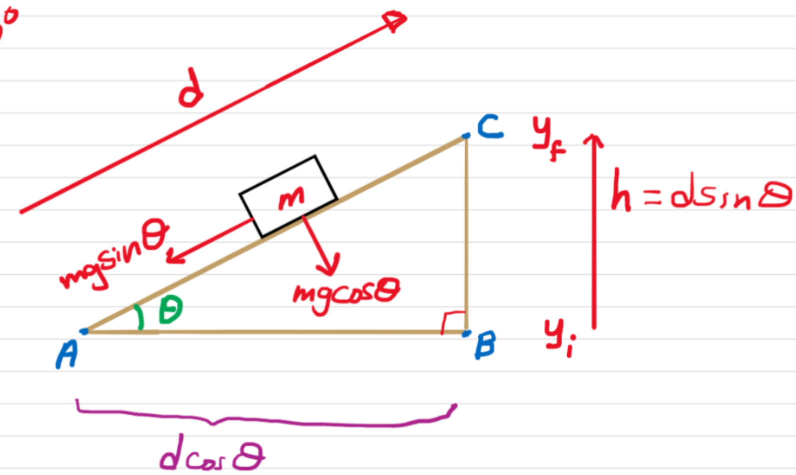
① $A \to C$ directly.

$W_{mg}(A \to C) = (mg\sin\theta)(d)\cos 180^\circ$

$W_{mg}(A \to C) = -mgd\sin\theta$

But $h = d\sin\theta \Rightarrow$

$W_{mg}(A \to C) = -mgh$.



② Find the work done by the force of gravity in moving the mass m from $A \to B \to C$

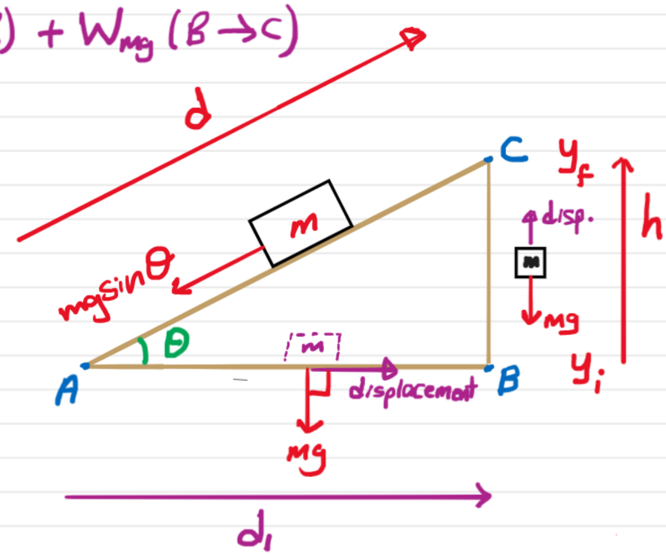
$W_{mg}(A \to B \to C) = W_{mg}(A \to B) + W_{mg}(B \to C)$

$W_{mg}(A \to B) = (mg)(d_1)\cos 90^\circ$

$\therefore$ $W_{mg}(A \to B) = 0$

$W_{mg}(B \to C) = (mg)(h)\cos 180^\circ$

$= -mgh$



$\therefore W_{mg}(A \to B \to C) = 0 + (-mgh)$

$W_{mg}(A \to B \to C) = -mgh$ as before.

$\therefore W_{mg}(A \xrightarrow{\text{directly}} C) = W_{mg}(A \to B \to C) = -mgh$

$\therefore$ Work of gravity does NOT depend on the path.

③ Find the work done by gravity when the object moves from $A \to B \to C \to A$ (closed path).

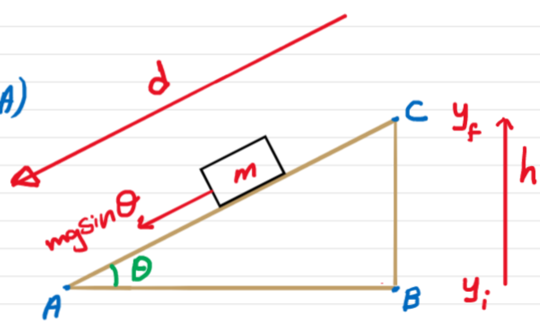
$W_{mg}(A \to B \to C \to A)$

$= W_{mg}(A \to B \to C) + W_{mg}(C \to A)$

$= -mgh + mgh = 0$

Note: $mg\sin\theta$ is parallel to $\vec{d}$

$\Rightarrow$ angle between them $= 0$



$W(C \to A) = (mg\sin\theta)(d)\cos(0)$

$= mgd\sin\theta$

$= mgh$

$\therefore W_{mg}(A \to B \to C \to A) = 0$

$\therefore$ Work of gravity round a closed path $= 0$.