

# Chapter 6: Work and Energy

## Lecture 3

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### Conservative Forces:

- Their work does NOT depend on the path.
- Their work round a closed path is zero.

### Examples of conservative forces

- Gravitational force (REQUIRED in this course)
- Spring force (NOT required in this course)
- electric force (Not required in this course)

## Nonconservative Forces

- Their work depends on the path.
- Their work round a closed path  $\neq 0$

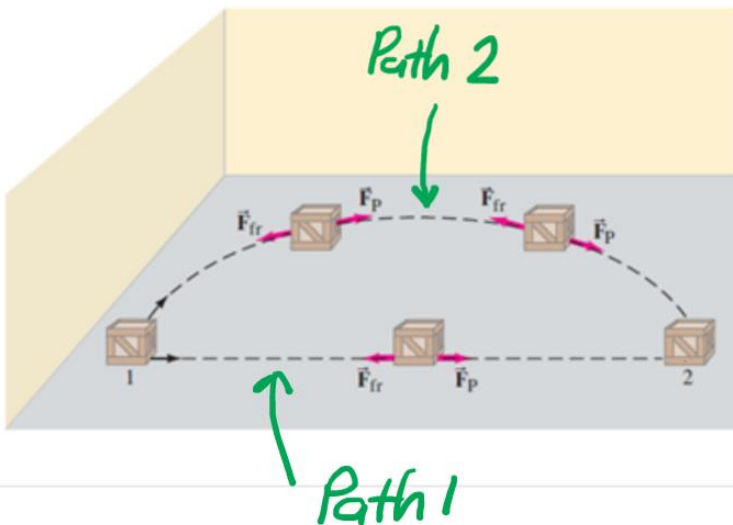


FIGURE 6-16 A crate is pushed slowly at constant speed across a rough floor from position 1 to position 2 via two paths, one straight and one curved. The pushing force  $\vec{F}_p$  is in the direction of motion at each point. (The friction force opposes the motion.) Hence for a constant magnitude pushing force, the work it does is  $W = F_p d$ , so if the distance traveled  $d$  is greater (as for the curved path), then  $W$  is greater. The work done does not depend only on points 1 and 2; it also depends on the path taken.

$W_{fr}$  along the path 1  $\neq$   $W_{fr}$  along path 2  
 $\uparrow$  work done by friction.

$W_{fr}$  from 1  $\rightarrow$  2 back to 1 is NOT 0.

When you raise an object up to a height  $h$  above the ground, you do work against gravity. This work is stored as gravitational potential energy in the system that consists of the object-earth system.

What is the evidence that energy is stored?  
 When you release the object it falls back to the ground.

In the figure above, work is done by the force  $\vec{F}_p$  against the force of friction to move the box from point 1  $\rightarrow$  point 2. But this energy is NOT stored; it is lost as heat for example. The box does NOT move back from point 1  $\rightarrow$  point 2 when  $\vec{F}_p$  is removed.

You do more work on the curved path because the distance is greater and, unlike the gravitational force, the pushing force  $\vec{F}_p$  is in the direction of motion at each point. Thus the work done by the person in Fig. 6-16 does not depend *only* on points 1 and 2; it depends also on the path taken. The force of kinetic friction, also shown in Fig. 6-16, always opposes the motion; it too is a nonconservative force, and we discuss how to treat it later in this Chapter (Section 6-9). Table 6-1 lists a few conservative and nonconservative forces.

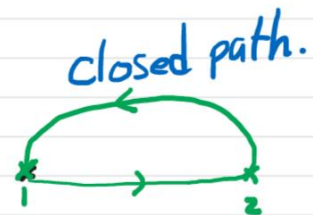
Because potential energy is energy associated with the position or configuration of objects, potential energy can only make sense if it can be stated uniquely for a given point. This cannot be done with nonconservative forces because the work done depends on the path taken (as in Fig. 6-16). Hence, *potential energy can be defined only for a conservative force*. Thus, although potential energy is always associated with a force, not all forces have a potential energy. For example, there is no potential energy for friction.

**EXERCISE E** An object acted on by a constant force  $F$  moves from point 1 to point 2 and back again. The work done by the force  $F$  in this round trip is 60 J. Can you determine from this information if  $F$  is a conservative or nonconservative force?

### Spring force

TABLE 6-1 Conservative and Nonconservative Forces

| Conservative Forces | Nonconservative Forces     |
|---------------------|----------------------------|
| Gravitational       | Friction                   |
| Elastic             | Air resistance             |
| Electric            | Tension in cord            |
|                     | Motor or rocket propulsion |
|                     | Push or pull by a person   |



Answer:  $F$  is NOT a conservative force since its work round a closed path  $\neq 0$ .

## Work-Energy Extended

$$W_{\text{net}} = \Delta K$$
$$W_{\text{nc}} + W_{\text{c}} = \Delta K$$

$W_{\text{net}}$ : work done on the object by ALL forces acting on it (conservative and nonconservative).



But  $W_{\text{c}} = -\Delta U$

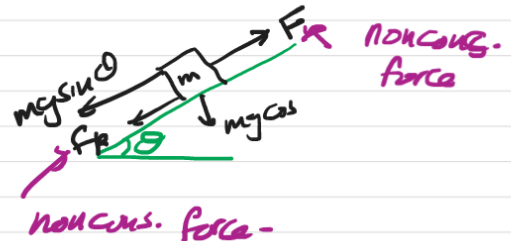
$W_{\text{nc}}$ : work done on object by nonconservative forces only.

$W_{\text{c}}$ : work done on object by conservative forces only.

$\therefore W_{\text{nc}} - \Delta U = \Delta K$

$$W_{\text{nc}} = \Delta U + \Delta K$$

$\therefore \Delta U + \Delta K = W_{\text{nc}}$



Change in potential energy + Change in Kinetic Energy  
= work done by nonconservative forces.

## 6-6] Conservation of total mechanical energy

Total mechanical energy  $E$  is defined as

$$\begin{array}{c} \text{Total} \\ \text{mechanical} \\ \text{energy} \end{array} \rightarrow E = \begin{array}{c} \text{kinetic} \\ \text{energy} \end{array} K + \begin{array}{c} \text{potential} \\ \text{energy} \end{array} U$$

If no nonconservative forces are present (e.g. free fall)

⇒ No work is done by nonconservative forces

i.e.  $W_{nc} = 0$

∴  $\Delta K + \Delta U = 0$  ↙  $W_{nc} = 0$  since we have NO nonconservative forces act

$(K_f - K_i) + (U_f - U_i) = 0$

$K_f + U_f - (K_i + U_i) = 0$

$E_f - E_i = 0 \Rightarrow E_f = E_i = \text{Constant}$  i.e. conserved.

⇒ There is NO loss of total mechanical energy  $E \Rightarrow$  Total mechanical energy is conserved.

$E_f - E_i = \Delta E = 0$  (when No nonconservative forces are present)

$v_i = 0$   
 $U_i = mgh, K_i = 0$   
 $E_i = mgh + 0 = mgh$   


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 $U_f = 0, K_f = \frac{1}{2}mv_f^2$   
 $E_f = \frac{1}{2}mv_f^2 + 0$   
 $E_f = E_i \Rightarrow mgh = \frac{1}{2}mv_f^2$

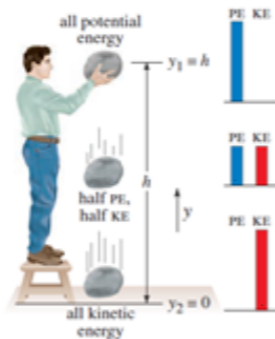


FIGURE 6-17 The rock's potential energy changes to kinetic energy as it falls. Note bar graphs representing potential energy PE and kinetic energy KE for the three different positions.

$PE \equiv U, KE \equiv K$

Since ONLY conservative forces act ⇒ total mechanical energy is conserved i.e.

$E = K + U = \text{constant}$

As the ball falls only force (mg) which is a conservative force acts ⇒ U decreases while K increases BUT  $K + U = E$  is constant.

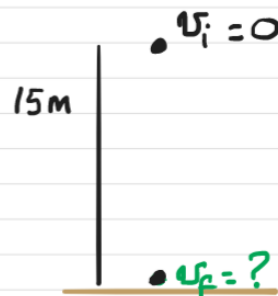
$E = K + U = \frac{1}{2}mv^2 + mgy$

$\Delta K + \Delta U = 0$

Example: An object of mass  $m$  is dropped from a height of 15 m above the earth's surface. Ignoring air resistance, Find:

i) Its speed just before hitting the ground.

Only the weight does work. It is a conservative force  $\Rightarrow$  total mechanical energy is conserved.



$$\Delta K + \Delta U = 0$$

For  $\Delta U$ : If object rises  $\Rightarrow \Delta U = +mgh$   $\uparrow h$   
 If object descends (falls)  $\Rightarrow \Delta U = -mgh$   $\downarrow h$

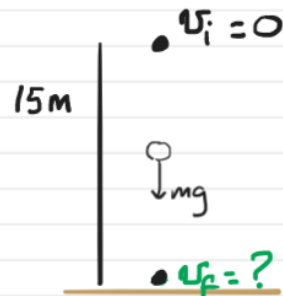
If object remains on the same level  $\Rightarrow \Delta U = 0$   $\text{---}$

$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}m(u_f^2 - (0)^2) - mg(15) = 0$$

$$\frac{1}{2}u_f^2 = 15(g)$$

$$\therefore u_f = \sqrt{30g} \sim 17.1 \text{ m/s}$$



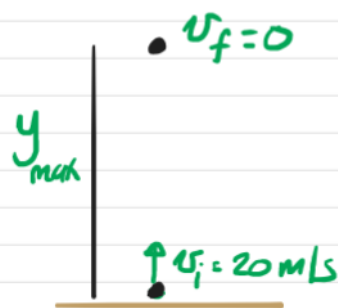


Example: A stone is projected vertically upwards from ground level with an initial speed of 20 m/s. Ignoring air resistance, find its maximum height.

Only the gravitational force acts  $\Rightarrow \Delta K + \Delta U = 0$

$$\frac{1}{2}m(0 - (20)^2) + mgy_{\max} = 0$$

$$\therefore y_{\max} = \frac{(20)^2}{2g} = 20 \text{ m.}$$



Example:

Find the speed of the of  $m_1$  when it has fallen a vertical distance of 2m. All surfaces are smooth.

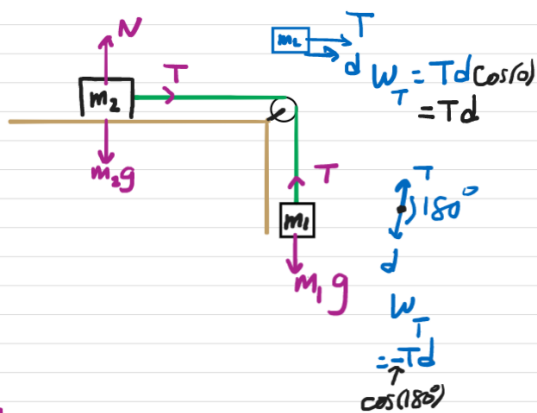
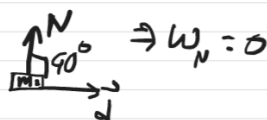
Assume system started from rest.

$$\Delta K + \Delta U = 0$$

$$\underbrace{(\Delta K_1 + \Delta U_1)}_{\text{for mass 1}} + \underbrace{(\Delta K_2 + \Delta U_2)}_{\text{for mass 2}} = 0$$

$$\frac{1}{2}m_1(v_{1f}^2 - v_{1i}^2) - mg(2) + \frac{1}{2}m_2(v_{2f}^2 - v_{2i}^2) + 0 = 0$$

$$\frac{1}{2}m_1(v_{1f}^2 - 0) - 2m_1g + \frac{1}{2}m_2 v_{2f}^2 = 0$$



$\uparrow$  Since  $m_2$  does NOT change height.

$m_1$  and  $m_2$  are connected by an inextensible string

$$\Rightarrow v_{1f} = v_{2f} \equiv v_f$$

$$\frac{1}{2} (m_1 + m_2) v_f^2 = 2 m_1 g$$

$$\therefore v_f^2 = \frac{4 m_1 g}{m_1 + m_2} \Rightarrow v_f = \sqrt{\frac{4 m_1 g}{m_1 + m_2}}$$

NOTE:  $T$  is a nonconservative force. But the work done by the vertical tension  $T$  is negative ( $T$  opposite to the downward displacement of  $m_1$ , i.e.  $W_T^{\text{vertical}} = Td \cos(180) = -Td$ )

while the tension in the horizontal spring does positive work as it is parallel to the displacement of  $m_2$  which is to right  $W_T^{\text{horizontal}} = Td \cos(0) = Td$

$\therefore$  Total work done by the tensions is  $Td + (-Td) = 0$ .