Chapter 6:Work and Energy

Lecture 4

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6-9] Energy conservation with dissipitave forces

We live in a world were nonconservative forces like friction, tension, applied forces do work. The work done by such nonconservative forces (W_{nc}) is taken into account using

$$\Delta K + \Delta U = W_{nc}$$

$$(K_f - K_i) + (U_f - U_i) = W_{nc}$$

$$(K_f + U_f) - (K_i + U_i) = W_{nc}$$

$$E_f - E_i = W_{nc}$$

$\therefore \quad \Delta E = W_{nc}$

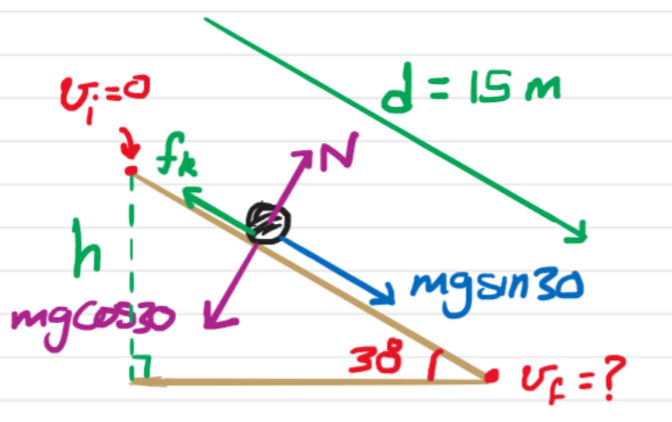
change in total mechanical energy

work done by nonconservative forces

$\therefore$ change in total mechanical energy equals the work by nonconservative forces.

Example: Starting from rest a skier slides down a 30° inclined plane a distance of 15 m. If the coefficient of kinetic friction is 0.1 find his speed at the bottom of the slide.

<u>Note</u> N and $mg\cos 30$ do No work.



$$\Delta K + \Delta U = W_{nc}$$

$$\frac{1}{2}m(v_f^2 - 0) - mg\overbrace{\underbrace{(d\sin 30)}}^{h} = W_{nc}$$

h: vertical distance desended by the skier

$$\frac{1}{2}m(v_f^2-0) - mg(\underbrace{d\sin 30}_{h}) = W_{nc}$$

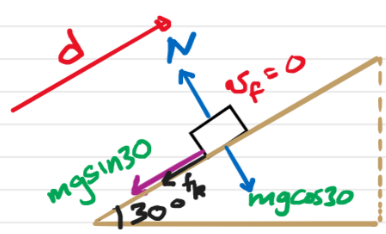
h: vertical distance desended by the skier

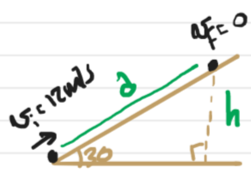
$$\frac{1}{2}\cancel{m}v_f^2 - \cancel{m}g(15\times\tfrac{1}{2}) = f_k\, d\cos 180^\circ$$
$$=(\mu_k \cancel{m} g\cos 30)(15)(-1)$$

$$\therefore\ v_f^2 = \frac{159}{2} - (0.1)(9)\left(\frac{\sqrt{3}}{2}\right)(15) \Rightarrow \underline{\underline{v_f}} \approx 7.8 \text{ m/s}.$$

<u>Example</u> A box of mass m is given an initial speed of 12 m/s up a 30° inclined plane. If the coefficient of kinetic friction between the box and the plane is $\mu_k = 0.15$ find the maximum distance the box moves up the inclined plane.

N and mg cos30 do No work.





$$\Delta K + \Delta U = W_{nc}$$

$$\frac{1}{2}m(v_f^2 - v_i^2) + mg(\overbrace{d\sin 30^\circ}^{h}) = f_k\, d\cos 180^\circ$$

$$\frac{1}{2}\cancel{m}(0-(12)^2) + \cancel{m}g\left(\frac{d}{2}\right) = (\mu_k \overbrace{\cancel{m}g\cos 30}^{N})(d)(-1)$$

$$-72 = -d(\mu_k g\cos 30 + \tfrac{1}{2}g) \Rightarrow d = 11.7 \text{ m}.$$

$h = d\sin\theta$
$= d\sin 30$
$h = \frac{1}{2}d$

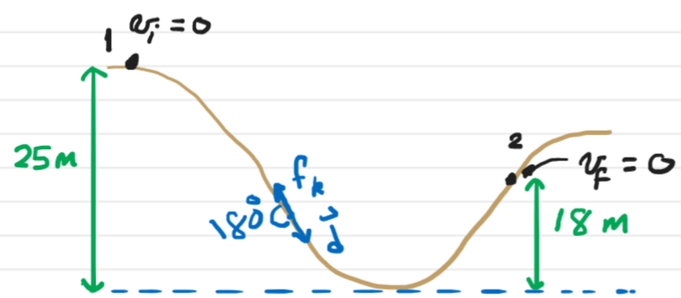
Example

A roller-coaster car of mass $m = 150$ kg starts from rest at point ① and slides down the roller-coaster. The car moves a dsitance of 60 m on the track before comming to rest at point ②. Find the magnitude of the force of kinetic friction f_k (Assume f_k to be constant)

$$\Delta K + \Delta U = W_{nc}$$

$$\tfrac{1}{2}m(v_2^2 - v_1^2) - mg(7) = f_k d \cos 180$$

$v_2^2 = v_f^2$, $v_1^2 = v_i^2$

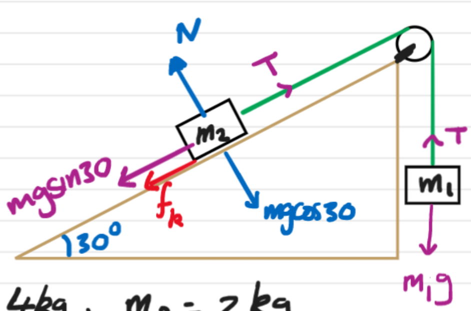


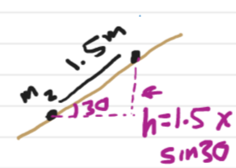
$$\tfrac{1}{2}m(0-0) - 7mg = -f_k(60)$$

$$\therefore f_k = \frac{7mg}{60} = \frac{(7)(150)(10)}{60} = 175 \text{ newtons.}$$

Example

In the figure μ_k between m_2 and the inclined plane is 0.2. Find the speed of m_1 after it has fallen a distance of 1.5 m. Assume system started from rest. Also $m_1 = 4$ kg, $m_2 = 2$ kg





$$\Delta K + \Delta U = W_{nc}$$

$$\left[\tfrac{1}{2}m_1(v_f^2 - v_i^2) - mg(1.5)\right] + \left[\tfrac{1}{2}m_2(v_f^2 - v_i^2) + mg(1.5\sin 30)\right] = f_k(1.5)\cos 180$$

Note # since m_1 and m_2 are connected by an inextensible string ⇒ the have the same speed.

when determining ΔU we only need the vertical distance $\underbrace{\text{ascended (for } m_2)}_{1.5\sin 30}$ or $\underbrace{\text{desended for } (m_1)}_{1.5\text{ m}}$

$$\frac{1}{2}(m_1+m_2)v_f^2 + 1.5g[m_2\sin 30 - m_1] = \mu_k m_2 g\cos 30 \times (-1)$$

$$3v_f^2 + [-34.02] = -3.46$$

$$\therefore \quad v_f \sim 3.2 \text{ m/s}.$$

6-10] Power (P)

Power : Work done per unit time (rate of doing work)

Average Power $\bar{P} = \dfrac{\text{Work done}}{\text{time taken}}$

Unit of power $[P] = \dfrac{J}{s} \equiv$ Watt (W)

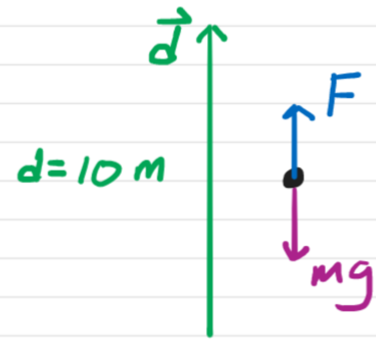
$\therefore$ 1 W = 1 J/s.

Another unit is the horsepower (hp)

1hp = 746 Watt .

Example

A 60-kg firefighter climbs a 10 m vertical wire (rop) in 10 seconds at constant speed. Find his average power output. ($\bar{P}$)



The firefighter exerts a force F as he climbs the rope.

$$\Rightarrow \bar{P} = \frac{W_F}{t} = \frac{Fd\cos(0)}{t} = \frac{Fd}{t}$$

How can we find F? He moves upwards at constant speed ($a_y = 0$).

$$\bar{P} = F\left(\frac{d}{t}\right) = Fv$$

Using Newton's second law

$$\uparrow^{+} \quad \Sigma F_y = ma_y \Rightarrow F - mg = m(0)$$

$$\therefore F = mg$$

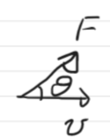
$$\bar{P} = \frac{W}{t}$$
$$W = \bar{P}t$$

$$\therefore \bar{P} = \frac{\overbrace{mg}^{F}d}{t} = \frac{(60)(10)(10)}{10} = 600\ W.$$

NOTE: using $\bar{P} = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv$

power of f_k $\rightarrow \bar{P} = f_k\, v \cos 180^\circ$
$= -f_k\, v$

$\bar{P} = Fv\cos\theta$ ONLY wheen speed is constant.
(θ: angle between $\vec{F}$ and $\vec{v}$.)



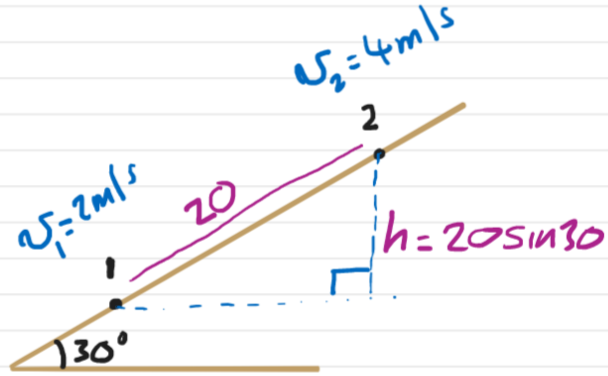
$\bar{P} = Fv\cos\theta$

$$\bar{P} = mg\left(\frac{10}{10}\right) = mg(1) = 60 \times 10 \times 1 = 600\ J$$

As before.

Example

A runner (mass = 50 kg) is going up a 30° inclined plane. At point 1 his speed is 2 m/s while at point 2 his speed is 4 m/s. The distance between points 1 and 2 is 20 m. He takes 10 s to move from point 1 → point 2.



If his mass is 55 kg find his average power output.

$$\bar{P} = \frac{W}{t}$$

How to find W?

Remember, the force of the runner is a nonconservative force ⇒

$$W = W_{nc} = \Delta K + \Delta U$$

$$= \underbrace{\tfrac{1}{2} m (v_f^2 - v_i^2)}_{} + mg(20 \sin 30)$$

when running at constant speed this term = 0 as in the above example of the fire fighter.

$\therefore$

$$W = W_{nc} = \frac{1}{2}(50)[16-4] + (60)(10)\underbrace{(20\sin 30)}_{\text{vertical distance ascended}}$$

$$= 5300\ J$$

$$\Rightarrow \bar{P} = \frac{5300}{10} = 530\ \text{Watt}$$

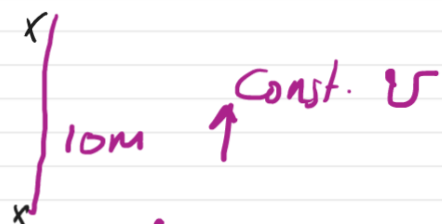
i.e $530\ J/s$.

$$W_F = W_{nc} = \Delta K + \Delta U$$

$$= 0 + mg(10)$$

$$= 60 \times 10 \times 10 = 6000\ J.$$

For fire fighter



$$\bar{P} = \frac{W_F}{t} = \frac{6000}{10} = \underline{600}\ \text{Watt as before.}$$

$$\bar{P} = F\,v\cos\theta$$

$$= F\frac{d}{t}\cos\theta$$