Chapter 7:Linear Momentum

Lecture 1
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7-8] Center of Mass (CM)

A box which has dimensions
We call it an extended object

- The dimensions of this of this object are very small ⇒ can treat it as a point ⇒ call it point object.

In the previous chapters we used to draw a free-body diagram were forces act on objects. For a point object it is easy to draw a force that acts on it like: $\bullet\!\longrightarrow F$

For an extended object like the box, at which part of the box should the force act?

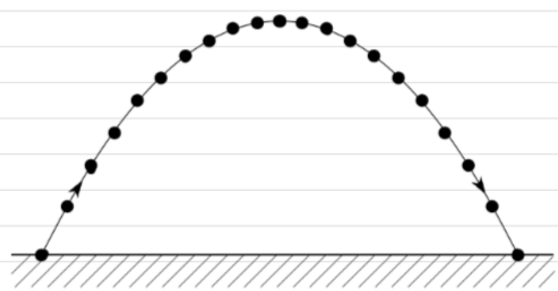
In all our free-body diagrams, we assumed a force to act on the center of mass (CM) of the box.

to act on the center of mass (CM) of the box.

How do we define the center of mass?

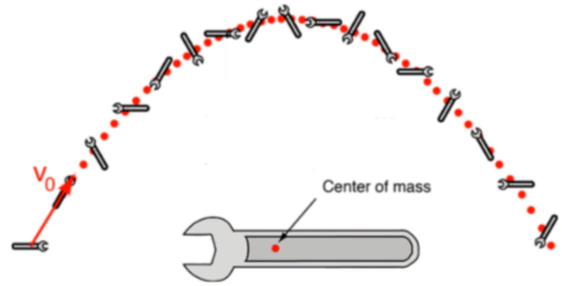
If you through a small metal sphere with an initial velocity that makes an angle $\theta > 0$ with the horizontal, it follows the following trajectory (path):

●: point object



If we through an extended objet like a wrench how does it move?

Which point of the wrench follows the same path?
It is the point of the center of mass.



The wrench behaves as if all its mass is concentrated at the point of center of mass.

How do we determine the position of center of mass?

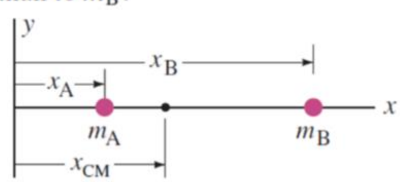
(i) system made up of two particles in one dimension

Both masses lie along the x-axis $\Rightarrow$

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{m_A x_A + m_B x_B}{M}$$

where $M = m_A + m_B$ is the total mass of the two particle system.

FIGURE 7–22 The center of mass of a two-particle system lies on the line joining the two masses. Here $m_A > m_B$, so the CM is closer to m_A than to m_B.



Example: $m_A = 6\,kg$, $m_B = 2\,kg$, $x_A = 2\,m$, $x_B = 4\,m$

Find the position of the center of mass.

$$x_{CM} = \frac{6 \times 2 + 2 \times 4}{10} = 2\,m$$

Clearly, the position of the CM is closer to the heavier mass, which is the 6 kg one.

Example: Find x_{CM} if $m_A = m_B$

$$x_{CM} = \frac{m\,x_A + m\,x_B}{m + m} = \frac{1}{2}(x_A + x_B)$$

→ x_{CM} is half way between the two equal masses.

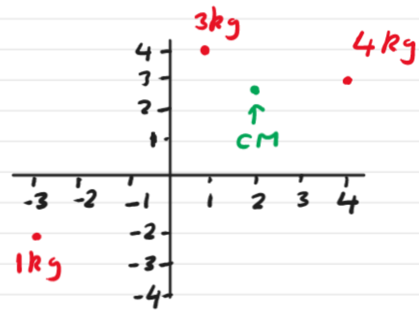
(ii) System made up of more than two particles in one dimension

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_c x_c + \cdots}{m_A + m_B + m_c + \cdots} = \frac{m_A x_A + m_B x_B + m_c x_c + \cdots}{M}$$

(iii) System particles in two dimensions

$$x_{CM} = \frac{1(-3) + 3(1) + 4(4)}{8} = \frac{16}{8} = 2\,m$$

$$y_{CM} = \frac{1(-2) + 3(4) + 4(3)}{8} = \frac{22}{8} = 2.75\,m$$



(iv) Symmetrical Extended Objects

For such objets, the position of CM is at the geometric center of the object.

Uniform ruler
CM in the middle.

Circular ring

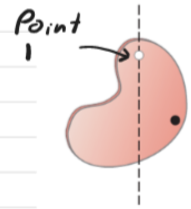
CM is at the center of the ring.

NOTE: the CM may be outside the mass distribution of the object, like this ring where the CM lies in the middle where there is no mass of the ring

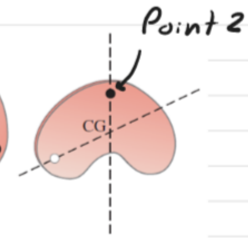
Determining the CM experimentally

① Suspend the object using a string tied at point 1 and draw a vertical line that passes through point of suspension (point 1) as shown in Fig(1)

② suspend the object using a string tied at point 2 and draw a vertical line that passes through point of suspension (point 2) as shown in Fig(2)



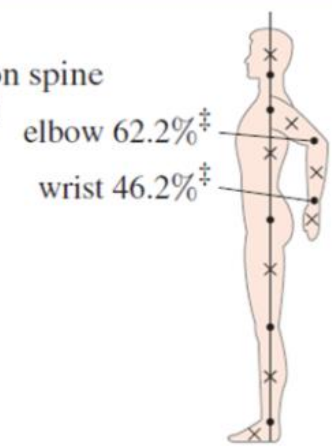
Fig(1)



Fig(2)

TABLE 7–1 Center of Mass of Parts of Typical Human Body, given as %
(full height and mass = 100 units)

Distance of Hinge Points from Floor (%)	Hinge Points (•) (Joints)	Center of Mass (×) (% Height Above Floor)		Percent Mass
91.2%	Base of skull on spine	Head	93.5%	6.9%
81.2%	Shoulder joint	Trunk and neck	71.1%	46.1%
		Upper arms	71.7%	6.6%
		Lower arms	55.3%	4.2%
52.1%	Hip joint	Hands	43.1%	1.7%
		Upper legs (thighs)	42.5%	21.5%
28.5%	Knee joint			
		Lower legs	18.2%	9.6%
4.0%	Ankle joint	Feet	1.8%	3.4%
		Body CM =	58.0%	100.0%

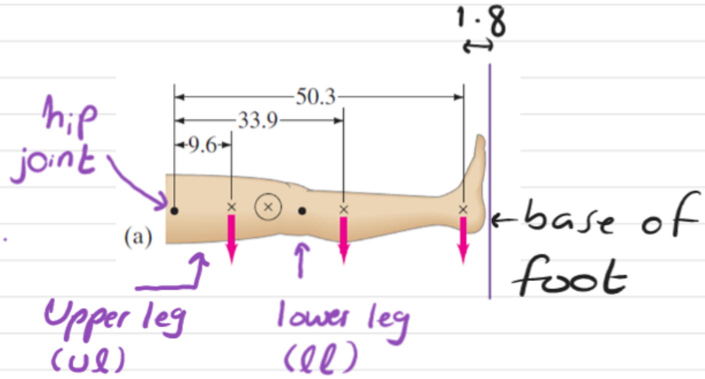


Example: CM of the leg.

Determine the center of mass of a whole leg in two cases

(a) When streached out.

(b) when bent at 90°.



a) When leg is streached

This is a one-dimensional problem

$$x_{CM} = \frac{m_{ul}\, x_{ul} + m_{ll}\, x_{ll} + m_f\, x_f}{m_{ul} + m_{ll} + m_f}$$

$$= \frac{(21.5)(9.6) + (9.6)(33.9) + (3.4)(50.3)}{21.5 + 9.6 + 3.4}$$

$= 20.4$ units from the hip

as shown by ⊗ in fig (a)

$= 52.1 - 20.4 = 31.7$ units from the base of the foot.

The person is 1.7 m tall ⇒

$\frac{31.7}{100} \times 1.7\text{ m} = 0.54\text{ m}$ above the base of the foot.

b] When leg is bent at 90°.

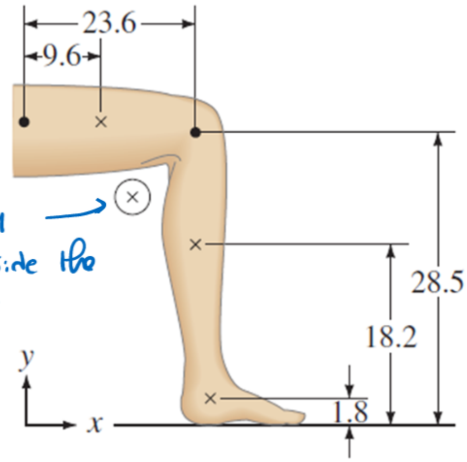
This is a two-dimensional problem

$$x_{CM} = \frac{(21.5)(9.6) + (9.6)(23.6) + (3.4)(23.6)}{21.5 + 9.6 + 3.4}$$

$= 14.9$ units.

$= \frac{14.9}{100} \times 1.7 \text{ m} = 0.25 \text{ m}$

CM outside the leg.



(b)

$$y_{CM} = \frac{(21.5)(28.5) + (9.6)(18.2) + (3.4)(1.8)}{21.5 + 9.6 + 3.4}$$

$= 23$ units $= \frac{23}{100} \times 1.7 = 0.39 \text{ m}$

Knowing the CM of the body when it is in various positions is of great use in studying body mechanics. One simple example from athletics is shown in Fig. 7–27. If high jumpers can get into the position shown, their CM can pass below the bar which their bodies go over, meaning that for a particular takeoff speed, they can clear a higher bar. This is indeed what they try to do.

FIGURE 7–27 A high jumper's CM may actually pass beneath the bar.