1. A bicycle odometer (which counts revolutions and is calibrated to report distance traveled) is attached near the wheel axle and is calibrated for 27 -inch wheels. What happens if you use it on a bicycle with 24 -inch wheels?
2. Suppose a disk rotates at constant angular velocity. (a) Does a point on the rim have radial and/or tangential acceleration? (b) If the disk's angular velocity increases uniformly, does the point have radial and/or tangential acceleration? (c) For which cases would the magnitude of either component of linear acceleration change?
3. Can a small force ever exert a greater torque than a larger force? Explain.
4. Why is it more difficult to do a sit-up with your hands behind your head than when your arms are stretched out in front of you? A diagram may help you to answer this.
5. If the net force on a system is zero, is the net torque also zero? If the net torque on a system is zero, is the net force zero? Explain and give examples.
6. Mammals that depend on being able to run fast have slender lower legs with flesh and muscle concentrated high, close to the body (Fig. 8-33). On the basis of rotational dynamics, explain why this distribution of mass is advantageous.

FIGURE 8-33
Question 6.
A gazelle.

7. This book has three symmetry axes through its center, all mutually perpendicular. The book's moment of inertia would be smallest about which of the three? Explain.
8. Can the mass of a rigid object be considered concentrated at its CM for rotational motion? Explain.
9. The moment of inertia of a rotating solid disk about an axis through its CM is $\frac{1}{2} M R^{2}$ (Fig. 8-20c). Suppose instead that a parallel axis of rotation passes through a point on the edge of the disk. Will the moment of inertia be the same, larger, or smaller? Explain why.
10. Two inclines have the same height but make different angles with the horizontal. The same steel ball rolls without slipping down each incline. On which incline will the speed of the ball at the bottom be greater? Explain.
11. Two spheres look identical and have the same mass. However, one is hollow and the other is solid. Describe an experiment to determine which is which.
12. A sphere and a cylinder have the same radius and the same mass. They start from rest at the top of an incline. (a) Which reaches the bottom first? (b) Which has the greater speed at the bottom? (c) Which has the greater total kinetic energy at the bottom? (d) Which has the greater rotational kinetic energy? Explain your answers.
13. Why do tightrope walkers (Fig. 8-34) carry a long, narrow rod?


FIGURE 8-34 Question 13.
14. We claim that momentum and angular momentum are conserved. Yet most moving or rotating objects eventually slow down and stop. Explain.
15. Can the diver of Fig. 8-28 do a somersault without having any initial rotation when she leaves the board? Explain.
16. When a motorcyclist leaves the ground on a jump and leaves the throttle on (so the rear wheel spins), why does the front of the cycle rise up?
17. A shortstop may leap into the air to catch a ball and throw it quickly. As he throws the ball, the upper part of his body rotates. If you look quickly you will notice that his hips and legs rotate in the opposite direction (Fig. 8-35). Explain.

FIGURE 8-35 Question 17.
A shortstop in the air, throwing the ball.

*18. The angular velocity of a wheel rotating on a horizontal axle points west. In what direction is the linear velocity of a point on the top of the wheel? If the angular acceleration points east, describe the tangential linear acceleration of this point at the top of the wheel. Is the angular speed increasing or decreasing?
*19. In what direction is the Earth's angular velocity vector as it rotates daily about its axis, north or south?
*20. On the basis of the law of conservation of angular momentum, discuss why a helicopter must have more than one rotor (or propeller). Discuss one or more ways the second propeller can operate in order to keep the helicopter stable.

## MisConceptual Questions

1. Bonnie sits on the outer rim of a merry-go-round, and Jill sits midway between the center and the rim. The merry-go-round makes one complete revolution every 2 seconds. Jill's linear velocity is:
(a) the same as Bonnie's.
(b) twice Bonnie's.
(c) half of Bonnie's.
(d) one-quarter of Bonnie's.
(e) four times Bonnie's.
2. An object at rest begins to rotate with a constant angular acceleration. If this object rotates through an angle $\theta$ in time $t$, through what angle did it rotate in the time $\frac{1}{2} t$ ?
(a) $\frac{1}{2} \theta$.
(b) $\frac{1}{4} \theta$.
(c) $\theta$.
(d) $2 \theta$.
(e) $4 \theta$.
3. A car speedometer that is supposed to read the linear speed of the car uses a device that actually measures the angular speed of the tires. If larger-diameter tires are mounted on the car instead, how will that affect the speedometer reading? The speedometer
(a) will still read the speed accurately.
(b) will read low.
(c) will read high.
4. The solid dot shown in Fig. $8-36$ is a pivot point. The board can rotate about the pivot. Which force shown exerts the largest magnitude torque on the board?

(a) 1000 N

FIGURE 8-36 MisConceptual Question 4.
5. Consider a force $F=80 \mathrm{~N}$ applied to a beam as shown in Fig. 8-37. The length of the beam is $\ell=5.0 \mathrm{~m}$, and $\theta=37^{\circ}$, so that $x=3.0 \mathrm{~m}$ and $y=4.0 \mathrm{~m}$. Of the following expressions, which ones give the correct torque produced by the force $\overrightarrow{\mathbf{F}}$ around point P ?
(a) 80 N .
(b) $(80 \mathrm{~N})(5.0 \mathrm{~m})$.
(c) $(80 \mathrm{~N})(5.0 \mathrm{~m})\left(\sin 37^{\circ}\right)$.
(d) $(80 \mathrm{~N})(4.0 \mathrm{~m})$.
(e) $(80 \mathrm{~N})(3.0 \mathrm{~m})$.
(f) $(48 \mathrm{~N})(5.0 \mathrm{~m})$.
$(g)(48 \mathrm{~N})(4.0 \mathrm{~m})\left(\sin 37^{\circ}\right)$.


FIGURE 8-37
MisConceptual Question 5.
6. Two spheres have the same radius and equal mass. One sphere is solid, and the other is hollow and made of a denser material. Which one has the bigger moment of inertia about an axis through its center?
(a) The solid one.
(b) The hollow one.
(c) Both the same.
7. Two wheels having the same radius and mass rotate at the same angular velocity (Fig. 8-38). One wheel is made with spokes so nearly all the mass is at the rim. The other is a solid disk. How do their rotational kinetic energies compare?
(a) They are nearly the same.
(b) The wheel with spokes has about twice the KE.
(c) The wheel with spokes has higher KE, but not twice as high.
(d) The solid wheel has about twice the KE.
(e) The solid wheel has higher KE, but not twice as high.

FIGURE 8-38
MisConceptual Question 7.

8. If you used 1000 J of energy to throw a ball, would it travel faster if you threw the ball (ignoring air resistance)
(a) so that it was also rotating?
(b) so that it wasn't rotating?
(c) It makes no difference.
9. A small solid sphere and a small thin hoop are rolling along a horizontal surface with the same translational speed when they encounter a $20^{\circ}$ rising slope. If these two objects roll up the slope without slipping, which will rise farther up the slope?
(a) The sphere.
(b) The hoop.
(c) Both the same.
(d) More information about the objects' mass and diameter is needed.
10. A small mass $m$ on a string is rotating without friction in a circle. The string is shortened by pulling it through the axis of rotation without any external torque, Fig. 8-39. What happens to the angular velocity of the object?
(a) It increases.
(b) It decreases.
(c) It remains the same.

FIGURE 8-39
MisConceptual Questions 10 and 11.

11. A small mass $m$ on a string is rotating without friction in a circle. The string is shortened by pulling it through the axis of rotation without any external torque, Fig. 8-39. What happens to the tangential velocity of the object?
(a) It increases.
(b) It decreases.
(c) It remains the same.
12. If there were a great migration of people toward the Earth's equator, the length of the day would
(a) increase because of conservation of angular momentum.
(b) decrease because of conservation of angular momentum.
(c) decrease because of conservation of energy.
(d) increase because of conservation of energy.
(e) remain unaffected.
13. Suppose you are sitting on a rotating stool holding a $2-\mathrm{kg}$ mass in each outstretched hand. If you suddenly drop the masses, your angular velocity will
(a) increase.
(b) decrease.
(c) stay the same.

For assigned homework and other learning materials, go to the MasteringPhysics website.

## Problems

## 8-1 Angular Quantities

1. (I) Express the following angles in radians: (a) $45.0^{\circ}$, (b) $60.0^{\circ}$, (c) $90.0^{\circ}$, (d) $360.0^{\circ}$, and (e) $445^{\circ}$. Give as numerical values and as fractions of $\pi$.
2. (I) The Sun subtends an angle of about $0.5^{\circ}$ to us on Earth, 150 million km away. Estimate the radius of the Sun.
3. (I) A laser beam is directed at the Moon, $380,000 \mathrm{~km}$ from Earth. The beam diverges at an angle $\theta$ (Fig. 8-40) of $1.4 \times 10^{-5} \mathrm{rad}$. What diameter spot will

4. (I) The blades in a blender rotate at a rate of 6500 rpm . When the motor is turned off during operation, the blades slow to rest in 4.0 s . What is the angular acceleration as the blades slow down?
5. (II) The platter of the hard drive of a computer rotates at $7200 \mathrm{rpm}(\mathrm{rpm}=$ revolutions per minute $=\mathrm{rev} / \mathrm{min})$. (a) What is the angular velocity ( $\mathrm{rad} / \mathrm{s}$ ) of the platter? (b) If the reading head of the drive is located 3.00 cm from the rotation axis, what is the linear speed of the point on the platter just below it? (c) If a single bit requires $0.50 \mu \mathrm{~m}$ of length along the direction of motion, how many bits per second can the writing head write when it is 3.00 cm from the axis?
6. (II) A child rolls a ball on a level floor 3.5 m to another child. If the ball makes 12.0 revolutions, what is its diameter?
7. (II) (a) A grinding wheel 0.35 m in diameter rotates at 2200 rpm . Calculate its angular velocity in rad/s. (b) What are the linear speed and acceleration of a point on the edge of the grinding wheel?
8. (II) A bicycle with tires 68 cm in diameter travels 9.2 km . How many revolutions do the wheels make?
9. (II) Calculate the angular velocity (a) of a clock's second hand, $(b)$ its minute hand, and $(c)$ its hour hand. State in rad/s. (d) What is the angular acceleration in each case?
10. (II) A rotating merry-go-round makes one complete revolution in 4.0 s (Fig. 8-41). (a) What is the linear speed of a child seated 1.2 m from the center? (b) What is her acceleration (give components)?

FIGURE 8-41
Problem 10.

11. (II) What is the linear speed, due to the Earth's rotation, of a point $(a)$ on the equator, $(b)$ on the Arctic Circle (latitude $66.5^{\circ} \mathrm{N}$ ), and (c) at a latitude of $42.0^{\circ} \mathrm{N}$ ?
12. (II) Calculate the angular velocity of the Earth (a) in its orbit around the Sun, and (b) about its axis.
13. (II) How fast (in rpm) must a centrifuge rotate if a particle 8.0 cm from the axis of rotation is to experience an acceleration of $100,000 \mathrm{~g}$ 's?
14. (II) A $61-\mathrm{cm}$-diameter wheel accelerates uniformly about its center from 120 rpm to 280 rpm in 4.0 s . Determine (a) its angular acceleration, and (b) the radial and tangential components of the linear acceleration of a point on the edge of the wheel 2.0 s after it has started accelerating.
15. (II) In traveling to the Moon, astronauts aboard the Apollo spacecraft put the spacecraft into a slow rotation to distribute the Sun's energy evenly (so one side would not become too hot). At the start of their trip, they accelerated from no rotation to 1.0 revolution every minute during a $12-\mathrm{min}$ time interval. Think of the spacecraft as a cylinder with a diameter of 8.5 m rotating about its cylindrical axis. Determine ( $a$ ) the angular acceleration, and (b) the radial and tangential components of the linear acceleration of a point on the skin of the ship 6.0 min after it started this acceleration.
16. (II) A turntable of radius $R_{1}$ is turned by a circular rubber roller of radius $R_{2}$ in contact with it at their outer edges. What is the ratio of their angular velocities, $\omega_{1} / \omega_{2}$ ?

## 8-2 and 8-3 Constant Angular Acceleration; Rolling

17. (I) An automobile engine slows down from 3500 rpm to 1200 rpm in 2.5 s . Calculate (a) its angular acceleration, assumed constant, and (b) the total number of revolutions the engine makes in this time.
18. (I) A centrifuge accelerates uniformly from rest to $15,000 \mathrm{rpm}$ in 240 s. Through how many revolutions did it turn in this time?
19. (I) Pilots can be tested for the stresses of flying high-speed jets in a whirling "human centrifuge," which takes 1.0 min to turn through 23 complete revolutions before reaching its final speed. (a) What was its angular acceleration (assumed constant), and (b) what was its final angular speed in rpm?
20. (II) A cooling fan is turned off when it is running at $850 \mathrm{rev} / \mathrm{min}$. It turns 1250 revolutions before it comes to a stop. (a) What was the fan's angular acceleration, assumed constant? (b) How long did it take the fan to come to a complete stop?
21. (II) A wheel 31 cm in diameter accelerates uniformly from 240 rpm to 360 rpm in 6.8 s . How far will a point on the edge of the wheel have traveled in this time?
22. (II) The tires of a car make 75 revolutions as the car reduces its speed uniformly from $95 \mathrm{~km} / \mathrm{h}$ to $55 \mathrm{~km} / \mathrm{h}$. The tires have a diameter of 0.80 m . (a) What was the angular acceleration of the tires? If the car continues to decelerate at this rate, (b) how much more time is required for it to stop, and (c) how far does it go?
23. (II) A small rubber wheel is used to drive a large pottery wheel. The two wheels are mounted so that their circular edges touch. The small wheel has a radius of 2.0 cm and accelerates at the rate of $7.2 \mathrm{rad} / \mathrm{s}^{2}$, and it is in contact with the pottery wheel (radius 27.0 cm ) without slipping. Calculate (a) the angular acceleration of the pottery wheel, and (b) the time it takes the pottery wheel to reach its required speed of 65 rpm .

## 8-4 Torque

24. (I) A 52-kg person riding a bike puts all her weight on each pedal when climbing a hill. The pedals rotate in a circle of radius 17 cm . (a) What is the maximum torque she exerts? (b) How could she exert more torque?
25. (II) Calculate the net torque about the axle of the wheel shown in Fig. 8-42. Assume that a friction torque of $0.60 \mathrm{~m} \cdot \mathrm{~N}$ opposes the motion.


FIGURE 8-42 Problem 25.
26. (II) A person exerts a horizontal force of 42 N on the end of a door 96 cm wide. What is the magnitude of the torque if the force is exerted (a) perpendicular to the door and (b) at a $60.0^{\circ}$ angle to the face of the door?
27. (II) Two blocks, each of mass $m$, are attached to the ends of a massless rod which pivots as shown in Fig. 8-43. Initially the rod is held in the horizontal position and then released. Calculate the magnitude and direction of the net torque on this system when it is first released.


FIGURE 8-43 Problem 27.
28. (II) The bolts on the cylinder head of an engine require tightening to a torque of $95 \mathrm{~m} \cdot \mathrm{~N}$. If a wrench is 28 cm long, what force perpendicular to the wrench must the mechanic exert at its end? If the six-sided bolt head is 15 mm across (Fig. 8-44), estimate the force applied near each of the six points by a wrench.


FIGURE 8-44 Problem 28.
29. (II) Determine the net torque on the $2.0-\mathrm{m}$-long uniform beam shown in Fig. 8-45. All forces are shown. Calculate about (a) point C, the CM, and $(b)$ point P at one end.

FIGURE 8-45
Problem 29.


## 8-5 and 8-6 Rotational Dynamics

30. (I) Determine the moment of inertia of a $10.8-\mathrm{kg}$ sphere of radius 0.648 m when the axis of rotation is through its center.
31. (I) Estimate the moment of inertia of a bicycle wheel 67 cm in diameter. The rim and tire have a combined mass of 1.1 kg . The mass of the hub (at the center) can be ignored (why?).
32. (II) A merry-go-round accelerates from rest to $0.68 \mathrm{rad} / \mathrm{s}$ in 34 s . Assuming the merry-go-round is a uniform disk of radius 7.0 m and mass $31,000 \mathrm{~kg}$, calculate the net torque required to accelerate it.
33. (II) An oxygen molecule consists of two oxygen atoms whose total mass is $5.3 \times 10^{-26} \mathrm{~kg}$ and whose moment of inertia about an axis perpendicular to the line joining the two atoms, midway between them, is $1.9 \times 10^{-46} \mathrm{~kg} \cdot \mathrm{~m}^{2}$. From these data, estimate the effective distance between the atoms.
34. (II) A grinding wheel is a uniform cylinder with a radius of 8.50 cm and a mass of 0.380 kg . Calculate (a) its moment of inertia about its center, and $(b)$ the applied torque needed to accelerate it from rest to 1750 rpm in 5.00 s . Take into account a frictional torque that has been measured to slow down the wheel from 1500 rpm to rest in 55.0 s .
35. (II) The forearm in Fig. 8-46 accelerates a $3.6-\mathrm{kg}$ ball at $7.0 \mathrm{~m} / \mathrm{s}^{2}$ by means of the triceps muscle, as shown. Calculate (a) the torque needed, and (b) the force that must be exerted by the triceps muscle. Ignore the mass of the arm.
36. (II) Assume that a $1.00-\mathrm{kg}$ ball is thrown solely by the action of the forearm, which rotates about the elbow joint under the action of the triceps muscle, Fig. 8-46. The ball is accelerated uniformly from rest to $8.5 \mathrm{~m} / \mathrm{s}$ in 0.38 s , at which point it is released. Calculate (a) the angular acceleration of the arm, and $(b)$ the force required of the triceps muscle. Assume that the forearm has a mass of 3.7 kg and rotates like a uniform rod about an axis at its end.
37. (II) A softball player swings a bat, accelerating it from rest to $2.6 \mathrm{rev} / \mathrm{s}$ in a time of 0.20 s . Approximate the bat as a $0.90-\mathrm{kg}$ uniform rod of length 0.95 m , and compute the torque the player applies to one end of it.
38. (II) A small 350 -gram ball on the end of a thin, light rod is rotated in a horizontal circle of radius 1.2 m . Calculate (a) the moment of inertia of the ball about the center of the circle, and (b) the torque needed to keep the ball rotating at constant angular velocity if air resistance exerts a force of 0.020 N on the ball. Ignore air resistance on the rod and its moment of inertia.
39. (II) Calculate the moment of inertia of the array of point objects shown in Fig. 8-47 about (a) the $y$ axis, and (b) the $x$ axis. Assume $m=2.2 \mathrm{~kg}, M=3.4 \mathrm{~kg}$, and the objects are wired together by very light, rigid pieces of wire. The array is rectangular and is split through the middle by the $x$ axis. (c) About which axis would it be harder to accelerate this array?


FIGURE 8-47 Problem 39.
40. (II) A potter is shaping a bowl on a potter's wheel rotating at constant angular velocity of $1.6 \mathrm{rev} / \mathrm{s}$ (Fig. 8-48). The friction force between her hands and the clay is 1.5 N total. (a) How large is her torque on the wheel, if the diameter of the bowl is 9.0 cm ? (b) How long would it take for the potter's wheel to stop if the only torque acting on it is due to the potter's hands? The moment of inertia of the wheel and the bowl is $0.11 \mathrm{~kg} \cdot \mathrm{~m}^{2}$.

FIGURE 8-48
Problem 40.

41. (II) A 0.72 -m-diameter solid sphere can be rotated about an axis through its center by a torque of $10.8 \mathrm{~m} \cdot \mathrm{~N}$ which accelerates it uniformly from rest through a total of 160 revolutions in 15.0 s . What is the mass of the sphere?
42. (II) Let us treat a helicopter rotor blade as a long thin rod, as shown in Fig. 8-49. (a) If each of the three rotor helicopter blades is 3.75 m long and has a mass of 135 kg , calculate the moment of inertia of the three rotor blades about the axis of rotation. (b) How much torque must the motor apply to bring the blades from rest up to a speed of $6.0 \mathrm{rev} / \mathrm{s}$ in 8.0 s ?

FGURE 8-49
Problem 42.

43. (II) To get a flat, uniform cylindrical satellite spinning at the correct rate, engineers fire four tangential rockets as shown in Fig. 8-50. Suppose that the satellite has a mass of 3600 kg and a radius of 4.0 m , and that the rockets each add a mass of 250 kg . What is the steady force required of each rocket if the satellite is to reach 32 rpm in 5.0 min , starting from rest?

FIGURE 8-50
Problem 43.

44. (III) Two blocks are connected by a light string passing over a pulley of radius 0.15 m and moment of inertia $I$. The blocks move (towards the right) with an acceleration of $1.00 \mathrm{~m} / \mathrm{s}^{2}$ along their frictionless inclines (see Fig. 8-51). (a) Draw free-body diagrams for each of the two blocks and the pulley. (b) Determine $F_{\mathrm{TA}}$ and $F_{\mathrm{TB}}$, the tensions in the two parts of the string. (c) Find the net torque acting on the pulley, and determine its moment of inertia, $I$.


FIGURE 8-51 Problem 44.
45. (III) An Atwood machine consists of two masses, $m_{\mathrm{A}}=65 \mathrm{~kg}$ and $m_{\mathrm{B}}=75 \mathrm{~kg}$, connected by a massless inelastic cord that passes over a pulley free to rotate, Fig. 8-52. The pulley is a solid cylinder of radius $R=0.45 \mathrm{~m}$ and mass 6.0 kg . (a) Determine the acceleration of each mass. (b) What \% error would be made if the moment of inertia of the pulley is ignored? [Hint: The tensions $F_{\mathrm{TA}}$ and $F_{\mathrm{TB}}$ are not equal. We discussed the Atwood machine in Example 4-13, assuming $I=0$ for the pulley.]

FIGURE 8-52 Problem 45.
Atwood machine.

## 8-7 Rotational Kinetic Energy

46. (I) An automobile engine develops a torque of $265 \mathrm{~m} \cdot \mathrm{~N}$ at 3350 rpm . What is the horsepower of the engine?
47. (I) A centrifuge rotor has a moment of inertia of $3.25 \times 10^{-2} \mathrm{~kg} \cdot \mathrm{~m}^{2}$. How much energy is required to bring it from rest to 8750 rpm ?
48. (I) Calculate the translational speed of a cylinder when it reaches the foot of an incline 7.20 m high. Assume it starts from rest and rolls without slipping.
49. (II) A bowling ball of mass 7.25 kg and radius 10.8 cm rolls without slipping down a lane at $3.10 \mathrm{~m} / \mathrm{s}$. Calculate its total kinetic energy.
50. (II) A merry-go-round has a mass of 1440 kg and a radius of 7.50 m . How much net work is required to accelerate it from rest to a rotation rate of 1.00 revolution per 7.00 s ? Assume it is a solid cylinder.
51. (II) A ball of radius $r$ rolls on the inside of a track of radius $R$ (see Fig. 8-53). If the ball starts from rest at the vertical edge of the track, what will be its speed when it reaches the lowest point of the track, rolling without slipping?


FIGURE 8-53 Problem 51.
52. (II) A rotating uniform cylindrical platform of mass 220 kg and radius 5.5 m slows down from $3.8 \mathrm{rev} / \mathrm{s}$ to rest in 16 s when the driving motor is disconnected. Estimate the power output of the motor (hp) required to maintain a steady speed of $3.8 \mathrm{rev} / \mathrm{s}$.
53. (II) Two masses, $m_{\mathrm{A}}=32.0 \mathrm{~kg}$ and $m_{\mathrm{B}}=38.0 \mathrm{~kg}$, are connected by a rope that hangs over a pulley (as in Fig. 8-54). The pulley is a uniform cylinder of radius $R=0.311 \mathrm{~m}$ and mass 3.1 kg . Initially $m_{\mathrm{A}}$ is on the ground and $m_{\mathrm{B}}$ rests 2.5 m above the ground. If the system is released, use conservation of energy to determine the speed of $m_{\mathrm{B}}$ just before it strikes the ground. Assume the pulley bearing is frictionless.

FIGURE 8-54
Problem 53.
54. (III) A $1.80-\mathrm{m}$-long pole is balanced vertically with its tip on the ground. It starts to fall and its lower end does not slip. What will be the speed of the upper end of the pole just before it hits the ground? [Hint: Use conservation of energy.]

## 8-8 Angular Momentum

55. (I) What is the angular momentum of a $0.270-\mathrm{kg}$ ball revolving on the end of a thin string in a circle of radius 1.35 m at an angular speed of $10.4 \mathrm{rad} / \mathrm{s}$ ?
56. (I) (a) What is the angular momentum of a $2.8-\mathrm{kg}$ uniform cylindrical grinding wheel of radius 28 cm when rotating at 1300 rpm ? (b) How much torque is required to stop it in 6.0 s?
57. (II) A person stands, hands at his side, on a platform that is rotating at a rate of $0.90 \mathrm{rev} / \mathrm{s}$. If he raises his arms to a horizontal position, Fig. 8-55, the speed of rotation decreases to $0.60 \mathrm{rev} / \mathrm{s}$. (a) Why? (b) By what factor has his moment of inertia changed?

FIGURE 8-55
Problem 57.

58. (II) A nonrotating cylindrical disk of moment of inertia $I$ is dropped onto an identical disk rotating at angular speed $\omega$. Assuming no external torques, what is the final common angular speed of the two disks?
59. (II) A diver (such as the one shown in Fig. 8-28) can reduce her moment of inertia by a factor of about 3.5 when changing from the straight position to the tuck position. If she makes 2.0 rotations in 1.5 s when in the tuck position, what is her angular speed (rev/s) when in the straight position?
60. (II) A figure skater can increase her spin rotation rate from an initial rate of 1.0 rev every 1.5 s to a final rate of $2.5 \mathrm{rev} / \mathrm{s}$. If her initial moment of inertia was $4.6 \mathrm{~kg} \cdot \mathrm{~m}^{2}$, what is her final moment of inertia? How does she physically accomplish this change?
61. (II) (a) What is the angular momentum of a figure skater spinning at $3.0 \mathrm{rev} / \mathrm{s}$ with arms in close to her body, assuming her to be a uniform cylinder with a height of 1.5 m , a radius of 15 cm , and a mass of 48 kg ? (b) How much torque is required to slow her to a stop in 4.0 s , assuming she does not move her arms?
62. (II) A person of mass 75 kg stands at the center of a rotating merry-go-round platform of radius 3.0 m and moment of inertia $820 \mathrm{~kg} \cdot \mathrm{~m}^{2}$. The platform rotates without friction with angular velocity $0.95 \mathrm{rad} / \mathrm{s}$. The person walks radially to the edge of the platform. (a) Calculate the angular velocity when the person reaches the edge. (b) Calculate the rotational kinetic energy of the system of platform plus person before and after the person's walk.
63. (II) A 4.2-m-diameter merry-go-round is rotating freely with an angular velocity of $0.80 \mathrm{rad} / \mathrm{s}$. Its total moment of inertia is $1360 \mathrm{~kg} \cdot \mathrm{~m}^{2}$. Four people standing on the ground, each of mass 65 kg , suddenly step onto the edge of the merry-go-round. (a) What is the angular velocity of the merry-go-round now? (b) What if the people were on it initially and then jumped off in a radial direction (relative to the merry-go-round)?
64. (II) A uniform horizontal rod of mass $M$ and length $\ell$ rotates with angular velocity $\omega$ about a vertical axis through its center. Attached to each end of the rod is a small mass $m$. Determine the angular momentum of the system about the axis.
65. (II) A uniform disk turns at $3.3 \mathrm{rev} / \mathrm{s}$ around a frictionless central axis. A nonrotating rod, of the same mass as the disk and length equal to the disk's diameter, is dropped onto the freely spinning disk, Fig. 8-56. They then turn together around the axis with their centers superposed. What is the angular frequency in rev/s of the combination?

FIGURE 8-56
Problem 65.

66. (III) An asteroid of mass $1.0 \times 10^{5} \mathrm{~kg}$, traveling at a speed of $35 \mathrm{~km} / \mathrm{s}$ relative to the Earth, hits the Earth at the equator tangentially, in the direction of Earth's rotation, and is embedded there. Use angular momentum to estimate the percent change in the angular speed of the Earth as a result of the collision.

## *8-9 Angular Quantities as Vectors

67. (III) Suppose a $65-\mathrm{kg}$ person stands at the edge of a $5.5-\mathrm{m}$ diameter merry-go-round turntable that is mounted on frictionless bearings and has a moment of inertia of $1850 \mathrm{~kg} \cdot \mathrm{~m}^{2}$. The turntable is at rest initially, but when the person begins running at a speed of $4.0 \mathrm{~m} / \mathrm{s}$ (with respect to the turntable) around its edge, the turntable begins to rotate in the opposite direction. Calculate the angular velocity of the turntable.

## General Problems

68. A merry-go-round with a moment of inertia equal to $1260 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ and a radius of 2.5 m rotates with negligible friction at $1.70 \mathrm{rad} / \mathrm{s}$. A child initially standing still next to the merry-go-round jumps onto the edge of the platform straight toward the axis of rotation, causing the platform to slow to $1.35 \mathrm{rad} / \mathrm{s}$. What is her mass?
69. A $1.6-\mathrm{kg}$ grindstone in the shape of a uniform cylinder of radius 0.20 m acquires a rotational rate of $24 \mathrm{rev} / \mathrm{s}$ from rest over a 6.0 -s interval at constant angular acceleration. Calculate the torque delivered by the motor.
70. On a $12.0-\mathrm{cm}$-diameter audio compact disc (CD), digital bits of information are encoded sequentially along an outward spiraling path. The spiral starts at radius $R_{1}=2.5 \mathrm{~cm}$ and winds its way out to radius $R_{2}=5.8 \mathrm{~cm}$. To read the digital information, a CD player rotates the CD so that the player's readout laser scans along the spiral's sequence of bits at a constant linear speed of $1.25 \mathrm{~m} / \mathrm{s}$. Thus the player must accurately adjust the rotational frequency $f$ of the CD as the laser moves outward. Determine the values for $f$ (in units of rpm) when the laser is located at $R_{1}$ and when it is at $R_{2}$.
71. A hollow cylinder (hoop) is rolling on a horizontal surface at speed $v=3.0 \mathrm{~m} / \mathrm{s}$ when it reaches a $15^{\circ}$ incline. (a) How far up the incline will it go? (b) How long will it be on the incline before it arrives back at the bottom?
72. A cyclist accelerates from rest at a rate of $1.00 \mathrm{~m} / \mathrm{s}^{2}$. How fast will a point at the top of the rim of the tire (diameter $=68.0 \mathrm{~cm}$ ) be moving after 2.25 s ? [Hint: At any moment, the lowest point on the tire is in contact with the ground and is at rest-see Fig. 8-57.]

73. Suppose David puts a $0.60-\mathrm{kg}$ rock into a sling of length 1.5 m and begins whirling the rock in a nearly horizontal circle, accelerating it from rest to a rate of 75 rpm after 5.0 s . What is the torque required to achieve this feat, and where does the torque come from?
74. Bicycle gears: (a) How is the angular velocity $\omega_{R}$ of the rear wheel of a bicycle related to the angular velocity $\omega_{\mathrm{F}}$ of the front sprocket and pedals? Let $N_{\mathrm{F}}$ and $N_{\mathrm{R}}$ be the number of teeth on the front and rear sprockets, respectively, Fig. 8-58. The teeth are spaced the same on both sprockets and the rear sprocket is firmly attached to the rear wheel. (b) Evaluate the ratio $\omega_{\mathrm{R}} / \omega_{\mathrm{F}}$ when the front and rear sprockets have 52 and 13 teeth, respectively, and (c) when they have 42 and 28 teeth.

75. Figure 8-59 illustrates an $\mathrm{H}_{2} \mathrm{O}$ molecule. The $\mathrm{O}-\mathrm{H}$ bond length is 0.096 nm and the $\mathrm{H}-\mathrm{O}-\mathrm{H}$ bonds make an angle of $104^{\circ}$. Calculate the moment of inertia of the $\mathrm{H}_{2} \mathrm{O}$ molecule (assume the atoms are points) about an axis passing through the center of the oxygen atom (a) perpendicular to the plane of the molecule, and (b) in the plane of the molecule, bisecting the $\mathrm{H}-\mathrm{O}-\mathrm{H}$ bonds.

FIGURE 8-59
Problem 75.
76. Determine the angular momentum of the Earth (a) about its rotation axis (assume the Earth is a uniform sphere), and (b) in its orbit around the Sun (treat the Earth as a particle orbiting the Sun).
77. A wheel of mass $M$ has radius $R$. It is standing vertically on the floor, and we want to exert a horizontal force $F$ at its axle so that it will climb a step against which it rests (Fig. 8-60). The step has height $h$, where $h<R$. What minimum force $F$ is needed?


FIGURE 8-60 Problem 77.
78. If the coefficient of static friction between a car's tires and the pavement is 0.65 , calculate the minimum torque that must be applied to the $66-\mathrm{cm}$-diameter tire of a $1080-\mathrm{kg}$ automobile in order to "lay rubber" (make the wheels spin, slipping as the car accelerates). Assume each wheel supports an equal share of the weight.
79. A $4.00-\mathrm{kg}$ mass and a $3.00-\mathrm{kg}$ mass are attached to opposite ends of a very light $42.0-\mathrm{cm}$-long horizontal rod (Fig. 8-61). The system is rotating at angular speed $\omega=5.60 \mathrm{rad} / \mathrm{s}$ about a vertical axle at the center of the rod. Determine (a) the kinetic energy KE of the system, and (b) the net force on each mass.


FIGURE 8-61 Problem 79.
80. A small mass $m$ attached to the end of a string revolves in a circle on a frictionless tabletop. The other end of the string passes through a hole in the table (Fig. 8-62). Initially, the mass revolves with a speed $v_{1}=2.4 \mathrm{~m} / \mathrm{s}$ in a circle of radius $r_{1}=0.80 \mathrm{~m}$. The string is then pulled slowly through the hole so that the radius is reduced to $r_{2}=0.48 \mathrm{~m}$. What is the speed, $v_{2}$, of the mass now?

81. A uniform rod of mass $M$ and length $\ell$ can pivot freely (i.e., we ignore friction) about a hinge attached to a wall, as in Fig. 8-63. The rod is held horizontally and then released. At the moment of release, determine ( $a$ ) the angular acceleration of the rod, and (b) the linear acceleration of the tip of the rod. Assume that the force of gravity acts at the center of mass of the rod, as shown. [Hint: See Fig. 8-20g.]

FIGURE 8-63
Problem 81.

82. Suppose a star the size of our Sun, but with mass 8.0 times as great, were rotating at a speed of 1.0 revolution every 9.0 days. If it were to undergo gravitational collapse to a neutron star of radius 12 km , losing $\frac{3}{4}$ of its mass in the process, what would its rotation speed be? Assume the star is a uniform sphere at all times. Assume also that the thrownoff mass carries off either (a) no angular momentum, or (b) its proportional share $\left(\frac{3}{4}\right)$ of the initial angular momentum.
83. A large spool of rope rolls on the ground with the end of the rope lying on the top edge of the spool. A person grabs the end of the rope and walks a distance $\ell$, holding onto it, Fig. 8-64. The spool rolls behind the person without slipping. What length of rope unwinds from the spool? How far does the spool's center of mass move?

FIGURE 8-64
Problem 83.

84. The Moon orbits the Earth such that the same side always faces the Earth. Determine the ratio of the Moon's spin angular momentum (about its own axis) to its orbital angular momentum. (In the latter case, treat the Moon as a particle orbiting the Earth.)
85. Most of our Solar System's mass is contained in the Sun, and the planets possess almost all of the Solar System's angular momentum. This observation plays a key role in theories attempting to explain the formation of our Solar System. Estimate the fraction of the Solar System's total angular momentum that is possessed by planets using a simplified model which includes only the large outer planets with the most angular momentum. The central Sun (mass $1.99 \times 10^{30} \mathrm{~kg}$, radius $6.96 \times 10^{8} \mathrm{~m}$ ) spins about its axis once every 25 days and the planets Jupiter, Saturn, Uranus, and Neptune move in nearly circular orbits around the Sun with orbital data given in the Table below. Ignore each planet's spin about its own axis.

| Planet | Mean Distance from <br> Sun $\left(\times \mathbf{1 0}^{\mathbf{6}} \mathbf{~ k m}\right)$ | Orbital Period <br> $($ Earth Years $)$ | Mass <br> $\left(\times \mathbf{1 0}^{\mathbf{2 5}} \mathbf{~ k g )}\right.$ |
| :--- | :---: | :---: | :---: |
| Jupiter | 778 | 11.9 | 190 |
| Saturn | 1427 | 29.5 | 56.8 |
| Uranus | 2870 | 84.0 | 8.68 |
| Neptune | 4500 | 165 | 10.2 |

86. Water drives a waterwheel (or turbine) of radius $R=3.0 \mathrm{~m}$ as shown in Fig. 8-65. The water enters at a speed $v_{1}=7.0 \mathrm{~m} / \mathrm{s}$ and exits from the waterwheel at a speed $v_{2}=3.8 \mathrm{~m} / \mathrm{s}$. (a) If 85 kg of water passes through per second, what is the rate at which the water delivers angular momentum to the waterwheel? (b) What is the torque the water applies to the waterwheel? (c) If the water causes the waterwheel to make one revolution every 5.5 s , how much power is delivered to the wheel?


FIGURE 8-65
Problem 86.

## Search and Learn

1. Why are Eqs. 8-4 and $8-5$ valid for radians but not for revolutions or degrees? Read Section 8-1 and follow the derivations carefully to find the answer.
2. Total solar eclipses can happen on Earth because of amazing coincidences: for one, the sometimes near-perfect alignment of Earth, Moon, and Sun. Secondly, using the information inside the front cover, calculate the angular diameters (in radians) of the Sun and the Moon, as seen from Earth, and then comment.
3. Two uniform spheres simultaneously start rolling (from rest) down an incline. One sphere has twice the radius and twice the mass of the other. (a) Which reaches the bottom of the incline first? (b) Which has the greater speed there? (c) Which has the greater total kinetic energy at the bottom? Explain your answers.
4. Model a figure skater's body as a solid cylinder and her arms as thin rods, making reasonable estimates for the dimensions. Then calculate the ratio of the angular speeds for a spinning skater with outstretched arms, and with arms held tightly against her body. Check Sections 8-5 and 8-8.
5. One possibility for a low-pollution automobile is for it to use energy stored in a heavy rotating flywheel. Suppose such a car has a total mass of 1100 kg , uses a uniform cylindrical flywheel of diameter 1.50 m and mass 270 kg , and should be able to travel 350 km without needing a flywheel "spinup." (a) Make reasonable assumptions (average frictional retarding force on car $=450 \mathrm{~N}$, thirty acceleration periods from rest to $95 \mathrm{~km} / \mathrm{h}$, equal uphill and downhill, and that energy can be put back into the flywheel as the car goes downhill), and estimate what total energy needs to be stored in the flywheel. (b) What is the angular velocity of the flywheel when it has a full "energy charge"? (c) About how long would it take a 150 -hp motor to give the flywheel a full energy charge before a trip?
*6. A person stands on a platform, initially at rest, that can rotate freely without friction. The moment of inertia of the person plus the platform is $I_{\mathrm{P}}$. The person holds a spinning bicycle wheel with its axis horizontal. The wheel has moment of inertia $I_{\mathrm{W}}$ and angular velocity $\omega_{\mathrm{W}}$. What will be the angular velocity $\omega_{\mathrm{P}}$ of the platform if the person moves the axis of the wheel so that it points (a) vertically upward, (b) at a $60^{\circ}$ angle to the vertical, (c) vertically downward? (d) What will $\omega_{\mathrm{P}}$ be if the person reaches up and stops the wheel in part (a)? See Sections 8-8 and 8-9.

## ANSWERS TO EXERCISES

A: $f=0.076 \mathrm{~Hz} ; T=13 \mathrm{~s}$.
B: $\overrightarrow{\mathbf{F}}_{\mathrm{A}}$.
C: (c).

D: Yes; she does work to pull in her arms.
E: (b).
F: (b).

