

1-5 Units, Standards, and the SI System:

The measurement of any quantity is made relative to a particular standard or unit, and this unit must be specified along with the numerical value of the quantity.

For example, when we measure the height of a table, we may write 2 m.

↳ Unit (meter)

We shall write the unit because meters is very different from inches (in). or feet.

For any unit we use, such as distance or time or Mass we need to define a standard so any one need to make a measurement can refer to the standard and to communicate.

In the International System (SI system) by the French Academy of Sciences, the units for length, Mass, Time are :-

meter (m), Kilogram (kg), Second (s)

before this was called the mKS

Sometimes people use the CGS,
 cm^{mass} , gram, second

but we shall mostly use the SI (mKS) and in (1-6) we will learn how to convert from mKS \rightleftarrows CGS easily.

Scientists, in the interest of simplicity, want the smallest number of base quantities possible consistent with a full description of the physical world. This number turns out to be seven, and those are :-

Quantity	Unit	Abbreviation
Length	meter	m
Time	Second	s
Mass	Kilogram	kg
Electric current	ampere	A
Temperature	Kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

* When another quantities can be expressed in terms of the base quantities we call them Derived quantities

Example (1)

Show that Force which has units of (newton) can be expressed in terms of L, M, T.

Sol. $F = \overset{\text{mass}}{ma}$ - acceleration

$$1 \text{ N} = 1 \text{ kg} \cdot 1 \frac{\text{m}}{\text{s}^2} = \text{M} \cdot \frac{\text{L}}{\text{T}^2}$$

Therefore the Newton is a derived quantity, since we can express it in terms of a combination of the base units L, M, T.

1-6 converting Units :-

Often we are given a quantity in one set of units, but we want it expressed in another set of units. For example, suppose we measure the length of a pen that is 3 (in) and we want to express this in cm →

$$1(\text{in}) = 2.54 \text{ cm}$$

$$1. = 2.54 \frac{\text{cm}}{(\text{in})} \leftarrow \text{num} = 1$$

$$\begin{aligned} \text{length of pen} &= 3(\text{in}) = 3(\text{in}) \cdot 1 \\ [\text{conversion factor}] \leftarrow &= 3 \left(\cancel{\text{in}} \cdot 2.54 \right) \frac{\text{cm}}{\text{cm}} \\ &= 7.62 \text{ cm}. \end{aligned}$$

Example (1)

The speed of a car is 100 km/h. Express the speed in terms of m/s.

$$1 \text{ km} = 1000 \text{ m} \rightarrow 1. = 1000 \frac{\text{m}}{\text{km}} \quad \boxed{1}$$

$$1 \text{ h} = 3600 \text{ s} \rightarrow 1. = 3600 \frac{\text{s}}{\text{h}} \quad \boxed{2}$$

$$\text{Speed of a car is} = 100 \text{ km/h} \cdot (1)$$

$$= 100 \text{ km/h} \cdot \left(1000 \frac{\text{m}}{\text{km}} \right) \cdot \left(\frac{1}{3600 \text{ s}} \right)$$

$$= \frac{1000}{36} \text{ m/s}$$

$$\approx 27.8 \text{ m/s}$$

* Note: 1 ft = 12 (in)

$$1(\text{in}) = 2.54 \text{ cm}$$

$$1 \text{ mi} = 5280 \text{ ft}$$

$$1 \text{ h} = 3600 \text{ s}$$

Metric (SI) prefixes :-

Prefix	Abbreviation	Value
Yotta	Y	10^{24}
Zetta	Z	10^{21}
Exa	E	10^{18}
Peta	P	10^{15}
Tera	T	10^{12}
Giga	G	10^9
Mega	M	10^6
Kilo	K	10^3
Hecto	h	10^2
Deka	da	10^1
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

Example (2)

The speed limit is 55 mi/h or mph. Find:

(a) This speed in meters per second.

(b) This speed in Kilometers per hour.

Sol. (a) The speed = $55 \text{ mi/h} \cdot (1) \cdot (1) \cdot (1)$

$$= 55 \text{ mi/h} \cdot \left(5280 \frac{\text{ft}}{\text{mi}} \right) \cdot \left(\frac{1 \text{ m}}{3 \text{ ft}} \right) \cdot \left(\frac{1 \text{ h}}{3600 \text{ s}} \right)$$

$$= 885 \frac{\text{m}}{\text{s}} \cdot \left(\frac{1 \text{ m}}{3600 \text{ s}} \right)$$

$$\approx 25 \text{ m/s.}$$

$$(b) 885 \frac{\text{m}}{\text{s}} \cdot \left(\frac{1 \text{ km}}{1000 \text{ m}} \right) = 88.5 \frac{\text{km}}{\text{h}}$$

$$* 1 \text{ m} = 100 \text{ cm} \rightarrow 1 \text{ m}^3 = 10^6 \text{ cm}^3 \rightarrow 1 = \frac{1 \text{ m}^3}{10^6 \text{ cm}^3}$$

Density of water is 1000 kg/m^3 , find the density in g/cm^3 : $* 1 \text{ kg} = 1000 \text{ gm} \rightarrow 1 = \frac{1000 \text{ g}}{\text{kg}}$

$$\text{Density} = 1000 \frac{\text{kg}}{\text{m}^3} \times \frac{1000 \text{ g}}{1 \text{ kg}} \times \frac{1 \text{ m}^3}{10^6 \text{ cm}^3} = 1 \text{ g/cm}^3$$

1-8 Dimensions :-

When we speak of the dimensions of a quantity, we are referring to the type of base units or base quantities that make it up. Note that the formula for a quantity may be different in some cases. For example, the area of a triangle of base b and height h is $A = \frac{1}{2}bh$, whereas the area of a circle of radius r is $A = \pi r^2$. The formulas are different, but the dimensions are always $[L^2]$.

And the question is: Where we can use Dimensions?

1 Dimensional analysis: which can be used to check if relationships are incorrect. For example: $v = v_0 + \frac{1}{2}at^2$, let's do a dimensional check. Note that numerical factors like $\frac{1}{2}$ ~~here~~ do not affect the checking process.

$$v = v_0 + \frac{1}{2}at^2$$
$$\left[\frac{L}{T}\right] \stackrel{??}{=} \left[\frac{L}{T}\right] + \frac{1}{2} \left[\frac{L}{T^2}\right] \cdot [T^2]$$
$$\stackrel{??}{=} \left[\frac{L}{T}\right] + [L].$$

The dimensions are incorrect.

2 A quick check on an equation:

For example, consider a simple pendulum of length L . Suppose that you can't remember whether the equation for the period T is $T = 2\pi\sqrt{L/g}$ or $T = 2\pi\sqrt{g/L}$

$$[T] = \sqrt{\frac{[L]}{[L/T]^2}} = [T]$$

It shows the 1st former is correct.

*Note:- A dimensional check can only tell you when a relationship is wrong, it can't tell you if it is completely right, such as a dimensionless numerical factor could be missing like: ($\frac{1}{2}, 2\pi$).

* Do the following questions:-

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* Consider: $v_f = v_i + 2at$, check the dimensions.

$$\left[\frac{L}{T}\right] \stackrel{?}{=} \left[\frac{L}{T}\right] + \left[\frac{L}{T^2}\right] \times [T]$$

$\left[\frac{L}{T}\right] = \left[\frac{L}{T}\right] + \left[\frac{L}{T}\right] \rightarrow$ Dimensionally correct
but wrong from the physics point of view

* If an equation is dimensionally wrong it is physically wrong

* If an equation is dimensionally correct it's not necessary to be physically correct as mentioned above.