# CHAPTER (23) – PART 2

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# CONCEPTS

- 23-1: The Ray Model of Light
- 23-4: Index of Refraction
- 23-5: Refraction: Snell's Law
- 23-6: Total Internal Reflection; Fiber Optics
- 23-7: Thin Lenses; Ray Tracing
- 23-8: The Thin Lens Equation

# 23-8: THE THIN LENS EQUATION

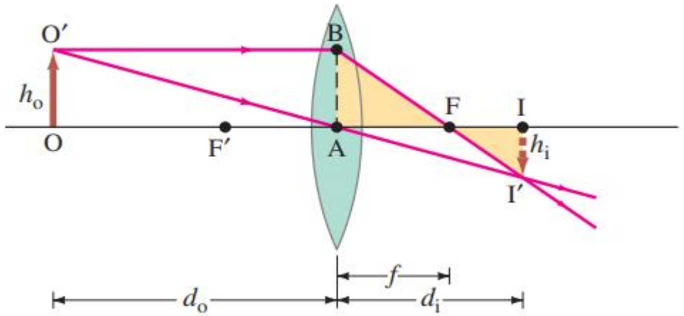


**FIGURE 23–40** Deriving the lens equation for a converging lens.

Let $h_o$ and $h_i$ refer to the heights of the object and image. Consider the two rays shown in Fig. 23–40 for a converging lens, assumed to be very thin. The right triangles FI′I and FBA (highlighted in yellow) are similar because angle AFB equals angle IFI′; so

$$\frac{h_i}{h_o} = \frac{d_i - f}{f},$$

since length $AB = h_o$. Triangles OAO′ and IAI′ are similar as well. Therefore,

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}.$$

We equate the right sides of these two equations (the left sides are the same), and divide by $d_i$ to obtain

$$\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o}$$

or

*THIN LENS EQUATION*

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \qquad (23\text{–}8)$$

This is called the **thin lens equation**. It relates the image distance $d_i$ to the object distance $d_o$ and the focal length $f$. It is the most useful equation in geometric optics. (Interestingly, it is exactly the same as the mirror equation, Eq. 23–2.)

If the object is at infinity, then $1/d_o = 0$, so $d_i = f$. Thus the focal length is the image distance for an object at infinity, as mentioned earlier.

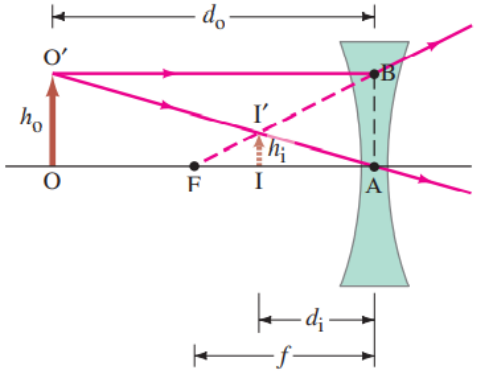


**FIGURE 23–41** Deriving the lens equation for a diverging lens.

We can derive the lens equation for a diverging lens using Fig. 23–41. Triangles IAI′ and OAO′ are similar; and triangles IFI′ and AFB are similar. Thus (noting that length AB = $h_o$)

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$

and

$$\frac{h_i}{h_o} = \frac{f - d_i}{f}.$$

When we equate the right sides of these two equations and simplify, we obtain

$$\frac{1}{d_o} - \frac{1}{d_i} = -\frac{1}{f}.$$

This equation becomes the same as Eq. 23–8 if we make $f$ and $d_i$ negative. That is, we take $f$ to be *negative for a diverging lens*, and $d_i$ negative when the image is on the same side of the lens as the light comes from. Thus Eq. 23–8 will be valid for both converging and diverging lenses, and for *all* situations, if we use the following **sign conventions**:

**CAUTION**
*Focal length is negative for diverging lens*

**PROBLEM SOLVING**
*SIGN CONVENTIONS for lenses*

1. The focal length is positive for converging lenses and negative for diverging lenses.
2. The object distance is positive if the object is on the side of the lens from which the light is coming (this is always the case for real objects; but when lenses are used in combination, it might not be so: see Example 23–16); otherwise, it is negative.
3. The image distance is positive if the image is on the opposite side of the lens from where the light is coming; if it is on the same side, $d_i$ is negative. Equivalently, the image distance is positive for a real image (Fig. 23–40) and negative for a virtual image (Fig. 23–41).
4. The height of the image, $h_i$, is positive if the image is upright, and negative if the image is inverted relative to the object. ($h_o$ is always taken as upright and positive.)

The **magnification**, $m$, of a lens is defined as the ratio of the image height to object height, $m = h_i/h_o$. From Figs. 23–40 and 23–41 and the conventions just stated (for which we will need a minus sign), we have

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \qquad \textbf{(23–9)}$$

For an upright image the magnification is positive, and for an inverted image the magnification is negative.

From sign convention 1, it follows that the power (Eq. 23–7) of a converging lens, in diopters, is positive, whereas the power of a diverging lens is negative. A converging lens is sometimes referred to as a **positive lens**, and a diverging lens as a **negative lens**.

Diverging lenses (see Fig. 23–41) always produce an upright virtual image for any real object, no matter where that object is. Converging lenses can produce real (inverted) images as in Fig. 23–40, or virtual (upright) images, depending on object position, as we shall see.

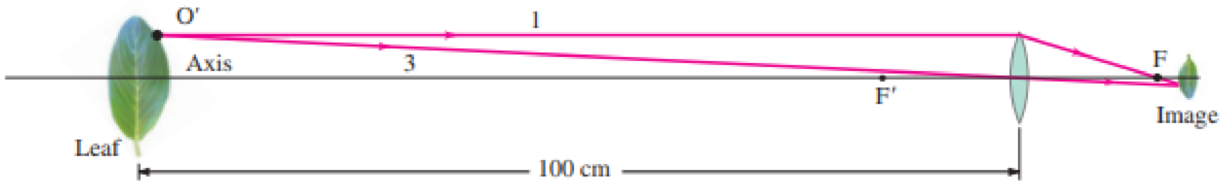


**FIGURE 23–42** Example 23–12. (Not to scale.)

**EXAMPLE 23–12** **Image formed by converging lens.** What is (*a*) the position, and (*b*) the size, of the image of a 7.6-cm-high leaf placed 1.00 m from a +50.0-mm-focal-length camera lens?

**APPROACH** We follow the steps of the Problem Solving Strategy explicitly.

**SOLUTION**

1. **Ray diagram.** Figure 23–42 is an approximate ray diagram, showing only rays 1 and 3 for a single point on the leaf. We see that the image ought to be a little behind the focal point F, to the right of the lens.
2. **Thin lens and magnification equations.** (*a*) We find the image position analytically using the thin lens equation, Eq. 23–8. The camera lens is converging, with $f = +5.00\text{ cm}$, and $d_o = 100\text{ cm}$, and so the thin lens equation gives

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{5.00\text{ cm}} - \frac{1}{100\text{ cm}} = \frac{20.0 - 1.0}{100\text{ cm}} = \frac{19.0}{100\text{ cm}}.$$

Then, taking the reciprocal,

$$d_i = \frac{100\text{ cm}}{19.0} = 5.26\text{ cm},$$

or 52.6 mm behind the lens.

(*b*) The magnification is

$$m = -\frac{d_i}{d_o} = -\frac{5.26\text{ cm}}{100\text{ cm}} = -0.0526,$$

so

$$h_i = mh_o = (-0.0526)(7.6\text{ cm}) = -0.40\text{ cm}.$$

The image is 4.0 mm high.

3. **Sign conventions.** The image distance $d_i$ came out positive, so the image is behind the lens. The image height is $h_i = -0.40\text{ cm}$; the minus sign means the image is inverted.
4. **Consistency.** The analytic results of steps 2 and 3 are consistent with the ray diagram, Fig. 23–42: the image is behind the lens and inverted.

**NOTE** Part (*a*) tells us that the image is 2.6 mm farther from the lens than the image for an object at infinity, which would equal the focal length, 50.0 mm. Indeed, when focusing a camera lens, the closer the object is to the camera, the farther the lens must be from the sensor or film.

**EXAMPLE 23–13** **Object close to converging lens.** An object is placed 10 cm from a 15-cm-focal-length converging lens. Determine the image position and size (*a*) analytically, and (*b*) using a ray diagram.

**APPROACH** The object is within the focal point—closer to the lens than the focal point F as $d_o < f$. We first use Eqs. 23–8 and 23–9 to obtain an analytic solution, and then confirm with a ray diagram using the special rays 1, 2, and 3 for a single object point.

**SOLUTION** (*a*) Given $f = 15\text{ cm}$ and $d_o = 10\text{ cm}$, then

$$\frac{1}{d_i} = \frac{1}{15\text{ cm}} - \frac{1}{10\text{ cm}} = \frac{2-3}{30\text{ cm}} = -\frac{1}{30\text{ cm}},$$

and $d_i = -30\text{ cm}$. (Remember to take the reciprocal!) Because $d_i$ is negative, the image must be virtual and on the same side of the lens as the object (sign convention 3, page 665). The magnification

$$m = -\frac{d_i}{d_o} = -\frac{-30\text{ cm}}{10\text{ cm}} = 3.0.$$

The image is three times as large as the object and is upright. This lens is being used as a magnifying glass, which we discuss in more detail in Section 25–3.

(*b*) The ray diagram is shown in Fig. 23–43 and confirms the result in part (*a*). We choose point O′ on the top of the object and draw ray 1. Ray 2, however, may take some thought: if we draw it heading toward F′, it is going the wrong way—so we have to draw it as if coming from F′ (and so dashed), striking the lens, and then going out parallel to the lens axis. We project it backward, with a dashed line, as we must do also for ray 1, in order to find where they cross. Ray 3 is drawn through the lens center, and it crosses the other two rays at the image point, I′.

**NOTE** From Fig. 23–43 we can see that, when an object is placed between a converging lens and its focal point, the image is virtual.

**CAUTION**
*Don't forget to take the reciprocal*

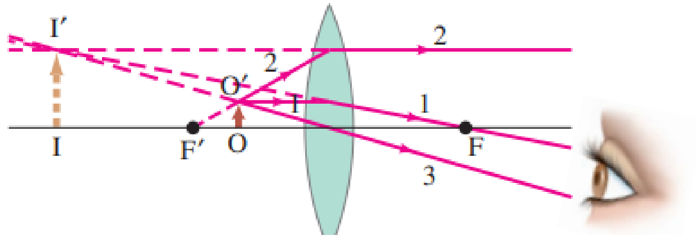


**FIGURE 23–43** An object placed within the focal point of a converging lens produces a virtual image. Example 23–13.