CHAPTER 30

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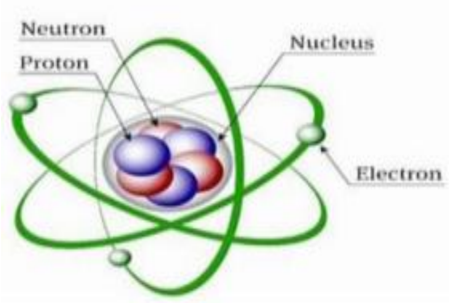
- 30-1 Structure and Properties of Nucleus
- 30-3 Radioactivity
- 30-8 Half-Life and Rate of Decay
- 30-9 Calculations Involving Decay Rates and Half-Life

30–1 STRUCTURE AND PROPERTIES OF THE NUCLEUS

- The atom is made of electrons moving around the nucleus
- nucleus is made up of two types of particles: protons and neutrons



The proton has a positive charge ($= +e = +1.60 \times 10^{-19}$ C, the same magnitude as for the electron) and its mass is measured to be

$$m_p = 1.67262 \times 10^{-27} \text{ kg}.$$

The **neutron**, whose existence was ascertained in 1932 by the English physicist James Chadwick (1891–1974), is electrically neutral ($q = 0$), as its name implies. Its mass is very slightly larger than that of the proton:

$$m_n = 1.67493 \times 10^{-27} \text{ kg}.$$

These two constituents of a nucleus, neutrons and protons, are referred to collectively as **nucleons**.

	Mass	Charge
Electron	9.110×10^{-31}	-1.6×10^{-19}

30–1 STRUCTURE AND PROPERTIES OF THE NUCLEUS

- Although a normal hydrogen nucleus consists of a single proton alone, the nuclei of all other elements consist of both neutrons and protons. The different nuclei are often referred to as nuclides

We refer to a given nuclide (nucleus) as:

mass number → A

$${}^{A}_{Z}X_{N}$$

atomic number → Z

N ← neutron number

Atomic number(Z): number of protons in the nucleus.

Neutron number(N) number of neutrons in the nucleus.

Mass number: the sum of proton and neutron numbers in the nucleus

$$A = Z + N$$

30–1 Structure and Properties of the Nucleus

For a particular type of atom (say, carbon), nuclei are found to contain different numbers of neutrons, although they all have the same number of protons. For example, carbon nuclei always have 6 protons, but they may have 5, 6, 7, 8, 9, or 10 neutrons. Nuclei that contain the same number of protons but different numbers of neutrons are called **isotopes**. Thus, ${}^{11}_{6}C$, ${}^{12}_{6}C$, ${}^{13}_{6}C$, ${}^{14}_{6}C$, ${}^{15}_{6}C$, and ${}^{16}_{6}C$ are all isotopes of carbon. The isotopes of a given element are not all equally common. For example, 98.9% of naturally occurring carbon (on Earth) is the isotope ${}^{12}_{6}C$, and about 1.1% is ${}^{13}_{6}C$. These percentages are referred to as the **natural abundances**.† Even hydrogen has isotopes: 99.99% of natural hydrogen is ${}^{1}_{1}H$, a simple proton, as the nucleus; there are also ${}^{2}_{1}H$, called **deuterium**, and ${}^{3}_{1}H$, **tritium**, which besides the proton contain 1 or 2 neutrons. (The bare nucleus in each case is called the **deuteron** and **triton**.)

Isotones: nuclides that have the same number of **neutrons**, but different number of protons. ${}^{12}_{5}B_{7}$, ${}^{13}_{6}C_{7}$

(same N different Z)

Isobars: nuclides that have the same mass number A. ${}^{40}_{18}Ar_{22}$; ${}^{40}_{19}K_{21}$; ${}^{40}_{20}Ca_{20}$

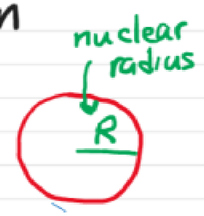
(same A)

30–1 STRUCTURE AND PROPERTIES OF THE NUCLEUS

Nuclei (nuclides) have roughly spherical shapes. The radius of a nucleus is given by:

$$R = 1.2 \times 10^{-15} \, A^{1/3} \text{ meters}$$

mass number $A = Z + N$



1 fermi $= 10^{-15}$ m

$$R = 1.2 \, A^{1/3} \text{ fermi}$$

Example: Determine the radius of ^{16}O.

$$R = 1.2 \times 10^{-15} (16)^{1/3} \simeq 3.02 \times 10^{-15} \text{ m}$$

$$R \sim 3.02 \text{ fm} \quad (\text{fm} \equiv \text{fermi}).$$

EXAMPLE 30–1 ESTIMATE **Nuclear sizes.** Estimate the diameter of the smallest and largest naturally occurring nuclei: (*a*) $^1_1\mathrm{H}$, (*b*) $^{238}_{92}\mathrm{U}$.

APPROACH The radius r of a nucleus is related to its number of nucleons A by Eq. 30–1. The diameter $d = 2r$.

SOLUTION (*a*) For hydrogen, $A = 1$, Eq. 30–1 gives

$$d = \text{diameter} = 2r \approx 2(1.2 \times 10^{-15}\,\mathrm{m})(A^{\frac{1}{3}}) = 2.4 \times 10^{-15}\,\mathrm{m}$$

since $A^{\frac{1}{3}} = 1^{\frac{1}{3}} = 1$.

(*b*) For uranium $d \approx (2.4 \times 10^{-15}\,\mathrm{m})(238)^{\frac{1}{3}} = 15 \times 10^{-15}\,\mathrm{m}$.

The range of nuclear diameters is only from 2.4 fm to 15 fm.

NOTE Because nuclear radii vary as $A^{\frac{1}{3}}$, the largest nuclei (such as uranium with $A = 238$) have a radius only about $\sqrt[3]{238} \approx 6$ times that of the smallest, hydrogen ($A = 1$).

EXAMPLE 30–2 ESTIMATE **Nuclear and atomic densities.** Compare the density of nuclear matter to the density of normal solids.

APPROACH The density of normal liquids and solids is on the order of 10^3 to $10^4\,\mathrm{kg/m^3}$ (see Table 10–1), and because the atoms are close packed, atoms have about this density too. We therefore compare the density (mass per volume) of a nucleus to that of its atom as a whole.

SOLUTION The mass of a proton is greater than the mass of an electron by a factor

$$\frac{1.67 \times 10^{-27}\,\mathrm{kg}}{9.1 \times 10^{-31}\,\mathrm{kg}} \approx 2000.$$

Thus, over 99.9% of the mass of an atom is in the nucleus, and for our estimate we can say the mass of the atom equals the mass of the nucleus, $m_{\mathrm{nucl}}/m_{\mathrm{atom}} = 1$. Atoms have a radius of about $10^{-10}\,\mathrm{m}$ (Chapter 27) and nuclei on the order of $10^{-15}\,\mathrm{m}$ (Eq. 30–1). Thus the ratio of nuclear density to atomic density is about

$$\frac{\rho_{\mathrm{nucl}}}{\rho_{\mathrm{atom}}} = \frac{(m_{\mathrm{nucl}}/V_{\mathrm{nucl}})}{(m_{\mathrm{atom}}/V_{\mathrm{atom}})} = \left(\frac{m_{\mathrm{nucl}}}{m_{\mathrm{atom}}}\right)\frac{\frac{4}{3}\pi r^3_{\mathrm{atom}}}{\frac{4}{3}\pi r^3_{\mathrm{nucl}}} \approx (1)\frac{(10^{-10})^3}{(10^{-15})^3} = 10^{15}.$$

The nucleus is 10^{15} times more dense than ordinary matter.

30–1 STRUCTURE AND PROPERTIES OF THE NUCLEUS

Nuclear masses can be specified in **unified atomic mass units** (u). On this scale, a neutral ${}^{12}_{6}C$ atom is given the exact value 12.000000 u. A neutron then has a measured mass of 1.008665 u, a proton 1.007276 u, and a neutral hydrogen atom ${}^{1}_{1}H$ (proton plus electron) 1.007825 u.

30–1 Structure and Properties of the Nucleus

Masses may be specified using the electron-volt energy unit, $1\text{ eV} = 1.6022 \times 10^{-19}\text{ J}$ (Section 17–4). This can be done because mass and energy are related, and the precise relationship is given by Einstein's equation $E = mc^2$ (Chapter 26). Since the mass of a proton is $1.67262 \times 10^{-27}\text{ kg}$, or 1.007276 u, then 1 u is equal to

$$1.0000\text{ u} = (1.0000\text{ u})\left(\frac{1.67262 \times 10^{-27}\text{ kg}}{1.007276\text{ u}}\right) = 1.66054 \times 10^{-27}\text{ kg};$$

this is equivalent to an energy (see Table inside front cover) in MeV ($= 10^6$ eV) of

$$E = mc^2 = \frac{(1.66054 \times 10^{-27}\text{ kg})(2.9979 \times 10^8\text{ m/s})^2}{(1.6022 \times 10^{-19}\text{ J/eV})} = 931.5\text{ MeV}.$$

Thus,

$$1\text{ u} = 1.6605 \times 10^{-27}\text{ kg} = 931.5\text{ MeV}/c^2.$$

The rest masses of some of the basic particles are given in Table 30–1. As a rule of thumb, to remember, the masses of neutron and proton are about $1\text{ GeV}/c^2$ ($= 1000\text{ MeV}/c^2$) which is about 2000 times the mass of an electron ($\approx \frac{1}{2}\text{MeV}/c^2$).

TABLE 30–1
Rest Masses in Kilograms, Unified Atomic Mass Units, and MeV/c^2

Object	Mass: kg	Mass: u	Mass: MeV/c^2
Electron	9.1094×10^{-31}	0.00054858	0.51100
Proton	1.67262×10^{-27}	1.007276	938.27
$^1_1$H atom	1.67353×10^{-27}	1.007825	938.78
Neutron	1.67493×10^{-27}	1.008665	939.57

30–3 RADIOACTIVITY

- Some nuclei (specially heavy ones) are unstable and emit neutrons, alpha particles, gamma radiation, x-rays .. .. Etc, to become more stable

The isotopes of uranium $^{235}_{92}U$ and $^{238}_{92}U$ are radioactive. Both are naturally occuring nuclides

$\Rightarrow$ natural radioactivity.

Manufactured radioactive nuclides lead to artificial radioactivity. (Californium) artificially radiactive manufactured element

30–3 RADIOACTIVITY

Unstable nuclei emit three main types of nuclear radiation, classified according to their penetrating power:

i) Alpha (α) particles [also can say α-radiation]

An α-particle is made up of 2 protons and 2 neutrons. So it is the nucleus of helium atom $\Rightarrow$ it is a doubly ionized helium atom

$\alpha \equiv {}^{4}_{2}He^{++}$ ← doubly ionized (two e^- are removed).

α

pp
nn

ii) $\beta^{\pm}$ particles [$\beta^{\pm}$ radiation]

β^- is an electron e^- $\quad -1.6\times10^{-19}$ J

β^+ is a positron e^+ $\quad +1.6\times10^{-19}$ J

e^+ is the antiparticle of the e^-.

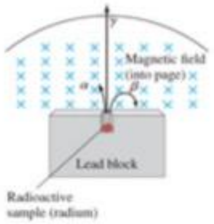
$m_{e^+} = m_{e^-}$ but $e^+ = +1.6022\times10^{-19}$ Coulomb.

30–3 RADIOACTIVITY

iii) γ-radiation

Very high frequency electromagnetic radiation.

FIGURE 30-4 Alpha and beta rays are bent in opposite directions by a magnetic field, whereas gamma rays are not bent at all.



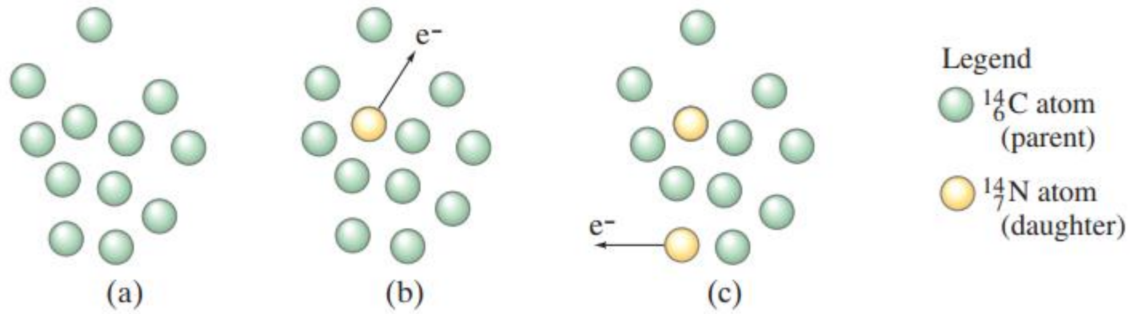
magnetic field only deflects charged particles.
It does not act with a force on neutral particles.

30–3 RADIOACTIVITY

Radiation	Penetrating Power
α	can be stopped using a piece of paper.
$\beta^{\pm}$	can be stopped using few mm of aluminum.
γ	has high penetrating power. can pass through several cm of lead.

30–8 Half-Life and Rate of Decay

FIGURE 30–9 Radioactive nuclei decay one by one. Hence, the number of parent nuclei in a sample is continually decreasing. When a $^{14}_{6}C$ nucleus emits an electron (b), the nucleus becomes a $^{14}_{7}N$ nucleus. Another decays in (c).



$^{14}_{6}C_8$ is an unstable nucleus ⇒ it decays by emitting β^- particles (e^-) and turns into $^{14}_{7}N_7$ which is stable.

$n \to e^- + \bar{\nu}_e + p$

$$\underbrace{^{14}_{6}C_8}_{\text{parent nucleus}} \longrightarrow \underbrace{^{14}_{7}N_7}_{\text{daughter nucleus}} + \underset{\text{electron } (\beta^-)}{e^-} + \underset{\text{antineutrino}}{\bar{\nu}_e}$$

$A = N + Z$

30–8 HALF-LIFE AND RATE OF DECAY

$$N = N_0 e^{-\lambda t},$$

where N_0 is the number of parent nuclei present at any chosen time $t = 0$, and N is the number remaining after a time t. The symbol e is the natural exponential (encountered earlier in Sections 19–6 and 21–12) whose value is $e = 2.718\cdots$. Thus the number of parent nuclei in a sample decreases exponentially in time. This is shown in Fig. 30–10a for the decay of ${}^{14}_{6}C$. Equation 30–4 is called the **radioactive decay law**.

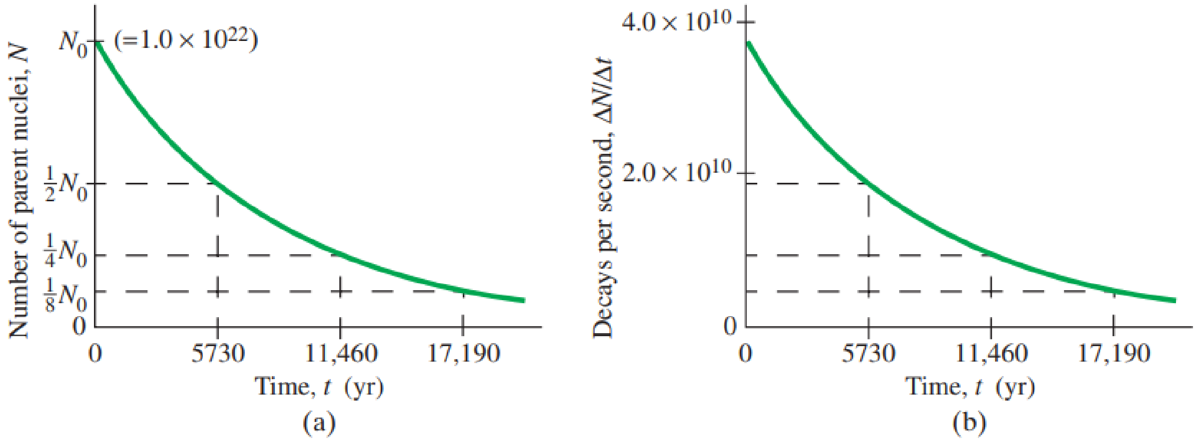


FIGURE 30–10 (a) The number N of parent nuclei in a given sample of ${}^{14}_{6}C$ decreases exponentially. We assume a sample that has $N_0 = 1.00 \times 10^{22}$ nuclei. (b) The number of decays per second also decreases exponentially. The half-life of ${}^{14}_{6}C$ is 5730 yr, which means that the number of parent nuclei, N, and the rate of decay, $\Delta N/\Delta t$, decrease by half every 5730 yr.

30–8 HALF-LIFE AND RATE OF DECAY

Half-Life

The rate of decay of any isotope is often specified by giving its "half-life" rather than the decay constant λ. The **half-life** of an isotope is defined as the time it takes for half the original amount of parent isotope in a given sample to decay.

* Deriving the Half-Life Formula

We can derive Eq. 30–6 starting from Eq. 30–4 by setting $N = N_0/2$ at $t = T_{\frac{1}{2}}$:

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{\frac{1}{2}}}$$

so

$$\frac{1}{2} = e^{-\lambda T_{\frac{1}{2}}}$$

and

$$e^{\lambda T_{\frac{1}{2}}} = 2.$$

We take natural logs of both sides ("ln" and "e" are inverse operations, meaning $\ln\left(e^x\right) = x$) and find

$$\ln\left(e^{\lambda T_{\frac{1}{2}}}\right) = \ln 2,$$

so

$$\lambda T_{\frac{1}{2}} = \ln 2 = 0.693$$

and

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda},$$

which is Eq. 30–6.

30–8 HALF-LIFE AND RATE OF DECAY

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$$

Each radioactive nuclide has its own λ.

large $\lambda \Rightarrow$ small $t_{1/2} \Rightarrow$ fast decay.

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

small $\lambda \Rightarrow$ large $t_{1/2} \Rightarrow$ slow decay.

For ^{238}U : $t_{1/2} \simeq 4.5 \times 10^{9}$ yr $\Rightarrow \lambda = \frac{\ln 2}{4.5} \times 10^{-9}$ yr^{-1}

Very small $\lambda \Rightarrow$ very large $t_{1/2} \Rightarrow$ slow decay.

For Iodine-131 (^{131}I) (used to treat thyroid gland cancer and also treatment of an overactiv thyroid gland).

$t_{1/2} \simeq 8$ days

$t_{1/2} \simeq 8$ days $\Rightarrow \lambda = \frac{\ln 2}{8} \simeq 0.087$ day^{-1}

large λ (compared to ^{235}U) $\Rightarrow$ small $t_{1/2} \Rightarrow$ fast decay.

30–8 HALF-LIFE AND RATE OF DECAY

The number of decays per second, or decay rate R, is the magnitude of $\Delta N/\Delta t$, and is also called the **activity** of the sample. The magnitude (always positive) of a quantity is often indicated using vertical lines. The magnitude of $\Delta N/\Delta t$ is written $|\Delta N/\Delta t|$ and it is proportional to N (see Eq. 30–3b). So it too decreases exponentially in time at the same rate (Fig. 30–10b). The activity of a pure sample at time t is

$$R = \left|\frac{\Delta N}{\Delta t}\right| = R_0 e^{-\lambda t}, \qquad \textbf{(30–5)}$$

where $R_0 = |\Delta N/\Delta t|_0$ is the activity at $t = 0$.

Equation 30–5 is also referred to as the **radioactive decay law** (as is Eq. 30–4).

The SI unit of activity is the Becquerel (Bq)

1 Bq = 1 decay/s or simply s^{-1}

Another more common unit of activity is the curie (Ci)

1 Ci = 3.7×10^{10} decays/s

Activity (A) : number of decays per second .
[rate of decay]

$$N = N_0 \, e^{-\lambda t}$$

$$A = -\frac{dN}{dt} = -\left[N_0 (-\lambda) \, e^{-\lambda t} \right]$$

$$\therefore \quad A = \lambda \underbrace{N_0 e^{-\lambda t}}$$

$$A = \lambda N$$

$$A = \lambda N_0 \, e^{-\lambda t} = A_0 \, e^{-\lambda t}$$

initial activity

activity at time t .

30–8 Half-Life and Rate of Decay

*Mean Life

Sometimes the **mean life** τ of an isotope is quoted, which is defined as $\tau = 1/\lambda$. Then Eq. 30–4 can be written $N = N_0 e^{-t/\tau}$, just as for RC and LR circuits (Chapters 19 and 21 where τ was called the time constant). The mean life of an isotope is then given by (see also Eq. 30–6)

$$\tau = \frac{1}{\lambda} = \frac{T_{\frac{1}{2}}}{0.693}. \qquad \text{[mean life]} \quad \textbf{(30–7)}$$

The mean life and half-life differ by a factor of 0.693, so confusing them can cause serious error (and has). The radioactive decay law, Eq. 30–5, can then be written as $R = R_0 e^{-t/\tau}$.

30-9 Calculations Involving Decay Rates and Half-Life

EXAMPLE 30–9 **Sample activity.** The isotope ${}^{14}_{6}C$ has a half-life of 5730 yr. If a sample contains 1.00×10^{22} carbon-14 nuclei, what is the activity of the sample?

APPROACH We first use the half-life to find the decay constant (Eq. 30–6), and use that to find the activity, Eq. 30–3b. The number of seconds in a year is $(60)(60)(24)(365\frac{1}{4}) = 3.156 \times 10^7$ s.

SOLUTION The decay constant λ from Eq. 30–6 is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{(5730\text{ yr})(3.156 \times 10^7\text{ s/yr})} = 3.83 \times 10^{-12}\text{ s}^{-1}.$$

From Eqs. 30–3b and 30–5, the activity or rate of decay is

$$R = \left|\frac{\Delta N}{\Delta t}\right| = \lambda N = (3.83 \times 10^{-12}\text{ s}^{-1})(1.00 \times 10^{22}) = 3.83 \times 10^{10}\text{ decays/s}.$$

Notice that the graph of Fig. 30–10b starts at this value, corresponding to the original value of $N = 1.0 \times 10^{22}$ nuclei in Fig. 30–10a.

NOTE The unit "decays/s" is often written simply as s^{-1} since "decays" is not a unit but refers only to the number. This simple unit of activity is called the becquerel: 1 Bq = 1 decay/s, as discussed in Chapter 31.

30-9 Calculations Involving Decay Rates and Half-Life

EXAMPLE 30–11 A sample of radioactive $^{13}_{7}N$. A laboratory has 1.49 μg of pure $^{13}_{7}N$, which has a half-life of 10.0 min (600 s). (*a*) How many nuclei are present initially? (*b*) What is the rate of decay (activity) initially? (*c*) What is the activity after 1.00 h? (*d*) After approximately how long will the activity drop to less than one per second $(= 1\,s^{-1})$?

APPROACH We use the definition of the mole and Avogadro's number (Sections 13–6 and 13–8) to find (*a*) the number of nuclei. For (*b*) we get λ from the given half-life and use Eq. 30–3b for the rate of decay. For (*c*) and (*d*) we use Eq. 30–5.

SOLUTION (*a*) The atomic mass is 13.0, so 13.0 g will contain 6.02×10^{23} nuclei (Avogadro's number). We have only 1.49×10^{-6} g, so the number of nuclei N_0 that we have initially is given by the ratio

$$\frac{N_0}{6.02 \times 10^{23}} = \frac{1.49 \times 10^{-6}\,\text{g}}{13.0\,\text{g}}.$$

Solving for N_0, we find $N_0 = 6.90 \times 10^{16}$ nuclei.

(*b*) From Eq. 30–6,

$$\lambda = 0.693/T_{\frac{1}{2}} = (0.693)/(600\,\text{s}) = 1.155 \times 10^{-3}\,\text{s}^{-1}.$$

Then, at $t = 0$ (see Eqs. 30–3b and 30–5)

$$R_0 = \left|\frac{\Delta N}{\Delta t}\right|_0 = \lambda N_0 = (1.155 \times 10^{-3}\,\text{s}^{-1})(6.90 \times 10^{16}) = 7.97 \times 10^{13}\,\text{decays/s}.$$

(*c*) After 1.00 h = 3600 s, the magnitude of the activity will be (Eq. 30–5)

$$R = R_0 e^{-\lambda t} = (7.97 \times 10^{13}\,\text{s}^{-1})e^{-(1.155\times 10^{-3}\,\text{s}^{-1})(3600\,\text{s})} = 1.25 \times 10^{12}\,\text{s}^{-1}.$$

(*d*) We want to determine the time t when $R = 1.00\,\text{s}^{-1}$. From Eq. 30–5, we have

$$e^{-\lambda t} = \frac{R}{R_0} = \frac{1.00\,\text{s}^{-1}}{7.97 \times 10^{13}\,\text{s}^{-1}} = 1.25 \times 10^{-14}.$$

We take the natural log (ln) of both sides $(\ln e^{-\lambda t} = -\lambda t)$ and divide by λ to find

$$t = -\frac{\ln(1.25 \times 10^{-14})}{\lambda} = 2.77 \times 10^{4}\,\text{s} = 7.70\,\text{h}.$$

Easy Alternate Solution to (c) 1.00 h = 60.0 minutes is 6 half-lives, so the activity will decrease to $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$ of its original value, or $(7.97 \times 10^{13})/(64) = 1.25 \times 10^{12}$ per second.