CHAPTER (4) – PART 2 (DYNAMICS: NEWTON'S LAWS OF MOTION)

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4 – 7 Solving Problems with Newton's Laws: Free-Body Diagrams

4 – 8 Problems Involving Friction, Inclines

Summary

Newton's three laws of motion are the basic classical laws describing motion.

Newton's first law (the **law of inertia**) states that if the net force on an object is zero, an object originally at rest remains at rest, and an object in motion remains in motion in a straight line with constant velocity.

Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it, and inversely proportional to its mass:

$$\Sigma \vec{\mathbf{F}} = m\vec{\mathbf{a}}. \tag{4–1}$$

Newton's second law is one of the most important and fundamental laws in classical physics.

Newton's third law states that whenever one object exerts a force on a second object, the second object always exerts a force on the first object which is equal in magnitude but opposite in direction:

$$\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA} \tag{4–2}$$

where $\vec{\mathbf{F}}_{BA}$ is the force on object B exerted by object A.

The tendency of an object to resist a change in its motion is called **inertia**. **Mass** is a measure of the inertia of an object.

Weight refers to the **gravitational force** on an object, and is equal to the product of the object's mass m and the acceleration of gravity $\vec{\mathbf{g}}$:

$$\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}. \tag{4–3}$$

Force, which is a vector, can be considered as a push or pull; or, from Newton's second law, force can be defined as an action capable of giving rise to acceleration. The **net force** on an object is the vector sum of all forces acting on that object.

When two objects slide over one another, the force of friction that each object exerts on the other can be written approximately as $F_{fr} = \mu_k F_N$, where F_N is the **normal force** (the force each object exerts on the other perpendicular to their contact surfaces), and μ_k is the coefficient of **kinetic friction**. If the objects are at rest relative to each other, then F_{fr} is just large enough to hold them at rest and satisfies the inequality $F_{fr} < \mu_s F_N$, where μ_s is the coefficient of **static friction**.

For solving problems involving the forces on one or more objects, it is essential to draw a **free-body diagram** for each object, showing all the forces acting on only that object. Newton's second law can be applied to the vector components for each object.

4–7 SOLVING PROBLEMS WITH NEWTON'S LAWS: FREE-BODY DIAGRAMS

PROBLEM SOLVING

Newton's Laws; Free-Body Diagrams

1. **Draw a sketch** of the situation, after carefully reading the Problem at least twice.
2. Consider only one object (at a time), and draw a **free-body diagram** for that object, showing *all* the forces acting *on* that object. Include any unknown forces that you have to solve for. Do not show any forces that the chosen object exerts on other objects.

   Draw the arrow for each force vector reasonably accurately for direction and magnitude. Label each force acting on the object, including forces you must solve for, according to its source (gravity, person, friction, and so on).

   If several objects are involved, draw a free-body diagram for each object *separately*. For each object, show all the forces acting *on that object* (and *only* forces acting on that object). For each (and every) force, you must be clear about: *on* what object that force acts, and *by* what object that force is exerted. Only forces acting *on* a given object can be included in $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ for that object.
3. Newton's second law involves vectors, and it is usually important to **resolve vectors** into components. **Choose** x and y **axes** in a way that simplifies the calculation. For example, it often saves work if you choose one coordinate axis to be in the direction of the acceleration (if known).
4. For each object, **apply Newton's second law** to the x and y components separately. That is, the x component of the net force on that object is related to the x component of that object's acceleration: $\Sigma F_x = ma_x$, and similarly for the y direction.
5. **Solve** the equation or equations for the unknown(s). Put in numerical values only at the end, and keep track of units.

EXAMPLE 4–6 **Weight, normal force, and a box.** A friend has given you a special gift, a box of mass 10.0 kg with a mystery surprise inside. The box is resting on the smooth (frictionless) horizontal surface of a table (Fig. 4–15a). (*a*) Determine the weight of the box and the normal force exerted on it by the table. (*b*) Now your friend pushes down on the box with a force of 40.0 N, as in Fig. 4–15b. Again determine the normal force exerted on the box by the table. (*c*) If your friend pulls upward on the box with a force of 40.0 N (Fig. 4–15c), what now is the normal force exerted on the box by the table?

APPROACH The box is at rest on the table, so the net force on the box in each case is zero (Newton's first or second law). The weight of the box has magnitude mg in all three cases.

SOLUTION (*a*) The weight of the box is $mg = (10.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 98.0\,\mathrm{N}$, and this force acts downward. The only other force on the box is the normal force exerted upward on it by the table, as shown in Fig. 4–15a. We chose the upward direction as the positive y direction; then the net force ΣF_y on the box is $\Sigma F_y = F_N - mg$; the minus sign means mg acts in the negative y direction (m and g are magnitudes). The box is at rest, so the net force on it must be zero (Newton's second law, $\Sigma F_y = ma_y$, and $a_y = 0$). Thus

$$\Sigma F_y = ma_y$$
$$F_N - mg = 0,$$

so we have

$$F_N = mg.$$

The normal force on the box, exerted by the table, is 98.0 N upward, and has magnitude equal to the box's weight.

(*b*) Your friend is pushing down on the box with a force of 40.0 N. So instead of only two forces acting on the box, now there are three forces acting on the box, as shown in Fig. 4–15b. The weight of the box is still $mg = 98.0\,\mathrm{N}$. The net force is $\Sigma F_y = F_N - mg - 40.0\,\mathrm{N}$, and is equal to zero because the box remains at rest ($a = 0$). Newton's second law gives

$$\Sigma F_y = F_N - mg - 40.0\,\mathrm{N} = 0.$$

We solve this equation for the normal force:

$$F_N = mg + 40.0\,\mathrm{N} = 98.0\,\mathrm{N} + 40.0\,\mathrm{N} = 138.0\,\mathrm{N},$$

which is greater than in (*a*). The table pushes back with more force when a person pushes down on the box. The normal force is not always equal to the weight!

(*c*) The box's weight is still 98.0 N and acts downward. The force exerted by your friend and the normal force both act upward (positive direction), as shown in Fig. 4–15c. The box doesn't move since your friend's upward force is less than the weight. The net force, again set to zero in Newton's second law because $a = 0$, is

$$\Sigma F_y = F_N - mg + 40.0\,\mathrm{N} = 0,$$

so

$$F_N = mg - 40.0\,\mathrm{N} = 98.0\,\mathrm{N} - 40.0\,\mathrm{N} = 58.0\,\mathrm{N}.$$

The table does not push against the full weight of the box because of the upward force exerted by your friend.

NOTE The weight of the box ($= mg$) does not change as a result of your friend's push or pull. Only the normal force is affected.

EXAMPLE 4–7 **Accelerating the box.** What happens when a person pulls upward on the box in Example 4–6*c* with a force equal to, or greater than, the box's weight? For example, let $F_P = 100.0\,\mathrm{N}$ (Fig. 4–16) rather than the 40.0 N shown in Fig. 4–15c.

APPROACH We can start just as in Example 4–6, but be ready for a surprise.

SOLUTION The net force on the box is

$$\begin{aligned}\Sigma F_y &= F_N - mg + F_P \\ &= F_N - 98.0\,\mathrm{N} + 100.0\,\mathrm{N},\end{aligned}$$

and if we set this equal to zero (thinking the acceleration might be zero), we would get $F_N = -2.0\,\mathrm{N}$. This is nonsense, since the negative sign implies F_N points downward, and the table surely cannot *pull* down on the box (unless there's glue on the table). The least F_N can be is zero, which it will be in this case. What really happens here is that the box accelerates upward $(a \neq 0)$ because the net force is not zero. The net force (setting the normal force $F_N = 0$) is

$$\begin{aligned}\Sigma F_y = F_P - mg &= 100.0\,\mathrm{N} - 98.0\,\mathrm{N} \\ &= 2.0\,\mathrm{N}\end{aligned}$$

upward. See Fig. 4–16. We apply Newton's second law and see that the box moves upward with an acceleration

$$\begin{aligned}a_y = \frac{\Sigma F_y}{m} &= \frac{2.0\,\mathrm{N}}{10.0\,\mathrm{kg}} \\ &= 0.20\,\mathrm{m/s^2}.\end{aligned}$$

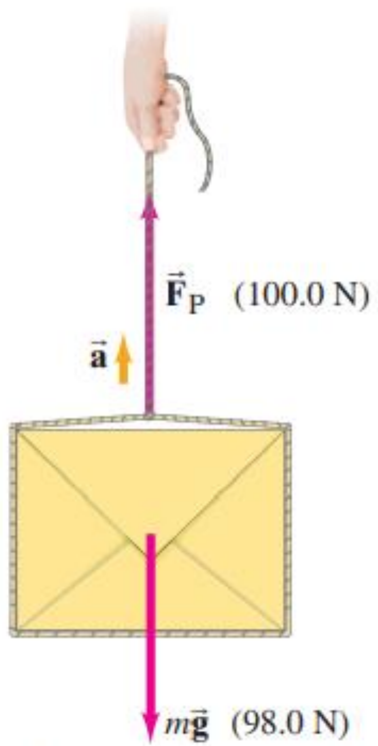
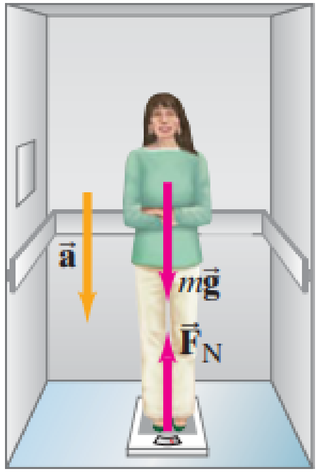


FIGURE 4–16 Example 4–7. The box accelerates upward because $F_P > mg$.

FIGURE 4–17 Example 4–8. The acceleration vector is shown in gold to distinguish it from the red force vectors.



EXAMPLE 4–8 **Apparent weight loss.** A 65-kg woman descends in an elevator that briefly accelerates at $0.20g$ downward. She stands on a scale that reads in kg. (*a*) During this acceleration, what is her weight and what does the scale read? (*b*) What does the scale read when the elevator descends at a constant speed of 2.0 m/s?

APPROACH Figure 4–17 shows all the forces that act on the woman (and *only* those that act on her). The direction of the acceleration is downward, so we choose the positive direction as down (this is the opposite choice from Examples 4–6 and 4–7).

SOLUTION (*a*) From Newton's second law,

$$\begin{aligned} \Sigma F &= ma \\ mg - F_N &= m(0.20g). \end{aligned}$$

We solve for F_N:

$$\begin{aligned} F_N &= mg - 0.20mg \\ &= 0.80mg, \end{aligned}$$

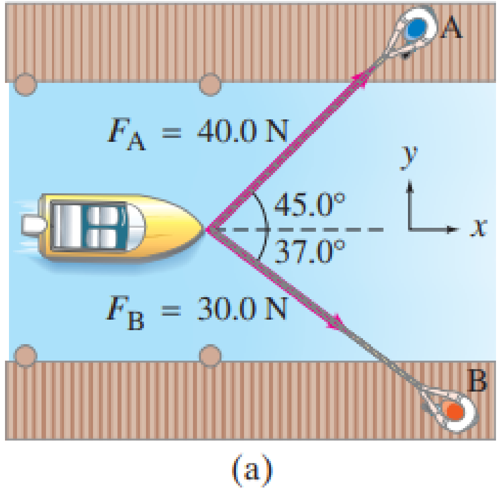
and it acts upward. The normal force $\vec{\mathbf{F}}_N$ is the force the scale exerts on the person, and is equal and opposite to the force she exerts on the scale: $F'_N = 0.80mg$ downward. Her weight (force of gravity on her) is still $mg = (65\,\text{kg})(9.8\,\text{m/s}^2) = 640\,\text{N}$. But the scale, needing to exert a force of only $0.80mg$, will give a reading of $0.80m = 52\,\text{kg}$.

(*b*) Now there is no acceleration, $a = 0$, so by Newton's second law, $mg - F_N = 0$ and $F_N = mg$. The scale reads her true mass of 65 kg.

NOTE The scale in (*a*) gives a reading of 52 kg (as an "apparent mass"), but her mass doesn't change as a result of the acceleration: it stays at 65 kg.

EXAMPLE 4–9 **Adding force vectors.** Calculate the sum of the two forces exerted on the boat by workers A and B in Fig. 4–19a.

FIGURE 4–19 Example 4–9: Two force vectors act on a boat.



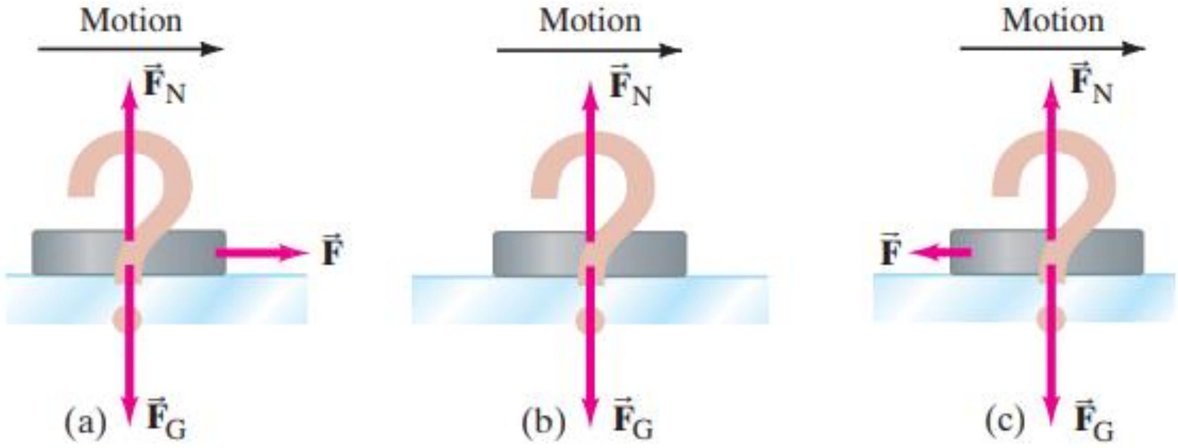


FIGURE 4–20 Example 4–10. Which is the correct free-body diagram for a hockey puck sliding across frictionless ice?

CONCEPTUAL EXAMPLE 4–10 **The hockey puck.** A hockey puck is sliding at constant velocity across a flat horizontal ice surface that is assumed to be frictionless. Which of the sketches in Fig. 4–20 is the correct free-body diagram for this puck? What would your answer be if the puck slowed down?

RESPONSE Did you choose (*a*)? If so, can you answer the question: what exerts the horizontal force labeled $\vec{\mathbf{F}}$ on the puck? If you say that it is the force needed to maintain the motion, ask yourself: what exerts this force? Remember that another object must exert any force—and there simply isn't any possibility here. Therefore, (*a*) is wrong. Besides, the force $\vec{\mathbf{F}}$ in Fig. 4–20a would give rise to an acceleration by Newton's second law. It is (*b*) that is correct. No net force acts on the puck, and the puck slides at constant velocity across the ice.

In the real world, where even smooth ice exerts at least a tiny friction force, then (*c*) is the correct answer. The tiny friction force is in the direction opposite to the motion, and the puck's velocity decreases, even if very slowly.

EXAMPLE 4–11 **Pulling the mystery box.** Suppose a friend asks to examine the 10.0-kg box you were given (Example 4–6, Fig. 4–15), hoping to guess what is inside; and you respond, "Sure, pull the box over to you." She then pulls the box by the attached cord, as shown in Fig. 4–21a, along the smooth surface of the table. The magnitude of the force exerted by the person is $F_P = 40.0\ \text{N}$, and it is exerted at a 30.0° angle as shown. Calculate (*a*) the acceleration of the box, and (*b*) the magnitude of the upward force F_N exerted by the table on the box. Assume that friction can be neglected.

SOLUTION

1. **Draw a sketch**: The situation is shown in Fig. 4–21a; it shows the box and the force applied by the person, F_P.
2. **Free-body diagram**: Figure 4–21b shows the free-body diagram of the box. To draw it correctly, we show *all* the forces acting on the box and *only* the forces acting on the box. They are: the force of gravity $m\vec{\mathbf{g}}$; the normal force exerted by the table $\vec{\mathbf{F}}_N$; and the force exerted by the person $\vec{\mathbf{F}}_P$. We are interested only in translational motion, so we can show the three forces acting at a point, Fig. 4–21c.
3. **Choose axes and resolve vectors**: We expect the motion to be horizontal, so we choose the x axis horizontal and the y axis vertical. The pull of 40.0 N has components

$$F_{Px} = (40.0\ \text{N})(\cos 30.0°) = (40.0\ \text{N})(0.866) = 34.6\ \text{N},$$
$$F_{Py} = (40.0\ \text{N})(\sin 30.0°) = (40.0\ \text{N})(0.500) = 20.0\ \text{N}.$$

In the horizontal (x) direction, $\vec{\mathbf{F}}_N$ and $m\vec{\mathbf{g}}$ have zero components. Thus the horizontal component of the net force is F_{Px}.

4. (*a*) **Apply Newton's second law** to get the x component of the acceleration:

$$F_{Px} = ma_x.$$

5. (*a*) **Solve**:

$$a_x = \frac{F_{Px}}{m} = \frac{(34.6\ \text{N})}{(10.0\ \text{kg})} = 3.46\ \text{m/s}^2.$$

The acceleration of the box is 3.46 m/s² to the right.

(*b*) Next we want to find F_N.

4′. (*b*) **Apply Newton's second law** to the vertical (y) direction, with upward as positive:

$$\Sigma F_y = ma_y$$
$$F_N - mg + F_{Py} = ma_y.$$

5′. (*b*) **Solve**: We have $mg = (10.0\ \text{kg})(9.80\ \text{m/s}^2) = 98.0\ \text{N}$ and, from point 3 above, $F_{Py} = 20.0\ \text{N}$. Furthermore, since $F_{Py} < mg$, the box does not move vertically, so $a_y = 0$. Thus

$$F_N - 98.0\ \text{N} + 20.0\ \text{N} = 0,$$

so

$$F_N = 78.0\ \text{N}.$$

NOTE F_N is less than mg: the table does not push against the full weight of the box because part of the pull exerted by the person is in the upward direction.

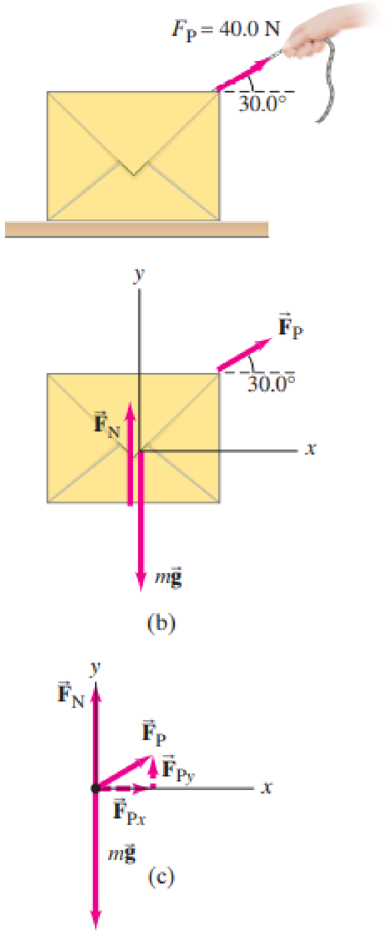


FIGURE 4–21 (a) Pulling the box, Example 4–11; (b) is the free-body diagram for the box, and (c) is the free-body diagram considering all the forces to act at a point (translational motion only, which is what we have here).

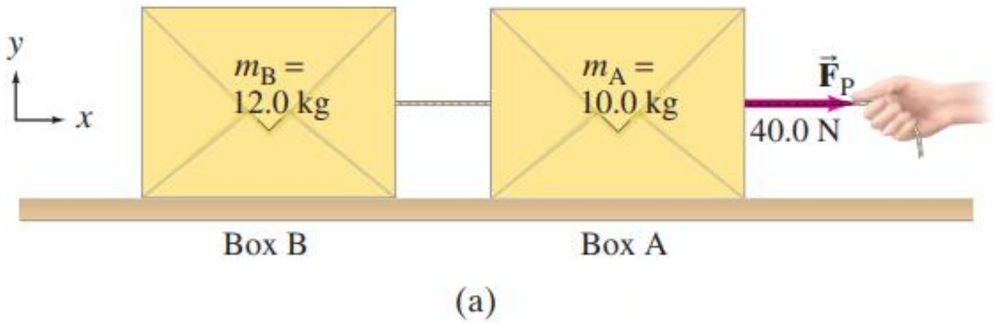


FIGURE 4–22 Example 4–12. (a) Two boxes, A and B, are connected by a cord. A person pulls horizontally on box A with force $F_P = 40.0$ N. (b) Free-body diagram for box A. (c) Free-body diagram for box B.

EXAMPLE 4–12 **Two boxes connected by a cord.** Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth (frictionless) table. The boxes have masses of 12.0 kg and 10.0 kg. A horizontal force F_P of 40.0 N is applied to the 10.0-kg box, as shown in Fig. 4–22a. Find (*a*) the acceleration of each box, and (*b*) the tension in the cord connecting the boxes.

SOLUTION (*a*) We apply $\Sigma F_x = ma_x$ to box A:

$$\Sigma F_x = F_P - F_T = m_A a_A. \qquad \text{[box A]}$$

For box B, the only horizontal force is F_T, so

$$\Sigma F_x = F_T = m_B a_B. \qquad \text{[box B]}$$

The boxes are connected, and if the cord remains taut and doesn't stretch, then the two boxes will have the same acceleration a. Thus $a_A = a_B = a$. We are given $m_A = 10.0\,\text{kg}$ and $m_B = 12.0\,\text{kg}$. We can add the two equations above to eliminate an unknown (F_T) and obtain

$$(m_A + m_B)a = F_P - F_T + F_T = F_P$$

or

$$a = \frac{F_P}{m_A + m_B} = \frac{40.0\,\text{N}}{22.0\,\text{kg}} = 1.82\,\text{m/s}^2.$$

This is what we sought.

(*b*) From the equation for box B above $(F_T = m_B a_B)$, the tension in the cord is

$$F_T = m_B a = (12.0\,\text{kg})(1.82\,\text{m/s}^2) = 21.8\,\text{N}.$$

Thus, $F_T < F_P$ $(= 40.0\,\text{N})$, as we expect, since F_T acts to accelerate only m_B.

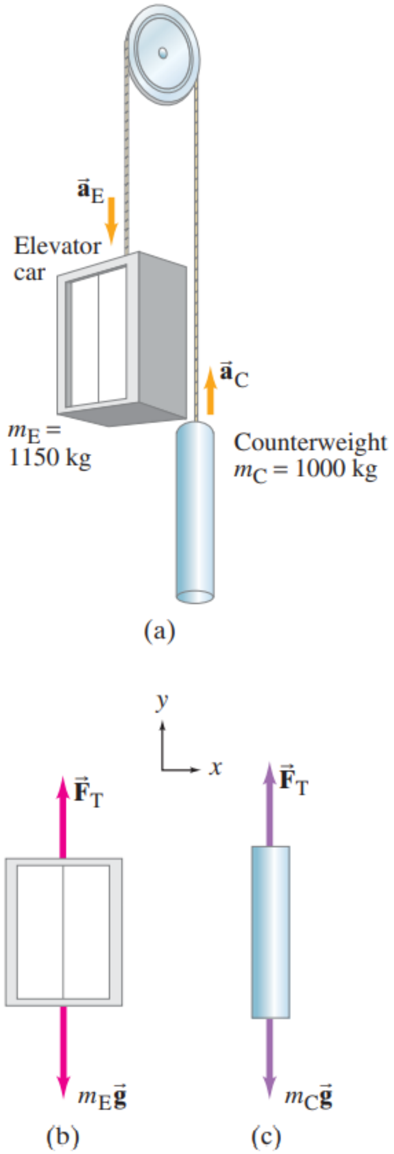
Alternate Solution to (a) We would have obtained the same result had we considered a single system, of mass $m_A + m_B$, acted on by a net horizontal force equal to F_P. (The tension forces F_T would then be considered internal to the system as a whole, and summed together would make zero contribution to the net force on the *whole* system.)

NOTE It might be tempting to say that the force the person exerts, F_P, acts not only on box A but also on box B. It doesn't. F_P acts only on box A. It affects box B via the tension in the cord, F_T, which acts on box B and accelerates it. (You could look at it this way: $F_T < F_P$ because F_P accelerates *both* boxes whereas F_T only accelerates box B.)

CAUTION

For any object, use only the forces on that object in calculating $\Sigma F = ma$

EXAMPLE 4–13 **Elevator and counterweight (Atwood machine).** A system of two objects suspended over a pulley by a flexible cable, as shown in Fig. 4–23a, is sometimes referred to as an *Atwood machine*. Consider the real-life application of an elevator (m_E) and its counterweight (m_C). To minimize the work done by the motor to raise and lower the elevator safely, m_E and m_C are made similar in mass. We leave the motor out of the system for this calculation, and assume that the cable's mass is negligible and that the mass of the pulley, as well as any friction, is small and ignorable. These assumptions ensure that the tension F_T in the cable has the same magnitude on both sides of the pulley. Let the mass of the counterweight be $m_C = 1000\,\text{kg}$. Assume the mass of the empty elevator is 850 kg, and its mass when carrying four passengers is $m_E = 1150\,\text{kg}$. For the latter case ($m_E = 1150\,\text{kg}$), calculate (*a*) the acceleration of the elevator and (*b*) the tension in the cable.



SOLUTION (*a*) To find F_T as well as the acceleration a, we apply Newton's second law, $\Sigma F = ma$, to each object. We take upward as the positive y direction for both objects. With this choice of axes, $a_C = a$ because m_C accelerates upward, and $a_E = -a$ because m_E accelerates downward. Thus

$$F_T - m_E g = m_E a_E = -m_E a$$
$$F_T - m_C g = m_C a_C = +m_C a.$$

We can subtract the first equation from the second to get

$$(m_E - m_C)g = (m_E + m_C)a,$$

where a is now the only unknown. We solve this for a:

$$a = \frac{m_E - m_C}{m_E + m_C} g = \frac{1150\text{ kg} - 1000\text{ kg}}{1150\text{ kg} + 1000\text{ kg}} g = 0.070g = 0.68\text{ m/s}^2.$$

The elevator (m_E) accelerates downward (and the counterweight m_C upward) at $a = 0.070g = 0.68\text{ m/s}^2$.

(*b*) The tension in the cable F_T can be obtained from either of the two $\Sigma F = ma$ equations at the start of our solution, setting $a = 0.070g = 0.68\text{ m/s}^2$:

$$\begin{aligned} F_T &= m_E g - m_E a = m_E(g - a) \\ &= 1150\text{ kg}\,(9.80\text{ m/s}^2 - 0.68\text{ m/s}^2) = 10{,}500\text{ N}, \end{aligned}$$

or

$$\begin{aligned} F_T &= m_C g + m_C a = m_C(g + a) \\ &= 1000\text{ kg}\,(9.80\text{ m/s}^2 + 0.68\text{ m/s}^2) = 10{,}500\text{ N}, \end{aligned}$$

which are consistent. As predicted, our result lies between 9800 N and 11,300 N.

NOTE We can check our equation for the acceleration a in this Example by noting that if the masses were equal $(m_E = m_C)$, then our equation above for a would give $a = 0$, as we should expect. Also, if one of the masses is zero (say, $m_C = 0$), then the other mass $(m_E \neq 0)$ would be predicted by our equation to accelerate at $a = g$, again as expected.

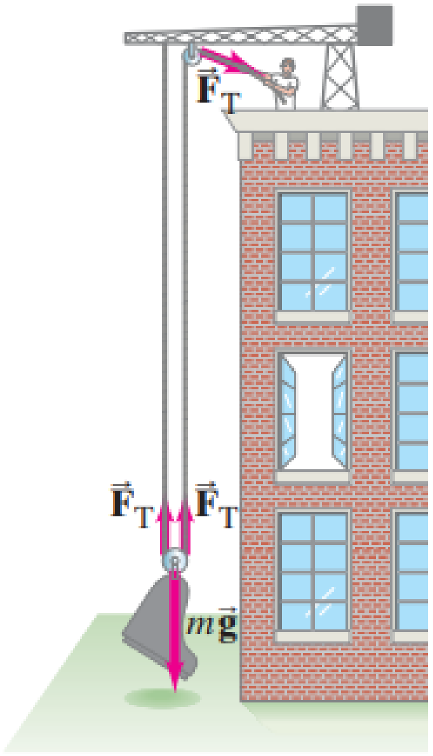


FIGURE 4–24 Example 4–14.

CONCEPTUAL EXAMPLE 4–14 **The advantage of a pulley.** A mover is trying to lift a piano (slowly) up to a second-story apartment (Fig. 4–24). He is using a rope looped over two pulleys as shown. What force must he exert on the rope to slowly lift the piano's 1600-N weight?

RESPONSE The magnitude of the tension force F_T within the rope is the same at any point along the rope if we assume we can ignore its mass. First notice the forces acting on the lower pulley at the piano. The weight of the piano $(= mg)$ pulls down on the pulley. The tension in the rope, looped through this pulley, pulls up *twice*, once on each side of the pulley. Let us apply Newton's second law to the pulley–piano combination (of mass m), choosing the upward direction as positive:

$$2F_T - mg = ma.$$

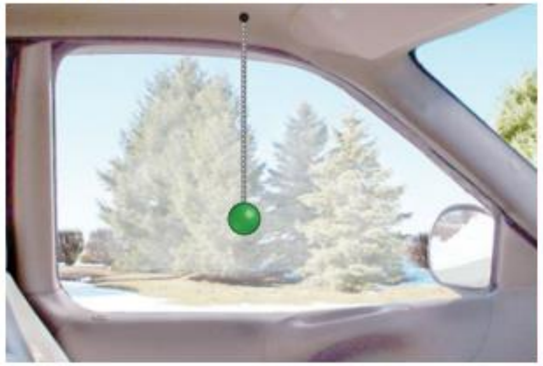
To move the piano with constant speed (set $a = 0$ in this equation) thus requires a tension in the rope, and hence a pull on the rope, of $F_T = mg/2$. The piano mover can exert a force equal to half the piano's weight.

NOTE We say the pulley has given a **mechanical advantage** of 2, since without the pulley the mover would have to exert twice the force.

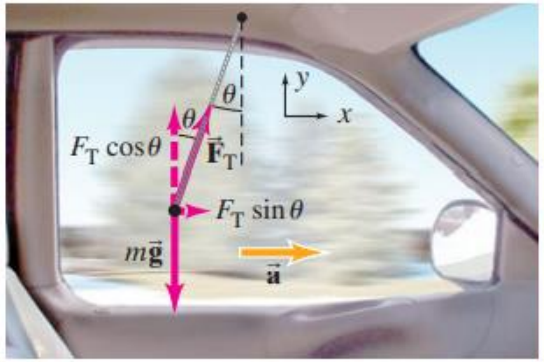
PHYSICS APPLIED
Accelerometer

EXAMPLE 4–15 **Accelerometer.** A small mass m hangs from a thin string and can swing like a pendulum. You attach it above the window of your car as shown in Fig. 4–25a. When the car is at rest, the string hangs vertically. What angle θ does the string make (a) when the car accelerates at a constant $a = 1.20\ \mathrm{m/s^2}$, and (b) when the car moves at constant velocity, $v = 90\ \mathrm{km/h}$?

FIGURE 4–25 Example 4–15.



(a)



(b)

4–8 PROBLEMS INVOLVING FRICTION, INCLINES

4–8 PROBLEMS INVOLVING FRICTION, INCLINES

Friction

Objects look smooth to the naked eye. But under a microscope objects have irregularities.

As the surfaces which are in contact move against each other the irregularities impede the motion.

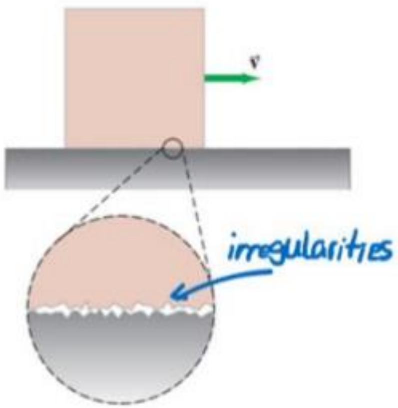
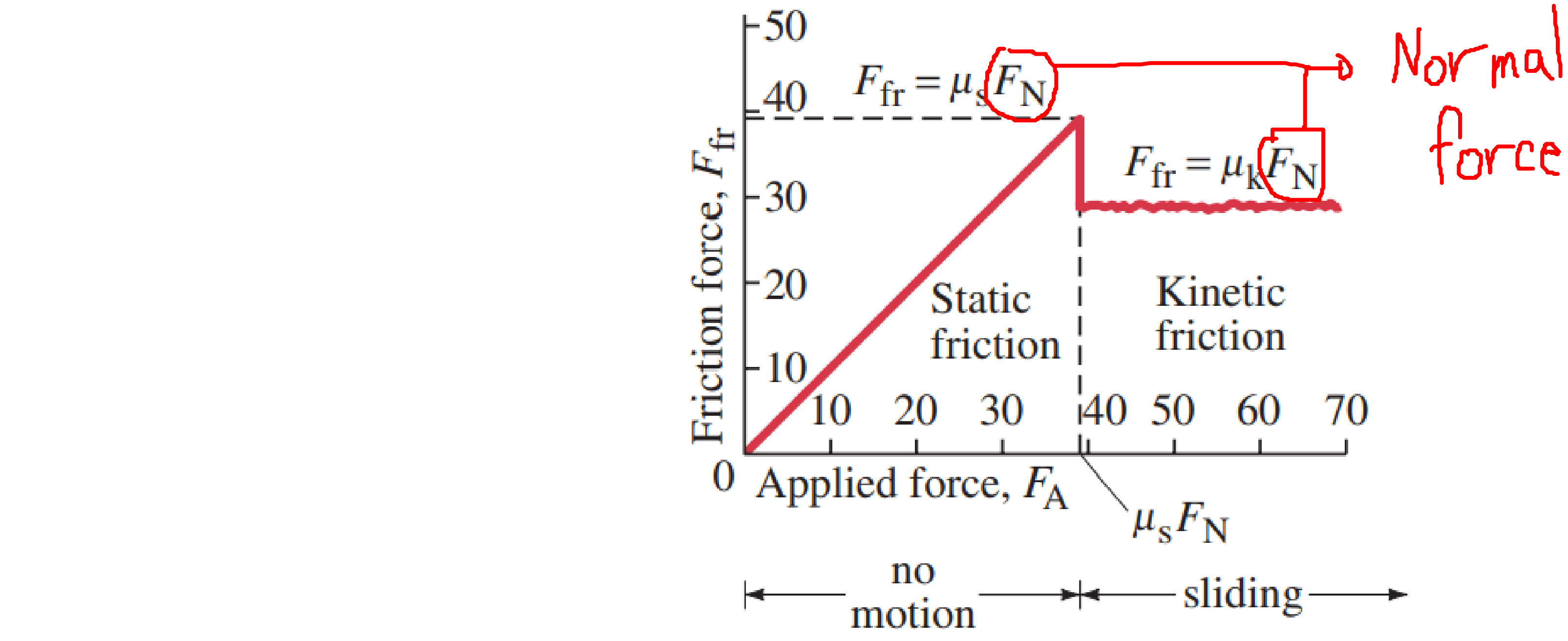


FIGURE 4–26 An object moving to the right on a table. The two surfaces in contact are assumed smooth, but are rough on a microscopic scale.

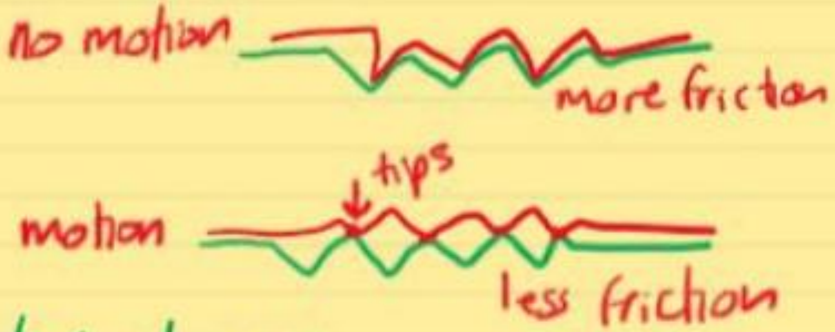
FIGURE 4–28 Example 4–16. Magnitude of the force of friction as a function of the external force applied to an object initially at rest. As the applied force is increased in magnitude, the force of static friction increases in proportion until the applied force equals $\mu_s F_N$. If the applied force increases further, the object will begin to move, and the friction force drops to a roughly constant value characteristic of kinetic friction.



$f_k = \mu_k N \longleftarrow$ normal force

$\uparrow$

coefficient of
kinetic friction (has no units)



Note that $f_k < f_{s,max}$. $\Rightarrow \mu_k < \mu_s$

when the object starts moving the contact between the irregularities is mostly at the tips which reduces the surface area of contact and hence reduces the force of friction.

NOTE THAT THE COEFFICIENT OF FRICTIONS DOESN'T DEPEND ON THE SURFACE AREA

TABLE 4–2 Coefficients of Friction†

Surfaces	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	0.01	0.01

† Values are approximate and intended only as a guide.

CONCEPTUAL EXAMPLE 4–17 **A box against a wall.** You can hold a box against a rough wall (Fig. 4–29) and prevent it from slipping down by pressing hard horizontally. How does the application of a horizontal force keep an object from moving vertically?

RESPONSE This won't work well if the wall is slippery. You need friction. Even then, if you don't press hard enough, the box will slip. The horizontal force you apply produces a normal force on the box exerted by the wall (the net force horizontally is zero since the box doesn't move horizontally). The force of gravity mg, acting downward on the box, can now be balanced by an upward static friction force whose maximum magnitude is proportional to the normal force. The harder you push, the greater F_N is and the greater F_{fr} can be. If you don't press hard enough, then $mg > \mu_s F_N$ and the box begins to slide down.

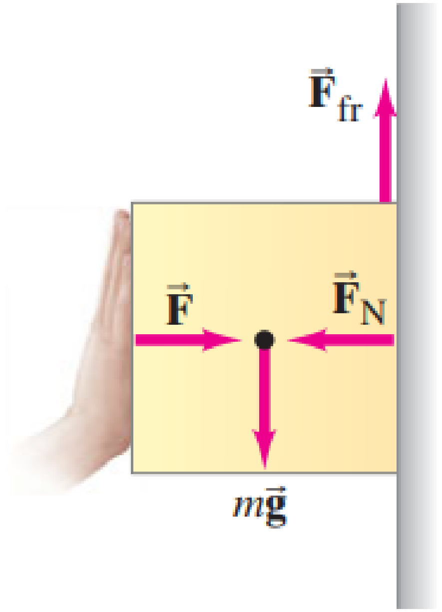


FIGURE 4–29 Example 4–17.

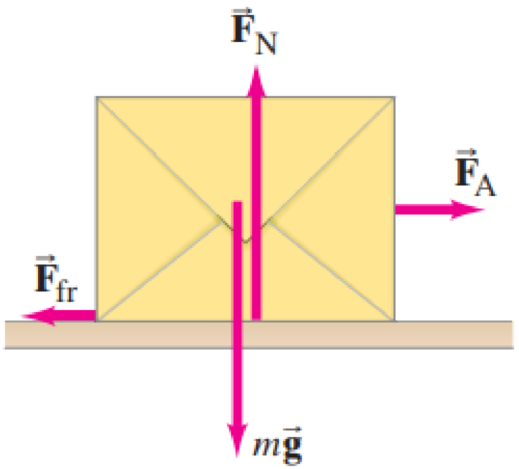


FIGURE 4–27

EXAMPLE 4–16 **Friction: static and kinetic.** Our 10.0-kg mystery box rests on a horizontal floor. The coefficient of static friction is $\mu_s = 0.40$ and the coefficient of kinetic friction is $\mu_k = 0.30$. Determine the force of friction, F_{fr}, acting on the box if a horizontal applied force F_A is exerted on it of magnitude: (*a*) 0, (*b*) 10 N, (*c*) 20 N, (*d*) 38 N, and (*e*) 40 N.

APPROACH We don't know, right off, if we are dealing with static friction or kinetic friction, nor if the box remains at rest or accelerates. We need to draw a free-body diagram, and then determine in each case whether or not the box will move: the box starts moving if F_A is greater than the maximum static friction force (Newton's second law). The forces on the box are gravity $m\vec{g}$, the normal force exerted by the floor $\vec{F}_N$, the horizontal applied force $\vec{F}_A$, and the friction force $\vec{F}_{fr}$, as shown in Fig. 4–27.

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SOLUTION The free-body diagram of the box is shown in Fig. 4–27. In the vertical direction there is no motion, so Newton's second law in the vertical direction gives $\Sigma F_y = ma_y = 0$, which tells us $F_N - mg = 0$. Hence the normal force is

$$F_N = mg = (10.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 98.0\,\mathrm{N}.$$

(*a*) Because $F_A = 0$ in this first case, the box doesn't move, and $F_{fr} = 0$.

(*b*) The force of static friction will oppose any applied force up to a maximum of

$$\mu_s F_N = (0.40)(98.0\,\mathrm{N}) = 39\,\mathrm{N}.$$

When the applied force is $F_A = 10\,\mathrm{N}$, the box will not move. Newton's second law gives $\Sigma F_x = F_A - F_{fr} = 0$, so $F_{fr} = 10\,\mathrm{N}$.

(*c*) An applied force of 20 N is also not sufficient to move the box. Thus $F_{fr} = 20\,\mathrm{N}$ to balance the applied force.

(*d*) The applied force of 38 N is still not quite large enough to move the box; so the friction force has now increased to 38 N to keep the box at rest.

(*e*) A force of 40 N will start the box moving since it exceeds the maximum force of static friction, $\mu_s F_N - (0.40)(98\,\mathrm{N}) - 39\,\mathrm{N}$. Instead of static friction, we now have kinetic friction, and its magnitude is

$$F_{fr} = \mu_k F_N = (0.30)(98.0\,\mathrm{N}) = 29\,\mathrm{N}.$$

There is now a net (horizontal) force on the box of magnitude $F = 40\,\mathrm{N} - 29\,\mathrm{N} = 11\,\mathrm{N}$, so the box will accelerate at a rate

$$a_x = \frac{\Sigma F}{m} = \frac{11\,\mathrm{N}}{10.0\,\mathrm{kg}} = 1.1\,\mathrm{m/s^2}$$

as long as the applied force is 40 N. Figure 4–28 shows a graph that summarizes this Example.

EXAMPLE 4–18 **Pulling against friction.** A 10.0-kg box is pulled along a horizontal surface by a force F_P of 40.0 N applied at a 30.0° angle above horizontal. This is like Example 4–11 except now there is friction, and we assume a coefficient of kinetic friction of 0.30. Calculate the acceleration.

APPROACH The free-body diagram is shown in Fig. 4–30. It is much like that in Fig. 4–21b, but with one more force, friction.

SOLUTION The calculation for the vertical (y) direction is just the same as in Example 4–11b, $mg = (10.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 98.0\,\mathrm{N}$ and $F_{Py} = (40.0\,\mathrm{N})(\sin 30.0°) = 20.0\,\mathrm{N}$. With y positive upward and $a_y = 0$, we have

$$F_N - mg + F_{Py} = ma_y$$
$$F_N - 98.0\,\mathrm{N} + 20.0\,\mathrm{N} = 0,$$

so the normal force is $F_N = 78.0\,\mathrm{N}$. Now we apply Newton's second law for the horizontal (x) direction (positive to the right), and include the friction force:

$$F_{Px} - F_{fr} = ma_x.$$

The friction force is kinetic friction as long as $F_{fr} = \mu_k F_N$ is less than $F_{Px} = (40.0\,\mathrm{N})\cos 30.0° = 34.6\,\mathrm{N}$, which it is:

$$F_{fr} = \mu_k F_N = (0.30)(78.0\,\mathrm{N}) = 23.4\,\mathrm{N}.$$

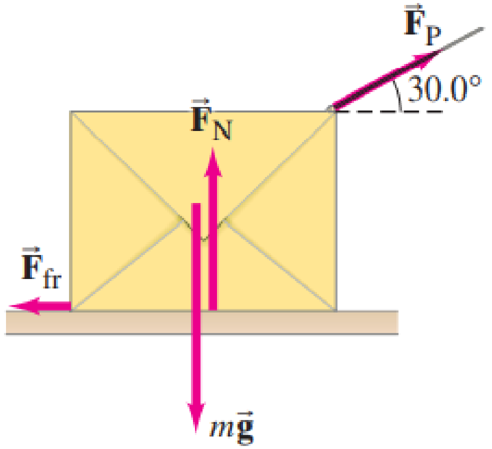
Hence the box does accelerate:

$$a_x = \frac{F_{Px} - F_{fr}}{m} = \frac{34.6\,\mathrm{N} - 23.4\,\mathrm{N}}{10.0\,\mathrm{kg}} = 1.1\,\mathrm{m/s^2}.$$

In the absence of friction, as we saw in Example 4–11, the acceleration would be much greater than this.

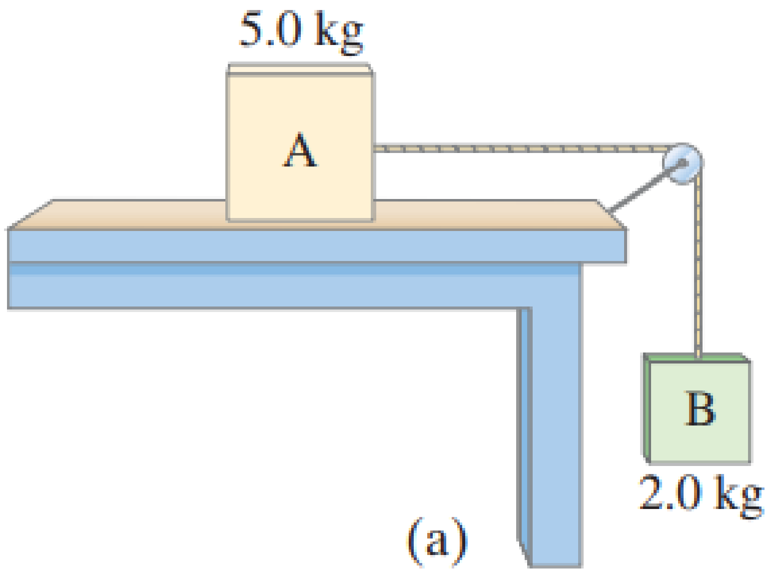
NOTE Our final answer has only two significant figures because our least significant input value $(\mu_k = 0.30)$ has two.

FIGURE 4–30 Example 4–18.



EXAMPLE 4–20 **Two boxes and a pulley.** In Fig. 4–32a, two boxes are connected by a cord running over a pulley. The coefficient of kinetic friction between box A and the table is 0.20. We ignore the mass of the cord and pulley and any friction in the pulley, which means we can assume that a force applied to one end of the cord will have the same magnitude at the other end. We wish to find the acceleration, a, of the system, which will have the same magnitude for both boxes assuming the cord doesn't stretch. As box B moves down, box A moves to the right.

FIGURE 4–32 Example 4–20.



APPROACH The free-body diagrams for each box are shown in Figs. 4–32b and c. The forces on box A are the pulling force of the cord F_T, gravity $m_A g$, the normal force exerted by the table F_N, and a friction force exerted by the table F_{fr}; the forces on box B are gravity $m_B g$, and the cord pulling up, F_T.

SOLUTION Box A does not move vertically, so Newton's second law tells us the normal force just balances the weight,

$$F_N = m_A g = (5.0\,\text{kg})(9.8\,\text{m/s}^2) = 49\,\text{N}.$$

In the horizontal direction, there are two forces on box A (Fig. 4–32b): F_T, the tension in the cord (whose value we don't know), and the force of friction

$$F_{fr} = \mu_k F_N = (0.20)(49\,\text{N}) = 9.8\,\text{N}.$$

The horizontal acceleration (box A) is what we wish to find; we use Newton's second law in the x direction, $\Sigma F_{Ax} = m_A a_x$, which becomes (taking the positive direction to the right and setting $a_{Ax} = a$):

$$\Sigma F_{Ax} = F_T - F_{fr} = m_A a. \qquad \text{[box A]}$$

Next consider box B. The force of gravity $m_B g = (2.0\,\text{kg})(9.8\,\text{m/s}^2) = 19.6\,\text{N}$ pulls downward; and the cord pulls upward with a force F_T. So we can write Newton's second law for box B (taking the downward direction as positive):

$$\Sigma F_{By} = m_B g - F_T = m_B a. \qquad \text{[box B]}$$

[Notice that if $a \neq 0$, then F_T is not equal to $m_B g$.]

We have two unknowns, a and F_T, and we also have two equations. We solve the box A equation for F_T:

$$F_T = F_{fr} + m_A a,$$

and substitute this into the box B equation:

$$m_B g - F_{fr} - m_A a = m_B a.$$

Now we solve for a and put in numerical values:

$$a = \frac{m_B g - F_{fr}}{m_A + m_B} = \frac{19.6\,\text{N} - 9.8\,\text{N}}{5.0\,\text{kg} + 2.0\,\text{kg}} = 1.4\,\text{m/s}^2,$$

which is the acceleration of box A to the right, and of box B down.

If we wish, we can calculate F_T using the third equation up from here:

$$F_T = F_{fr} + m_A a = 9.8\,\text{N} + (5.0\,\text{kg})(1.4\,\text{m/s}^2) = 17\,\text{N}.$$

NOTE Box B is not in free fall. It does not fall at $a = g$ because an additional force, F_T, is acting upward on it.

EXAMPLE 4–21 **The skier.** The skier in Fig. 4–34a has begun descending the 30° slope. If the coefficient of kinetic friction is 0.10, what is her acceleration?

APPROACH We choose the x axis along the slope, positive downslope in the direction of the skier's motion. The y axis is perpendicular to the surface. The forces acting on the skier are gravity, $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$, which points vertically downward (*not* perpendicular to the slope), and the two forces exerted on her skis by the snow—the normal force perpendicular to the snowy slope (*not* vertical), and the friction force parallel to the surface. These three forces are shown acting at one point in Fig. 4–34b, which is our free-body diagram for the skier.

SOLUTION We have to resolve only one vector into components, the weight $\vec{\mathbf{F}}_G$, and its components are shown as dashed lines in Fig. 4–34c. To be general, we use θ rather than 30° for now. We use the definitions of sine ("side opposite") and cosine ("side adjacent") to obtain the components:

$$\begin{aligned} F_{Gx} &= mg \sin\theta, \\ F_{Gy} &= -mg\cos\theta \end{aligned}$$

where F_{Gy} is in the negative y direction. To calculate the skier's acceleration down the hill, a_x, we apply Newton's second law to the x direction:

$$\begin{aligned} \Sigma F_x &= ma_x \\ mg\sin\theta - \mu_k F_N &= ma_x \end{aligned}$$

where the two forces are the x component of the gravity force ($+x$ direction) and the friction force ($-x$ direction). We want to find the value of a_x, but we don't yet know F_N in the last equation. Let's see if we can get F_N from the y component of Newton's second law:

$$\begin{aligned} \Sigma F_y &= ma_y \\ F_N - mg\cos\theta &= ma_y = 0 \end{aligned}$$

where we set $a_y = 0$ because there is no motion in the y direction (perpendicular to the slope). Thus we can solve for F_N:

$$F_N = mg\cos\theta$$

and we can substitute this into our equation above for ma_x:

$$mg\sin\theta - \mu_k(mg\cos\theta) = ma_x.$$

There is an m in each term which can be canceled out. Thus (setting $\theta = 30°$ and $\mu_k = 0.10$):

$$\begin{aligned} a_x &= g\sin 30° - \mu_k g\cos 30° \\ &= 0.50g - (0.10)(0.866)g = 0.41g. \end{aligned}$$

The skier's acceleration is 0.41 times the acceleration of gravity, which in numbers[†] is $a = (0.41)(9.8\,\mathrm{m/s^2}) = 4.0\,\mathrm{m/s^2}$.

NOTE The mass canceled out, so we have the useful conclusion that *the acceleration doesn't depend on the mass*. That such a cancellation sometimes occurs, and thus may give a useful conclusion as well as saving calculation, is a big advantage of working with the algebraic equations and putting in the numbers only at the end.