Chapter (5)

Continuous probability distribution

Sheet (1)

Q1 let X~$N(5, \sigma^2)$ & P(5<X≤8)=0.3413, then σ=

A) 5 B) 4 C) 3 D) 9 E) 2

Solution: $P(5 < x \leq 8) = 0.3413 \rightarrow P\left(\frac{5-5}{\sigma} < Z \leq \frac{8-5}{\sigma}\right) = 0.3413$

$P\left(0 < Z \leq \frac{3}{\sigma}\right) = 0.3413 \rightarrow P\left(Z \leq \frac{3}{\sigma}\right) - P(Z < 0) = 0.3413$

$\therefore P\left(Z \leq \frac{3}{\sigma}\right) = 0.8413 \rightarrow \frac{3}{\sigma} - 1 \rightarrow \sigma = 3 \rightarrow C$

Q2 In a class, the grades of students are normally distributed with mean 65 and variance 63 . If a student is selected randomly from this class, then the probability that grade of student is between 65 and 70 will be:

A) 0.3413 B) 0.2357 C) 0.4082 D) 0.3112 E) 0.2967

Solution:

$P(65 < x < 70) = P\left(\frac{65-65}{\sqrt{63}} < Z < \frac{70-65}{\sqrt{63}}\right) = P(0 < Z < 0.63) = P(Z < 0.63) - P(Z < 0)$

$= 0.7357 - 0.5 = 0.2357 \rightarrow B$

Q3 Let X~N(54 , σ^2) and P(X > 44)= 0.9772 .Then σ =

A) 1 B) 2 C) 3 D) 4 E) 5

Solution: $P(x > 44) = 0.9772 \rightarrow P(x \leq 44) = 0.0228$

$P\left(Z \leq \frac{44-54}{\sigma}\right) = 0.0228$

$\frac{44-54}{\sigma} = -2 \rightarrow -10 = -2\sigma \Rightarrow \sigma = 5 \rightarrow E$

Q4 If X~N(μ,25) and P(X>50) = 0.9452. Then μ =

A) 62 B) 56 C) 64 D) 58 E) 60

Solution:

$P(X > 50) = 0.9452 \rightarrow P\left(Z > \frac{50-\mu}{5}\right) = 0.9452 \rightarrow 1 - P\left(Z < \frac{50-\mu}{5}\right) = 0.9452$

$P\left(Z < \frac{50-\mu}{5}\right) = 0.0548 \rightarrow \frac{50-\mu}{5} = -1.60$ $\mu = 58 \rightarrow$ D

Q5 The grade of a math test are normally distributed with mean 70 and variance 100

i) The proportion of math grades that are greater than 80 equals

A) 0.0228 B) 0.0062 C) 0.1587 D) 0.0668

ii) if P(50 < X < a) = 0.50 , then a equals:

A) 0.4332 B) 70.6 C) 19.15 D) 47.72

Solution:

i) $P(x > 80) = P\left(Z > \frac{80-70}{10}\right) = P(Z > 1) = P(z < -1) = 0.1587 \rightarrow C$

ii) $P(50 < z < a) = 0.5 \rightarrow P\left(\frac{50-70}{10} < Z < \frac{a-70}{\sqrt{100}}\right) = 0.5$

$P\left(-2 < Z < \frac{a-70}{10}\right) = 0.5 \rightarrow P(Z < \frac{a-70}{10}) - P(Z < -2) = 0.5$

$P\left(\frac{a-70}{10}\right) = 0.5 + P(Z < -2) = 0.5 + 0.0228 = 0.5228$

$\therefore \frac{a-70}{10} = 0.06 \rightarrow a = 70.6$

Q6 if a group of students have test scores that are normally distributed with mean 82 & standard deviation 4 , then half of the students made a grade below :

A) 82 B) 86 C) 0.1355 D) 64 E) 16

Solution: 82 , because the normal distribution is symmetric so $Q_2 = \text{mean} = 82$

Q7 The shelf life of a particular dairy product is normally distributed with a mean of 12 days and a standard deviation of 3 days. About what percent of the products last 6 days or less?

A)68% **B) 34%** **C) 16%** **D) 2.5%**

Solution: $P(X< 6) = P(\frac{X-\mu}{\sigma} < \frac{6-12}{3}) = P(Z < -2) = 0.0228 \approx 2.5\% \rightarrow D$

Q8 A box has a large number of items which have mean weight 60 gm's and standard deviation 15 gm's. One item was picked at random. If its weight is denoted by X, then P(X > 57) is closest to:

A) 0.5793 **B) 0.2711** **C) 0.58** **D) 0.73** **E) 0.42**

Solution:

$$P(X > 57) = P\left(\frac{X-\mu}{\sigma} > \frac{57-60}{15}\right) = P(Z > -0.2) = P(Z < 0.2) = 0.5793 \rightarrow A$$

Q9 The heights of students are normally distributed with mean 1.65m and standard deviation 0.5m. A student whose height is more than 1.75m is selected at random.

The probability that this student has a height less than 1.95m equals:

A) 0.146 **B) 0.421** **C) 0.181** **D) 0.348** **E) 0.266**

Solution:

$$P(X<1.95 \setminus X>1.75) = \frac{P(1.75<X<1.95)}{P(X>1.75)} = \frac{P(\frac{1.75-1.65}{0.5} < \frac{x-\mu}{\sigma} < \frac{1.95-1.65}{0.5})}{1-P(\frac{x-\mu}{\sigma} < \frac{1.75-1.65}{0.5})} =$$

$$\frac{P(0.2<Z<0.6)}{1-P(Z<0.2)} = \frac{P(Z<0.6)-P(Z<0.2)}{1-P(Z<0.2)} = \frac{0.7257-0.5793}{1-0.5793} = 0.348 \rightarrow D$$

Q10 If P(-c < Z < c) =0.994 , then the value of c is :

A) 2.57 **B)2.75** **C)1.96** **D)2.32** **E)1.03**

Solution: $\frac{1-0.994}{2} = 0.003 \rightarrow C = 2.57 \rightarrow A$

Q11 Suppose a population of individuals has a mean weight of 160 pounds, with a population standard deviation of 30 pounds. what percent of the population would be between 100 and 220 pounds?

A) 10% B) 68% C) 95% D) 99.7%

Solution: $\mu = 160$, $\sigma = 30$

$P(100 < X < 220) = P(\frac{100-160}{30} < \frac{X-\mu}{\sigma} < \frac{220-160}{30}) = P(-2 < Z < 2) =$

$P(Z < 2) - P(Z < -2) = 0.9772 - 0.0228 = 0.9544 \cong 95\%$

Sheet (2)

Q1 suppose that X is normally distributed with mean μ=50 & standard deviation σ=6 , then the 90^{th} percentile of the distribution X is :

A) 57.4 B)56.68 C)53.44 D)57.68 E)58.68

Solution: $x \sim N(50, (6)^2)$

$$P(x < P_{90}) = 0.90 \Rightarrow P\left(Z < \frac{P_{90} - 50}{6}\right) = 0.90$$

$$\frac{P_{90} - 50}{6} = 1.28 \rightarrow P_{90} = 57.68 \rightarrow D$$

Q2 let X be distributed as normal (μ ,σ^2), then P($\mu - \sigma$ <x< μ)=

A) 0.3413 B)0.4332 C)0.1916 D)0.5001 E)none

Solution: $P(\mu - \sigma < z < \mu) = P\left(\frac{\mu-\sigma-\mu}{\sigma} < Z < \frac{\mu-\mu}{\sigma}\right) = P(-1 < Z < 0)$

$$= P(Z < 0) - P(Z < -1) = 0.5 - 0.1587 = 0.3413 \rightarrow A$$

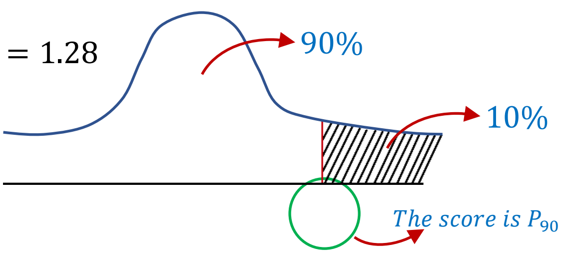
Q3 The IQ scores are normally distributed with mean 100 and standard deviation 15. A person is considered intelligent if his/her score is within the highest 10% of the IQ scores. The least intelligence IQ score is?

A)119.20 B)123.04 C)116.64 D)121.53 E)130.72

Solution: $P(x < P_{90}) = 0.90$

$$P\left(Z < \frac{P_{90} - 100}{15}\right) = 0.90 \rightarrow \frac{P_{90} - 100}{15} = 1.28$$

$$P_{90} = 119.2 \rightarrow A$$



Q4 for a continuous random variable the probability of a single value of X is

A) 1 B) 0 C) between 0 & 1 D) 0.5

Solution: $P(X = x_i)$ = zero → B

Q5 The lifetime of a certain brand of batteries is normally distributed with mean 30 hours and standard deviation 2 hours. Find the third quartile Q3 of the lifetime of this brand of batteries.

A) 32.34 B) 33.34 C) 34.34 D) 33.34 E) 31.34

Solution: $P(x < Q_3) = 0.75 \rightarrow P\left(Z < \frac{Q_3 - 30}{2}\right) = 0.75$

$\frac{Q_3 - 30}{2} = 0.67 \rightarrow Q_3 = 31.34 \rightarrow E$

Q6 In a certain population the weight (in KGs) of students are normally distributed with mean 62 KGs and variance 25 Kgs. A sample of 12 students is taken. The 90th percentile for the distribution of the sample mean is:

A) 62.97 B) 63.848 C) 64.97 D) 65.97 E) 66.97

Solution: $P(\overline{X} \leq P_{90}) = 0.90 \rightarrow P\left(Z \leq \frac{P_{90} - 62}{5/\sqrt{12}}\right) 0.90$

$\therefore \frac{P_{90} - 62}{5/\sqrt{12}} = 1.28 \rightarrow P_{90} = 63.848 \rightarrow B$

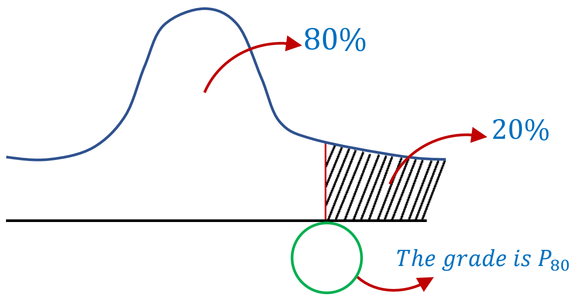
Q7 The Grades are normally distributed with mean 65.8 and variance 25, the minimum grade of the top 20% of the grades is:
A) 67 B) 64 C) 76 D) 73 E) 70

Solution:

$P(X < P_{80}) = 0.80$

$\Rightarrow P\left(Z < \frac{P_{80} - 65.8}{5}\right) = 0.80$

$\frac{P_{80} - 65.8}{5} = 0.84 \rightarrow P_{80} = 70 \rightarrow E$



Q8 Suppose that the time in minutes it takes a student to complete an assignment is normally distributed with a mean 50 and variance 100 then the 85th percentile of the average time it takes a random sample of 25 students to complete the assignment is closest to

A) 60 B) 48 C) 40 D) 52 E) 71

Solution: $P(\bar{X} < P_{85}) = 0.85 \rightarrow P\left(Z < \frac{P_{85}-50}{10/\sqrt{25}}\right) = 0.85$

$\rightarrow \frac{P_{85}-50}{10/\sqrt{25}} = 1.04 \rightarrow P_{85} = 52.08 \rightarrow D$

Q9 let X~ N (40, 25), then find the probability that X lies within 2 standard deviations about the mean:

A)0.678 B)0.98 C)0.9544 D)0.997 E)130.72

Solution: $P(\mu - S.\sigma \le x \le \mu + S.\sigma)$

$= P(40 - 2(5) \le x \le 40 + 2(5))$

$= P(30 \le x \le 50) = P\left(\frac{30-40}{5} \le Z \le \frac{50-40}{5}\right)$

$= P(-2 < Z < 2) = P(Z < 2) - P(Z < -2)$

$= 0.9772 - 0.0228 = 0.9544 \rightarrow C$

Q10 The recovery period from Corona follows a normal distribution with mean μ days and variance σ^2 days. One Corona patient is randomly selected, find the probability that this patient will recover after μ - 0.5 σ day.

A) 0.3085 B) 0.6911 C) 0.3242 D) 0.6915

Solution: P(X > μ-0.5 σ) = $P(Z > \frac{\mu-0.5\sigma-\mu}{\sigma})$ = P(Z > -0.5) = P(Z < 0.5) = 0.6915 → D

Q11 The weights of members of population are normally distributed. The distribution has a population mean (μ) weight 160 pounds and a population standard deviation (σ) 25 pounds. How many standard deviations from the mean is the weight of 185 pounds?

A) -1σ B) 1σ C) 2σ D) 0σ E) -2σ

Solution: $185 = \mu + \sigma.S \rightarrow 185 = 160 + 25\,S \rightarrow S = 1$

$1\,\sigma \rightarrow B$

Sheet (3)

Q1 If X is distributed as Binomial (100 , 0.4) by normal approximation P(x< 47) =

A) 0.8212 B) 0.973 C)0.6217 D) 0.9082 E)0.983

Solution: $x \sim \text{Bin}\ (100{,}0.4) = x \sim N(40{,}24)$

$P(x < 47) = P(x \le 46.5) = P\left(Z \le \frac{46.8-40}{\sqrt{24}}\right) = P(Z \le 1.33) = 0.9082 \rightarrow D$

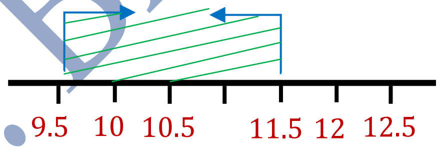
Q2 If X be distributed as Binomial (40, 0.2) by normal approximation P(10≤x<12) =

A) 0.1962 B)0.7224 C) 0.1938 D) 0.9827 E) none

Solution:

$x \sim \text{Bin}\ (40{,}0.2) = x \sim N(8{,}6.4)$

$P(10 \le x \le 12) = P(9.5 \le x \le 11.5)$



$= P\left(\frac{9.5-8}{\sqrt{6.4}} \le Z \le \frac{11.5-8}{\sqrt{6.4}}\right) = P(0.59 \le Z \le 1.38)$

$= p(Z \le 1.38) - p(Z \le 0.59) = 0.9162 - 0.7224 = 0.1938 \rightarrow C$

Q3 let X be distributed as Binomial (n, p), and after approximation X is distributed as
N (16, 3.2), then n =

A) 77 B)20 C) 85 D) 53 E)none

Solution:

$16 = n \times p \ldots$ ❶ & $3.2 = n \times p \times q \ldots$ ❷

Sub ❶ in ❷ $\Rightarrow 16 \times q = 3.2$

$\rightarrow q = 0.2 \quad \text{and} \quad p = 0.8$

$16 = n \times p \rightarrow 16 = 0.8 \times n$

$\rightarrow n = 20 \rightarrow B$

Q4 The distribution Binomial (50, 0.7) can be approximated by the distribution:
A) N(15 , 10.5) B) N (35 , 10.5) C) N(15 , 35) D) Poi(10.5)

Solution:

$X \sim \text{Bin}(50,0.7) = x \sim N(35,10.5) \rightarrow B$

Q5 If X~Bin(100, 0.2), then P($\mu - \sigma \leq X \leq \mu + 2\sigma$) =

A)0.8542 B)0.2694 C)0.2467 D)0.4145 E)0.8192

Solution:

$X \sim Bin(100,0.2) \rightarrow X \sim N(20,16)$

$E(x) = n * p = 100 * 0.2 = 20$

$Std(x) = \sqrt{n * p * q} = \sqrt{100 * 0.2 * 0.8} = 4$

$P(\mu - \sigma \leq x \leq \mu + 2\sigma) = P(20 - 4 \leq x \leq 20 + 2 * 4) = P(16 \leq x \leq 28)$

By C.C: $P(15.5 \leq x \leq 28.5) = P\left(\frac{15.5-20}{4} \leq \frac{x-\mu}{\sigma} \leq \frac{28.5-20}{4}\right) = P(-1.13 \leq Z \leq 2.13)$

$P(Z < 2.13) - P(Z < -1.13) = 0.9834 - 0.1292 = 0.8542 \rightarrow A$

Q6 Suppose that X ~ Bin (75, 0.2). using the normal approximation to the binomial distribution, P(14 < x ≤ 16) is closest to:

A) 0.28 B) 0.25 C) 0.17 D) 0.22 E) 0.33

Solution:

$X \sim Bin(75,0.2) \rightarrow X \sim N(15,12)$

By C.C : $P(14 < X \leq 16) = P(14.5 < X \leq 16.5) = P\left(\frac{14.5-15}{\sqrt{12}} \leq \frac{x-\mu}{\sigma} \leq \frac{16.5-15}{\sqrt{12}}\right)$

$= P(-0.14 < Z < 0.43) = P(Z < 0.43) - P(Z < -0.14)$

$= 0.6664 - 0.4443 = 0.2221 \rightarrow D$

Q7 Let X~Binomial (60,0.30). We wish to use normal approximation to this binomial distribution, the normal distribution that we use to approximate this binomial distribution is:

A) N(12,9.6) **B) N(18,12.6)** **C) N(24,14.4)** **D) N(15,10.5)**

Solution:

$x \sim \text{Bin}(60{,}0.30) = x \sim N(18{,}12.6) \rightarrow B$

Q8 If X ~ Binomial (50,0.2), then the normal approximation to P(10 < X ≤ 12) is closest to:

A) 0.8139 **B) 0.2305** **C) 0.2392** **D) 0.5701**

Solution:

$X \sim \text{Bin}(50, 0.02) \rightarrow X \sim N\,(10\,,8)$

Now, apply C . C:

$$P(10 < X \leq 12) = P(10.5 < X < 12.5\,) = P\left(\frac{10.5-10}{\sqrt{8}} < Z < \frac{12.5-10}{\sqrt{8}}\right)$$

$$= P(\,0.18 < Z < 0.88) = P(Z < 0.88) - P(Z < 0.18)$$

$$= 0.8106 - 0.5714 = 0.2392 \rightarrow C$$

Sheet (4)

Q1 let $\overline{X}$ be the mean of a random sample of size 25 selected from a normal population distribution with mean μ =3 & σ^2=100, then
$P(2< \overline{X} < 3)$ =

A) 0.0793 B)0.0648 C)0.4207 D)0.1915 E)0.4452

Solution:

$$P(2 < \overline{X} < 3) = P\left(\frac{2-3}{10/\sqrt{25}} < Z < \frac{3-3}{10/\sqrt{25}}\right) = P(-0.5 < Z < 0)$$

$$= P(Z < 0) - P(Z < -0.5) = 0.5 - 0.3085 =$$

$0.1915 \rightarrow D$

Q2 Let $\overline{X}$ be the mean of a random sample of size 64, selected from a population with mean 3 & variance 25, if $P(3 \leq \overline{X} \leq a)$ =0.4370 , then a=

A) 0.9370 B) 5.093 C)3.956 D)1.53 E) none

Solution:

$$P(3 \leq \overline{X} \leq a) = 0.437 \rightarrow P\left(\frac{3-3}{5/\sqrt{64}} \leq Z \leq \frac{a-3}{5/\sqrt{64}}\right) = 0.437 \rightarrow P\left(0 \leq Z \leq \frac{a-3}{5/8}\right) = 0.437$$

$$\rightarrow P\left(Z < \frac{a-3}{5/8}\right) - P(Z \leq 0) = 0.437 \rightarrow P\left(Z \leq \frac{a-3}{5/8}\right) = 0.937$$

$$\frac{a-3}{5/8} = 1.53 \rightarrow a = 3.956 \rightarrow C$$

Q3 the systolic blood pressure X for a healthy person is normally distributed with mean 120 & standard deviation 10. For a sample of 25 persons, the prob. That the average will be between 120 & 123:

A) 0.5 B)0.4332 C) 0.9332 D)0.25 E) none

Solution:

$$P(120 < \overline{X} < 123) = P\left(\frac{120-120}{10/\sqrt{25}} < Z < \frac{123-120}{10\sqrt{25}}\right)$$

$$P(0 < Z < 1.5) = P(Z < 1.5) - P(z < 0)$$

$$= 0.9332 - 0.5 = 0.4332 \rightarrow B$$

Q4 Let X be a random variable that is distributed according to the normal distribution with mean 30 and variance I00. A random sample of size 30 is taken, then the distribution of the sample average is:

A) Bin (30 ,6) B) N(30,100) C) N(30,3.33) D) N(30,36) E) none

Solution: $\bar{X} \sim N\left(30, \frac{100}{30}\right) = N(30\,, 3.33) \rightarrow C$

Q5 Let $\bar{X}$ be the mean of a random sample of size 64, selected from a population that has standard deviation 10. Then the variance of $\bar{X}$ is?

A)10/8 B)25/2 C)25/16 D)25/4 E)25/8

Solution: Variance $= \frac{\sigma^2}{n} = \frac{(10)^2}{64} = \frac{25}{16} \rightarrow C$

Q6 Let $\bar{X}$ be the mean of a random sample of size 81, selected from a population that has mean 100 standard deviation 10. Then the mean of $\bar{X}$ is?

A)10/8 B)25 C)1.23 D) 100 E)25/8

Solution: The mean 100 $\rightarrow$ D

Q7 Suppose the weights of a certain population are normally distributed with mean 70 and variance 100. if a random sample of size 50 is to be drawn, what is the probability their total weight exceeds 1500:

A) 28.3 B) 1 C) -28.3 D) 4.4 E) none

Solution:

$$P(\Sigma x_i > 1500) = P\left(\frac{\Sigma x_i}{50} > \frac{1500}{50}\right) = P(\bar{X} > 30) = P\left(z > \frac{30-70}{10\sqrt{50}}\right)$$

$$= P(Z > -28.3) \quad = P(z < 28.3) = 1 \rightarrow B$$

Q8 Which of the following properties is not true regarding the sampling distribution of $\overline{X}$:

A.$\mu_{\overline{X}}$ =μ_X no matter how large n is.

B. $\sigma_{\overline{X}}$ = σ_X /$\sqrt{n}$

C. By the central limit theorem, the distribution of $\overline{X}$ is normal no matter how large or small n is.

D. When the population being sampled follows a normal distribution, the distribution of $\overline{X}$ is normal no matter how large n is.

Solution: The answer is C