6-1 Work Done by a Constant Force :-

The work done on an object by a constant force is defined to be :-

"The product of the magnitude of the displacement times the component of the force parallel to the displacement."

$$* \; W = F d \cos\theta \; *$$

θ : is the angle between the directions of the force and the displacement.

* Work is a scalar quantity, its unit is Joul (J) → $1 J = 1 N \cdot 1m$.

* A force can be exerted on an object and yet do no work, when :- (even that $F_{net} \neq 0$)

1. The displacement is zero :-
   e.g: holding a bag in your hands at rest.
   $W = F d \cos\theta = 0$ (because $d = 0$)
   and pushing a rigid wall.

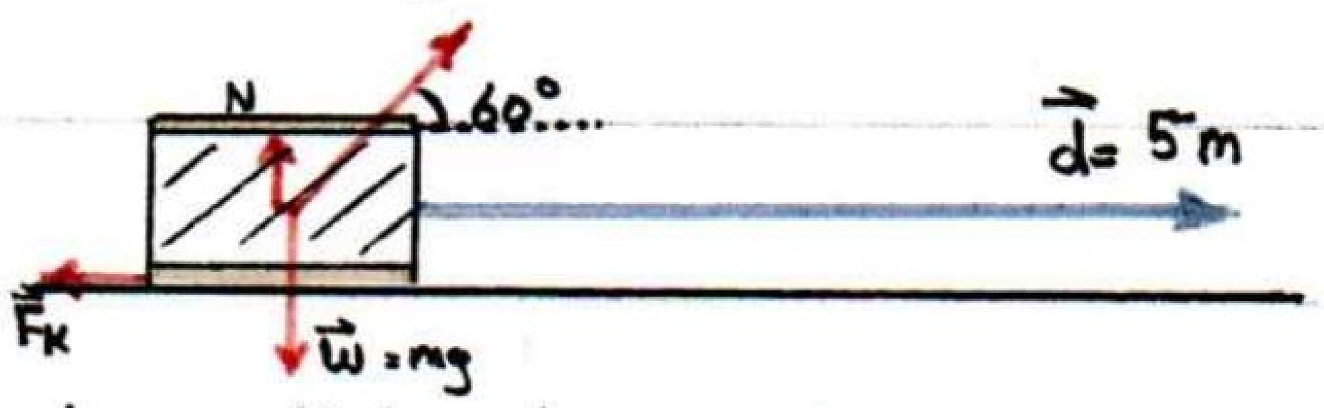
2. A force is perpendicular to the displacement :-
   e.g: Carrying an object at constant velocity by a force which acts at $\theta = 90°$ to (d).
   $W = F d \cos\theta = 0$ (because $\cos 90° = 0$)

* Note : when you stop or start walking, there is a horizontal acceleration, and a force, and thus do work on the bag.

* When we deal with work, it is necessary to specify the work is done on or by an object, also state whether the work done is due to one force or the NET FORCE on an object.
In this section we will deal with the work done by a constant force or net force.

Example (1)

A person pulls a 10-Kg box 5m along a horizontal floor by a constant force F = 60N, which acts at a 60° angle. The floor is rough and exerts a friction force F_k = 20 N. Determine :-



a) The work done by each force :-

$W_{\vec{N}} = 0 \quad (\theta = 90° \rightarrow \cos 90° = 0)$.

$W_{mg} = 0 \quad (\theta = 90° \rightarrow \cos 90° = 0)$.

$W_{\vec{F}_k} = (20).(5).(\cos 180°) = -100$ J.

$W_{\vec{F}} = (60).(5).(\cos 60°) = +150$ J.

b) The Net Force :-

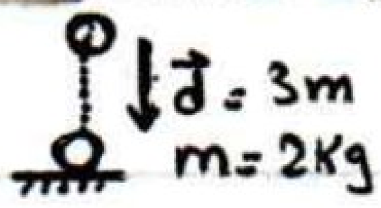
(1) → The algebraic sum of the work (Since it is a (scalar))
$W_{net} = W_{\vec{N}} + W_{mg} + W_{\vec{F}_k} + W_{\vec{F}} = 50$ J

or

(2) → Determining the Net Force First
$W_{net} = F_{net(x)} \cdot d = (60 \cos 60° - 20)(5) = 50$ J

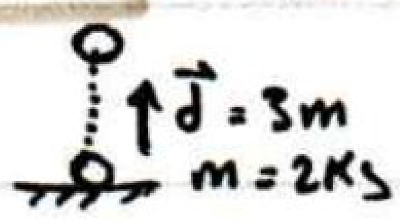
* In the previous example we saw that the net work could be :-

1. Positive: The object is accelerating.

e.g. 

$W = 60 J$ (Free fall)

2. Negative: The object is decelerating.

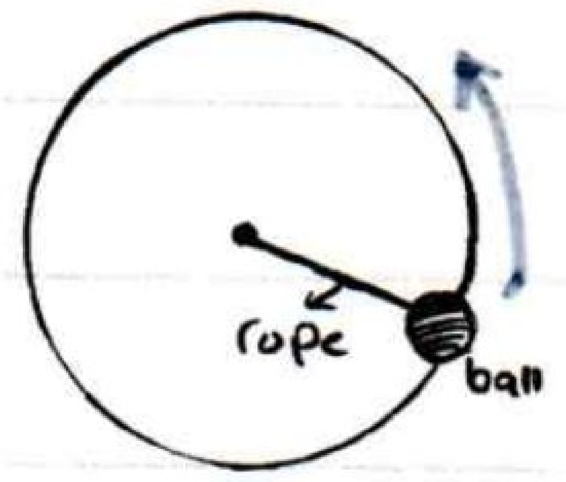
e.g. 

$W = -60 J$ (Free fall)

3. Zero: The object is at rest or moving at a constant speed.

* Note: Work is a scalar, the sign here does NOT mean direction.

* Note: when work equals to zero the object moves at a constant speed NOT velocity, such as, when a ball on a rope swings in a circle, the Moon orbits the Earth, and artificial satellites.



* Constant speed (the velocity is constant in magnitude but variable in direction).

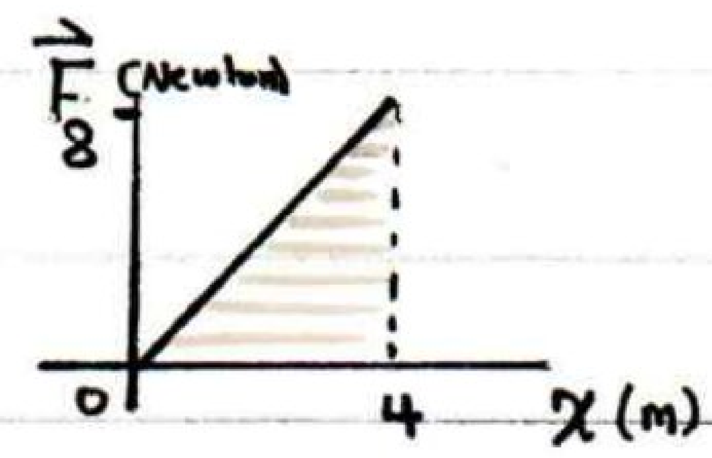
* The displacement and the velocity are perpendicular to the force.

6-2 Work Done by a Varying Force :-

In many cases the force which acts on an object could be variable in magnitude or direction. For example, the force exerted by a spring, which increases with the amount of stretch.

* The total work done by a varying force equal to the sum of areas under a curve. However, we will only deal with simple linear equations, so we are going to use basic formulas without the integration concept.

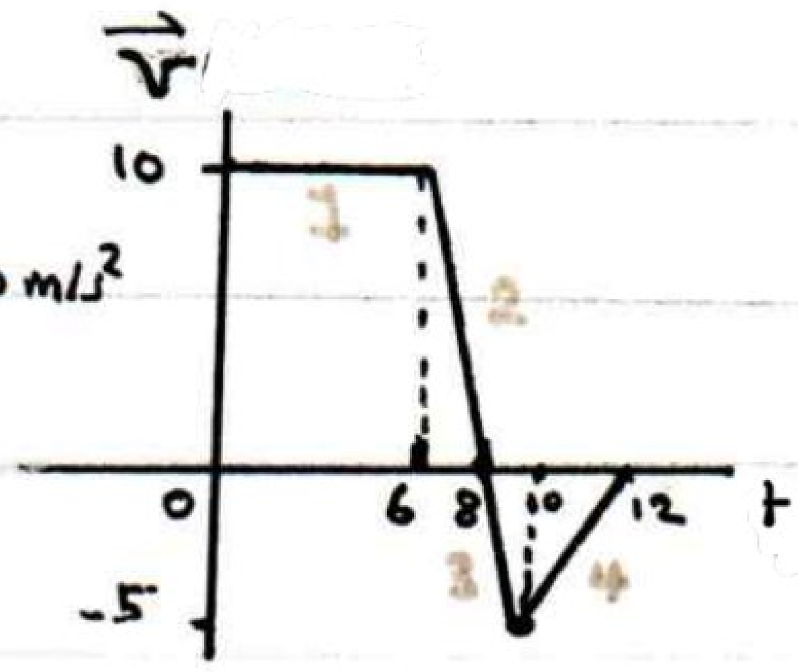
Example (1)



$W = \frac{1}{2} F \cdot x$

$= 16 J$

Example (2)



1 → Constant $\vec{v}$ → $\vec{a} = 0 \, m/s^2$

no (W) → work = 0

2 + 4 → The object is decelerating → The work is negative

3 → The object is accelerating → the work is positive.

6-9 Energy conservation with dissipitave Force :-

* In our previous applications of energy, we neglected friction, tension, Normal Force (nonconservative forces)

In real life they do work using :

$$W_{nc} = \Delta K + \Delta U$$

$$W_{nc} = (K_f - K_i) + (U_f - U_i)$$

$$W_{nc} = (K_f + U_f) - (K_i + U_i)$$

$$W_{nc} = E_f - E_i$$

$$W_{nc} = \Delta E \cdots *$$

"chang in total mechanical energy equals the work done by non conservative forces"

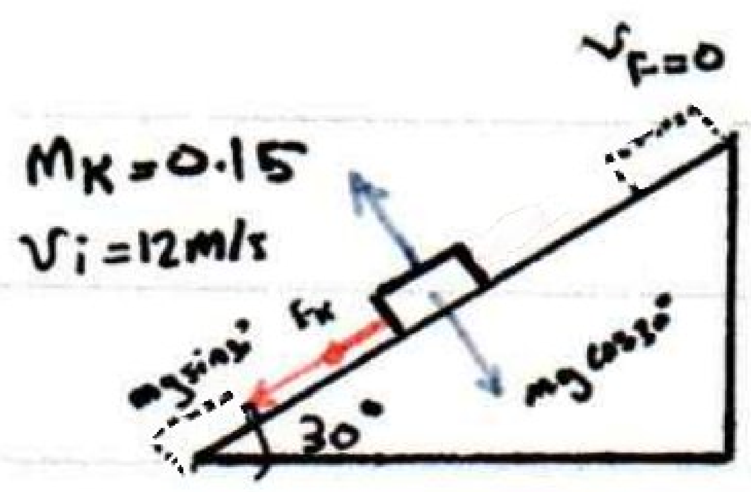
For example assume that $[E_i = K_i + U_i = 1000\,J]$

$[E_F = K_F + U_F = 800\,J]$ because this system includes non conservative forces $\Delta E \neq 0$

$[W_{nc} = \Delta E = K_f - K_i = -200\,J]$

Example (1)

A box moves up the inclined plane, Find the maximum distance.



$$W_{nc} = \Delta K + \Delta U$$

$$W_N + W_{fK} = \Delta K + \Delta U$$

$$0 + (\mu_K mg \cos 30°)(d)(\cos 180°) = -\frac{1}{2} m (12)(12) + mg(h)$$

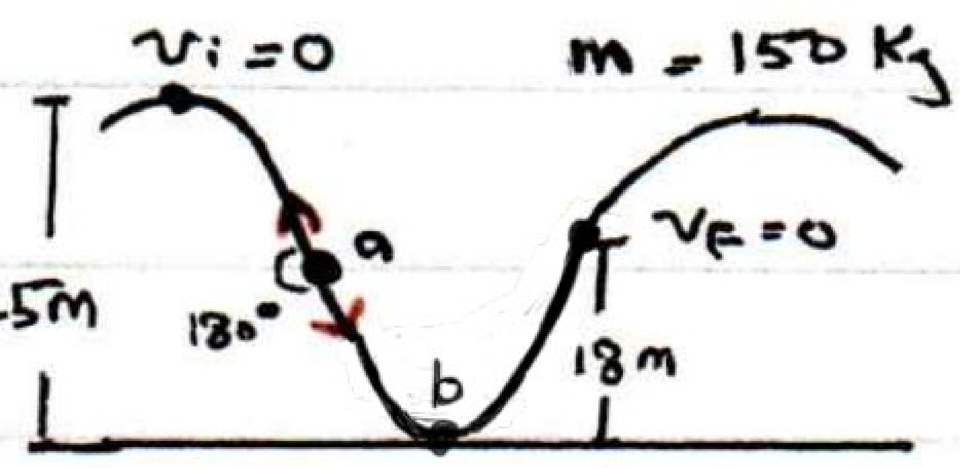
$$-0.15\, g \cdot \frac{\sqrt{3}}{2} \cdot d = -\frac{1}{2}(144) + g(d)\sin 30°$$

$$1.273\, d = 72 - 4.9\, d$$

$$6.173\, d = 72$$

$$d = 11.7\ m.$$

Example (2)



The car moves ($d = 60m$)

a) Find f_K (assume it a constant).

$$W_{nc} = \Delta K + \Delta U$$

$$W_N + W_{fK} = \Delta K + \Delta U$$

$$0 + f_K \cdot d \cdot \cos 180° = 0 - mg(25 - 18)$$

$$-60 f_K = -(150)\cdot(9.8)(7)$$

$$f_K = 171.5\ N.$$

b) Find the Velocity when the car $\underline{\underline{d}}$ is 40m and $\underline{\underline{h}}$ is 25m.

$$W_{nc} = \Delta K + \Delta U$$

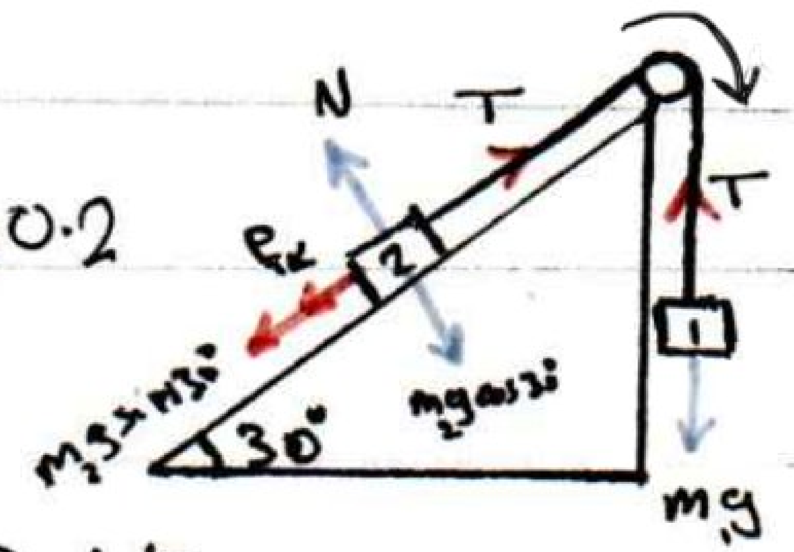
$$W_N + W_{fK} = \Delta K + \Delta U$$

$$0 + f_K \cdot d \cdot \cos 180° = \frac{1}{2} m V_f^2 - 0 - mg(25)$$

$$-6860 = \frac{150}{2}(V_f)^2 - (150)(9.8)(25)$$

$$V_f \approx 19.96 \approx 20\ m/s$$

Example (3)



$m_1 = 4\ Kg$ / $m_2 = 2\ Kg$ / $\mu_K = 0.2$

Find the speed of (m_1) after falling a distance of 1.5 m

* Note: T does work $= W_T = Td$ (For m_2)

$W_T = -Td$ (For m_1)

SO The NET WORK For (T) = 0

$$W_{nc} = \Delta K + \Delta U$$

$$-f_K(1.5) = \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2 - m_1 g(1.5) + m_2 g(1.5)\sin 30°$$

$$-5.09 = \frac{1}{2} V_f^2 (6) - 58.8 + 14.7$$

$V_f = 3.6\ m/s$