Chapter (6)

Sampling distribution

Sheet (1)

Q1 Suppose that 70% of the students of University of Jordan own laptops. In a random sample of 50 students, let $\widehat{P}$ be the proportion of students who own laptops, then the mean of the distribution of $\widehat{P}$ is:

A) 0.70 B) 0.006 C) 0.60 D) 0.40 E) N(0.70 , 24)

$\hat{P} \sim N\left(0.70 , \frac{0.70*0.30}{50}\right)$ → the mean is P = 0.70 → A

Q2 In certain population 60% of the people own cars. If a random sample of 100 persons is chosen, then the probability that the proportion of people who own cars is less than 0.63 will be:

A) 0.7939 B) 0.5988 C) 0.6109 D) 0.6591 E) 0.7291

$P(\hat{P} \leq 0.63) = P\left(\frac{\hat{P}-P}{\sqrt{\frac{P(1-P)}{n}}} \leq \frac{0.63-0.60}{\sqrt{\frac{0.60*0.40}{100}}}\right) = P(Z \leq 0.61) = 0.7291 \rightarrow$ E

Q3 Let $X_1, X_2, \ldots, X_{25}$ Be random sample from N(70 , 16) such that P(s^2 > B) =0.9 then the value of B is:

A) 26.1 B)10.44 C) 16.32 D) 40.79 E) NONE

$P(s^2 > B) = 0.9 \rightarrow P\left(\frac{S^2*(n-1)}{\sigma^2} > \frac{B*24}{16}\right) = 0.90$

$P\left(X^2 > \frac{B*24}{16}\right) = 0.90$ with d.f = 24

$\frac{B*24}{16} = 15.659 \rightarrow B = 10.44 \rightarrow B$

Q4 A sample of size 49 is chosen from a population with mean 60 and standard deviation 7. In such a case, the mean of the distribution of the sample mean is:

A)37.17 B)97.72 C)0.86 D)60 E)228

The mean is $\mu = 60 \rightarrow D$

Q5 A sample of size 10 is chosen from a normally distributed population. Let s² be the sample variance σ²=18. Then the 5th percentile of the distribution of s² is:

A) 8.402 B) 4.230 C) 5.466 D) 6.650 E) 7.880

$n = 10 \quad , \sigma^2 = 18$

$P(S^2 < P_5) = 0.05 \rightarrow P\left(x^2 < \frac{P_5(a)}{18}\right) = 0.05$

$1 - P\left(x^2 > \frac{9}{18}.P_5\right) = 0.05$

$P\left(x^2 > \frac{1}{2}.P_5\right) = 0.95 \quad \rightarrow \quad x^{2\ (9)}_{0.95} = 3.325$

$\frac{1}{2}P_5 = 3.325 \rightarrow P_5 = 6.65 \rightarrow D$

Q6 In a certain population the heights of students are normally distributed with mean 150 cm and standard deviation 4 cm. if a random sample of 16 students is chosen from this population, then the probability that the sample mean is greater than 152 cm will be:

A) 0.2103 B) 0.1587 C) 0.0228 D) 0.0013 E) 0.2451

$\mu = 150 \quad , \quad \sigma = 4 \quad , \quad n = 16$

$P(\overline{X} > 152) = P\left(\frac{\overline{X} - \mu}{\sigma/\sqrt{n}} > \frac{152 - 150}{4/\sqrt{160}}\right)$

$= P(Z > 2) = P(Z < -2) = 0.0228 \rightarrow C$

Q7 In certain population 80% of the people own houses. If a random sample of 200 persons is chosen, then the probability that the proportion of people who own houses is more than 0.83 will be:

A) 0.7939 B) 0.5988 C) 0.1446 D) 0.6591 E) 0.7291

$P = 0.80 \quad , \quad n = 200$

$$P(\hat{P} > 0.83) = P\left(\frac{\hat{P} - P}{\sqrt{\frac{P(1-P)}{n}}} > \frac{0.83 - 0.80}{\sqrt{\frac{0.8(0.2)}{200}}}\right) = P(Z > 1.06) = P(Z < -1.06)$$

$= 0.1446 \rightarrow C$

Q8 From normally distributed population with mean 42 and unknown variance , a random sample of size 16 showed variance 9 . The 90th percentile of the distribution of the sample mean is:
A) 43 B) 44 C) 45 D) 41 E) 42

$\mu = 42 , \; n = 16, \quad S = 9$, Since σ unknown and $n < 30 \rightarrow t$

$P(\bar{X} < P_{90}) = 0.90 \rightarrow P\left(\frac{\bar{X}-\mu}{5/\sqrt{n}} < \frac{P_{90}-42}{9/\sqrt{16}}\right) = 0.90$

$P\left(t > \frac{P_{90}-42}{9/\sqrt{16}}\right) = 0.10 \rightarrow t_{0.10}^{(15)} = 1.341$

$\therefore \frac{P_{90}-42}{9/4} = 1.341 \rightarrow P_{90} = 45 \rightarrow C$

Q9 Let $X_1,\ldots, X_9$ be a random sample of size 9 from $N(10,\sigma^2)$. Given the sample standard deviation S = 4. Find the value of c such that $P(\bar{X} < c) = 0.9$.

A) 1.397 B) 0.75 C) 9.8 D) 11.86

$X_9 \sim N\,(10\,, \sigma^2)\,, S = 4$, σ unknown and $n < 30 \;\rightarrow t-$ dis.

$$P(\bar{X} < c) = 0.9 \rightarrow P\left(t < \frac{C-10}{4/\sqrt{9}}\right) = 0.9 \;\rightarrow P\left(t > \frac{C-10}{4/3}\right) = 0.10 \quad d.f = 8$$

$\frac{C-10}{4/3} = 1.397 \rightarrow C = 11.86 \rightarrow D$

Q10 A random sample with size 9 and standard deviation S = 4 is selected from a population that is normally distributed with mean μ = 22. Find the probability that the sample mean is more than 25.861:

A)0.80 B) 0.90 C) 0.01 D) 0.99 E) 0.96

$\mu = 22 \quad n = 9 \quad , \ S = 4 \quad , \quad \sigma$ unknown and $n < 30 \rightarrow t$

$P(\bar{X} > 25.861) = P\left(t > \frac{25.861-22}{4\sqrt{9}}\right) = P(t > 2.896) = 0.01 \rightarrow C$

Q11 Let X1,X2,…X6 ~normal(μ, 10) and s^2 be the sample variance, the 99th percentile for s^2 equals:

A) 2.291 B) 1.145476 C) 22.141 D) 18.473 E) 30.172

$P(S^2 < P_{99}) = 0.99 \rightarrow P(\frac{(n-1)S^2}{\sigma^2} < \frac{5*P_{99}}{10}) = 0.99$

$P\left(X^2 > \frac{5*P_{99}}{10}\right) = 0.01\ with\ d.f = 5$

$\frac{5*P_{99}}{10} = 15.086 \rightarrow P_{99} = 30.172 \rightarrow E$

Q12 A random sample of size 12 is selected from Normal distribution with mean 22 and unknown variance. If the sample standard deviation S is 6, then the 95th percentile of the sample mean is closest to:

A) 19.60 B) 22.70 C) 23.20 D) 18.90 E) 25.11

$P(\bar{X} < P_{95}) = 0.95 \rightarrow P\left(\frac{\bar{X}-\mu}{S/\sqrt{n}} \le \frac{P_{95}-22}{6/\sqrt{12}}\right) = 0.95 \rightarrow P\left(t < \frac{P_{95}-22}{6/\sqrt{12}}\right) = 0.95$

$P\left(t > \frac{P_{95}-22}{6/\sqrt{12}}\right) = 0.05\ with\ d.f = 11$

$\frac{P_{95}-22}{6/\sqrt{12}} = 1.796 \rightarrow P_{95} = 25.11 \rightarrow E$

Sheet (2)

Q1 The average time in years to get an undergraduate degree in computer science was compared for men and women. Random samples of 100 male computer science majors and 100 female's computer science majores were taken. The appropriate parameter for this situation is:

A) One population proportion p

B) Difference between two population proportions p1-p2

C) One population mean μ1

D) Difference between two population means μ1 − μ2

Since it is the average time for men & women → D

Q2 Let P_1 = proportion of pop. I, and P_2 = proportion of pop. II. If P_1 = 0.2, P_2 = 0.36, n_1 = 40 and n_2 = 50, then the distribution of the difference between the two proportions is:

A) Bin(0.36 , 90) **B) Bin(-0.2 , 90)** **C) N (-0.16 , $\frac{269}{31250}$)**

$\hat{P}_1 - \hat{P}_2 \sim N\left(P_1 - P_2 , \frac{P_1(1-P_1)}{n} + \frac{P_2(1-P_2)}{m}\right) = \left(0.2 - 0.36 , \frac{0.2*0.8}{40} + \frac{0.36*0.64}{50}\right)$

$N\left(-0.16 , \frac{269}{31250}\right)$ → C

Q3 Let $X_1, \ldots, X_{25}$ distributed as N(20,45). Another independent sample $Y_1, \ldots, Y_{16}$ is taken from N(20,75). What is the standard deviation of ($\bar{X}$-$\bar{Y}$)?

A)3.125 **B)9** **C)4.34** **D)8** **E)2.55**

$\sigma = \sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}} = \sqrt{\frac{45}{25} + \frac{75}{16}} = 2.55$ → E

Q4 let X1,…,X25 distributed as N(20,45). Another independent random sample Y1,…,Y16 is taken from N(20,75). What is the mean of ($\bar{X}$-$\bar{Y}$)?

A)3.125 **B)9** **C)4.34** **D)8** **E)0**

The mean $= \mu_1 - \mu_2 = 20 - 20 = 0$ → E

Q5 Let $X_1, X_2, \ldots X_6$ ~Normal(20,5) and $Y_1, Y_2, \ldots Y_6$ ~Normal(20,5) be two independent sample . let $\bar{X}$ and $\bar{Y}$ be the two-sample means. Let S_1 and S_2 be the two sample standard deviation. The value of C for which $P(\bar{X} - \bar{Y} \le C) = 0.9850$ *is*:

A) 2.74 B) 0 C) 6.26 D) 2.80 E) 2.62

$$P(\bar{X} - \bar{Y} \le C) = 0.9850 \rightarrow P\left(\frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}} \le \frac{C-0}{\sqrt{\frac{5}{6}+\frac{5}{6}}} \right) = 0.9850$$

$$P\left(Z < \frac{C}{\sqrt{\frac{5}{6}+\frac{5}{6}}} \right) = 0.9850 \rightarrow \frac{C}{\sqrt{\frac{5}{6}+\frac{5}{6}}} = 2.17 \rightarrow C = 2.8 \rightarrow D$$

Q6 A sample of size 3 has a standard deviation of 9, and another sample of size 4 has a standard deviation of 5. Assuming the two populations are distributed normal, then the pooled standard deviation is closest to

A) 6.173 B) 6.885 C) 47.400 D) 5.944 E) None of these

$$S.P = \sqrt{\frac{S_1^2(n-1)+S_2^2(m-1)}{n+m-2}} = \sqrt{\frac{2*9^2+3*5^2}{3+4-2}} = 6.885 \rightarrow B$$

Q7 A study gave the following information about serum cholesterol levels in U.S females. Use these estimates as the mean μ and standard deviation σ for the respective population, suppose we select a simple random sample of Size 50 from each, what's the probability that the difference between two sample means $\overline{x_A} - \overline{x_B}$ will be more that 8?

Population	*Age*	*Mean*	*Standard deviation*
A	20 − 29	183	37.2
B	30 − 39	189	34.7

$$P(\overline{x_A} - \overline{x_b} > 8) = P\left(\frac{\overline{x_A}-\overline{x_B}-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_A^2}{n}+\frac{\sigma_B^2}{m}}} > \frac{8-(183-189)}{\sqrt{\frac{(37.2)^2}{50}+\frac{(34.7)^2}{50}}} \right) = P(z > 1.95)$$

$$= P(z < -1.95) = 0.0256$$

Q8 Suppose it has been established that for a certain type of client the average length of a home visit by a public health nurse is 45 minutes with a standard deviation of 15 minutes, and that for a second type of client the average home visit is 30 minutes long with a standard deviation of 20 minutes. If a nurse randomly visits 35 clients from the first and 40 from the second population, what is the probability that the average length of home visit will differ between the two groups by 20 or more minutes?

Type (1)	Type (2)
$\bar{X} = 45$	$\bar{X} = 30$
$\sigma = 15$	$\sigma = 20$
n = 35	n = 40

$$P(\bar{X} - \bar{Y} > 20) = P\left(\frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}} > \frac{20-(45-30)}{\sqrt{\frac{15^2}{35}+\frac{20^2}{40}}}\right) = P(Z > 1.23) = P(Z < -1.23) = 0.1093$$

Q9 The calcium levels in men and women ages 60 years or older are summarized in the following table:

	Mean	**Standard deviation**
Men	**797**	**482**
Women	**660**	**414**

Use these estimates as the mean μ and standard deviation σ for the U.S. populations for these age groups. If we take a random sample of 40 men and 35 women, what is the distribution of the difference between two sample means?

$$\bar{X} - \bar{Y} \sim N\left(\mu_1 - \mu_2 , \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}\right) = N\left(797 - 660, \frac{482^2}{40} + \frac{414^2}{35}\right)$$

$$= N(137, 10705.1)$$

Q10 In the 1999 National Health Interview Survey [A-7], researchers found that among U.S. adults ages 75 or older, 34 percent had lost all their natural teeth and for U.S. adults ages 65–74, 26 percent had lost all their natural teeth. Assume that these proportions are the parameters for the United States in those age groups. If a random sample of 200 adults ages 65–74 and an independent random sample of 250 adults ages 75 or older are drawn from these populations, find the probability that the difference in percent of total natural teeth loss is less than 5 percent between the two populations?

75 or more	65 – 74
P = 34% = 0.34	P = 26% = 0.26
m = 250	n=200

$$P\left(\hat{P}_1 - \hat{P}_2 < 0.05\right) =$$

$$P\left(\frac{\hat{P}_1 - \hat{P}_2 - (P_1 - P_2)}{\sqrt{\frac{P_1(1-P_1)}{n} + \frac{P_2(1-P_2)}{m}}} < \frac{0.05 - (0.34 - 0.26)}{\sqrt{\frac{0.34 * 0.66}{250} + \frac{0.26 * 0.74}{200}}}\right) = P(Z < -0.695)$$

$$= P(Z < -0.7) = 0.2420$$