CHAPTER 6 – PART 2

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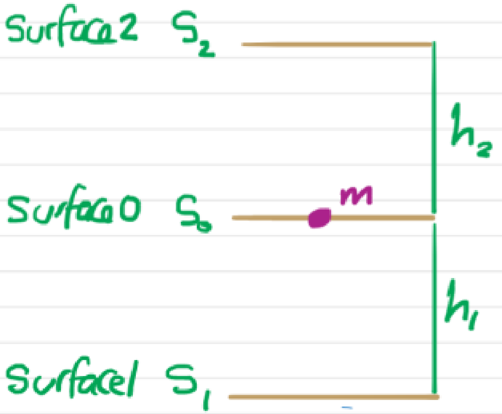
REVISION

- Work = F*d*cos θ
- KE = $\frac{1}{2}mv^2$
- $W_{net} = \Delta KE = \frac{1}{2}mv2^2 - \frac{1}{2}mv1^2$

6–4 POTENTIAL ENERGY

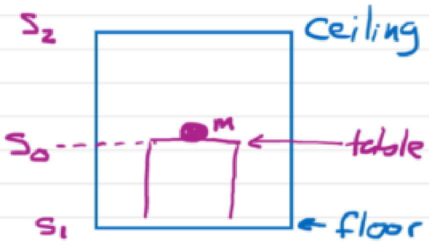
- potential energy: the energy associated with forces that depend on the position or configuration of an object (or objects) relative to the surroundings.
- Perhaps the most common example of potential energy is gravitational potential energy: If you raise an object to an "h" height above a ground and then release it, it will fall towards the ground, this shows that this object possessed energy when it was raised, which is represented by
  $U = mgh$ (Note that mg = gravitational force)

Gravitational potential energy is defined with respect to a surface.



$U_2 = -mgh_2$ potential energy of mass m relative to S_2. Negative U_2 means work must be done to raise m to S_2.

$U_1 = mgh_1$ potential energy of mass m relative to S_1. Positive value means if you release m it falls towards S_1.



$mgh = mg(0) = 0$

$U_0 = 0$ potential energy of mass m with respect to surface S_0. Note m is on the surface $S_0 \Rightarrow h = 0$.

Unlike kinetic energy K, the potential energy can be positive, negative or zero.

6–4 POTENTIAL ENERGY

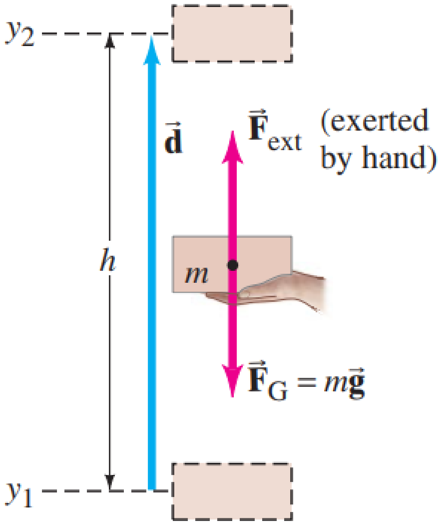


FIGURE 6–11 A person exerts an upward force $F_{ext} = mg$ to lift a brick from y_1 to y_2.

Let us seek the form for the gravitational potential energy of an object near the surface of the Earth. For an object of mass m to be lifted vertically, an upward force at least equal to its weight, mg, must be exerted on it, say by a person's hand. To lift the object without acceleration, the person exerts an "external force" $F_{ext} = mg$. If it is raised a vertical height h, from position y_1 to y_2 in Fig. 6–11 (upward direction chosen positive), a person does work equal to the product of the "external" force she exerts, $F_{ext} = mg$ upward, multiplied by the vertical displacement h. That is,

$$\begin{aligned} W_{ext} &= F_{ext}\, d \cos 0° = mgh \\ &= mg(y_2 - y_1). \end{aligned} \qquad \textbf{(6–5a)}$$

Gravity is also acting on the object as it moves from y_1 to y_2, and does work on the object equal to

$$W_G = F_G\, d \cos \theta = mgh \cos 180°,$$

where $\theta = 180°$ because $\vec{\mathbf{F}}_G$ and $\vec{\mathbf{d}}$ point in opposite directions. So

$$\begin{aligned} W_G &= -mgh \\ &= -mg(y_2 - y_1). \end{aligned} \qquad \textbf{(6–5b)}$$

Next, if we allow the object to start from rest at y_2 and fall freely under the action of gravity, it acquires a velocity given by $v^2 = 2gh$ (Eq. 2–11c) after falling a height h. It then has kinetic energy $\frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$, and if it strikes a stake, it can do work on the stake equal to mgh (Section 6–3).

Thus, to raise an object of mass m to a height h *requires* an amount of work equal to mgh (Eq. 6–5a). And once at height h, the object has the *ability* to do an amount of work equal to mgh. We can say that the work done in lifting the object has been stored as gravitational potential energy.

We therefore define the **gravitational potential energy** of an object, due to Earth's gravity, as the product of the object's weight mg and its height y above some reference level (such as the ground):

$$\mathrm{PE_G} = mgy. \tag{6–6}$$

The higher an object is above the ground, the more gravitational potential energy it has. We combine Eq. 6–5a with Eq. 6–6:

$$W_{\mathrm{ext}} = mg(y_2 - y_1)$$

$$W_{\mathrm{ext}} = \mathrm{PE_2} - \mathrm{PE_1} = \Delta\mathrm{PE_G}. \tag{6–7a}$$

CAUTION

$\Delta\mathrm{PE_G}$ = work done by net external force

That is, the change in potential energy when an object moves from a height y_1 to a height y_2 is equal to the work done by a net external force to move the object from position 1 to position 2 without acceleration.

Equivalently, we can define the change in gravitational potential energy, $\Delta\mathrm{PE_G}$, in terms of the work done by gravity itself. Starting from Eq. 6–5b, we obtain

$$W_{\mathrm{G}} = -mg(y_2 - y_1)$$

$$W_{\mathrm{G}} = -(\mathrm{PE_2} - \mathrm{PE_1}) = -\Delta\mathrm{PE_G}$$

or

$$\Delta\mathrm{PE_G} = -W_{\mathrm{G}}. \tag{6–7b}$$

CAUTION

$\Delta\mathrm{PE_G} = -W_{\mathrm{G}}$

$W_{net} = W_{mg} + W_{ext} = 0 \Rightarrow$ object moves at constant speed.

EXAMPLE 6–6 **Potential energy changes for a roller coaster.** A 1000-kg roller-coaster car moves from point 1, Fig. 6–12, to point 2 and then to point 3. (*a*) What is the gravitational potential energy at points 2 and 3 relative to point 1? That is, take $y = 0$ at point 1. (*b*) What is the change in potential energy when the car goes from point 2 to point 3? (*c*) Repeat parts (*a*) and (*b*), but take the reference point $(y = 0)$ to be at point 3.

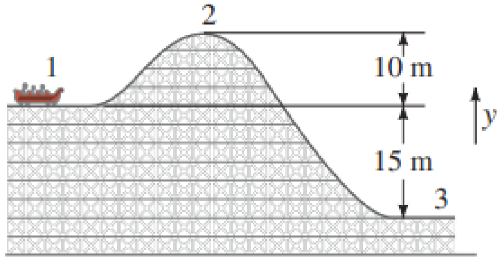


FIGURE 6–12 Example 6–6.

APPROACH We are interested in the potential energy of the car–Earth system. We take upward as the positive y direction, and use the definition of gravitational potential energy to calculate the potential energy.

SOLUTION (*a*) We measure heights from point 1 $(y_1 = 0)$, which means initially that the gravitational potential energy is zero. At point 2, where $y_2 = 10\text{ m}$,

$$\text{PE}_2 = mgy_2 = (1000\text{ kg})(9.8\text{ m/s}^2)(10\text{ m}) = 9.8 \times 10^4\text{ J}.$$

At point 3, $y_3 = -15\text{ m}$, since point 3 is below point 1. Therefore,

$$\text{PE}_3 = mgy_3 = (1000\text{ kg})(9.8\text{ m/s}^2)(-15\text{ m}) = -1.5 \times 10^5\text{ J}.$$

(*b*) In going from point 2 to point 3, the potential energy change $(\text{PE}_{\text{final}} - \text{PE}_{\text{initial}})$ is

$$\text{PE}_3 - \text{PE}_2 = (-1.5 \times 10^5\text{ J}) - (9.8 \times 10^4\text{ J}) = -2.5 \times 10^5\text{ J}.$$

The gravitational potential energy decreases by 2.5×10^5 J.

(*c*) Now we set $y_3 = 0$. Then $y_1 = +15\text{ m}$ at point 1, so the potential energy initially is

$$\text{PE}_1 = (1000\text{ kg})(9.8\text{ m/s}^2)(15\text{ m}) = 1.5 \times 10^5\text{ J}.$$

At point 2, $y_2 = 25\text{ m}$, so the potential energy is

$$\text{PE}_2 = 2.5 \times 10^5\text{ J}.$$

At point 3, $y_3 = 0$, so the potential energy is zero. The change in potential energy going from point 2 to point 3 is

$$\text{PE}_3 - \text{PE}_2 = 0 - 2.5 \times 10^5\text{ J} = -2.5 \times 10^5\text{ J},$$

which is the same as in part (*b*).

NOTE Work done by gravity depends only on the vertical height, so changes in gravitational potential energy do not depend on the path taken.

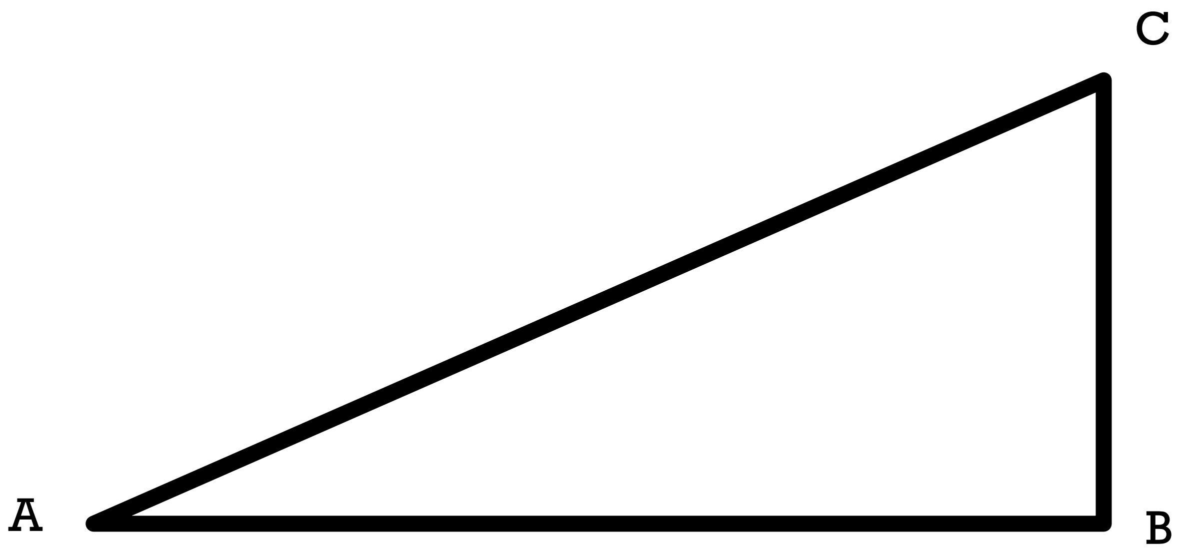
6–4 POTENTIAL ENERGY

Potential Energy Defined in General

There are other kinds of potential energy besides gravitational. Each form of potential energy is associated with a particular force, and can be defined analogously to gravitational potential energy. In general, the *change in potential energy associated with a particular force is equal to the negative of the work done by that force when the object is moved from one point to a second point* (as in Eq. 6–7b for gravity). Alternatively, we can define the *change in potential energy as the work required of an external force to move the object without acceleration between the two points*, as in Eq. 6–7a.

6–5 CONSERVATIVE AND NON-CONSERVATIVE FORCES

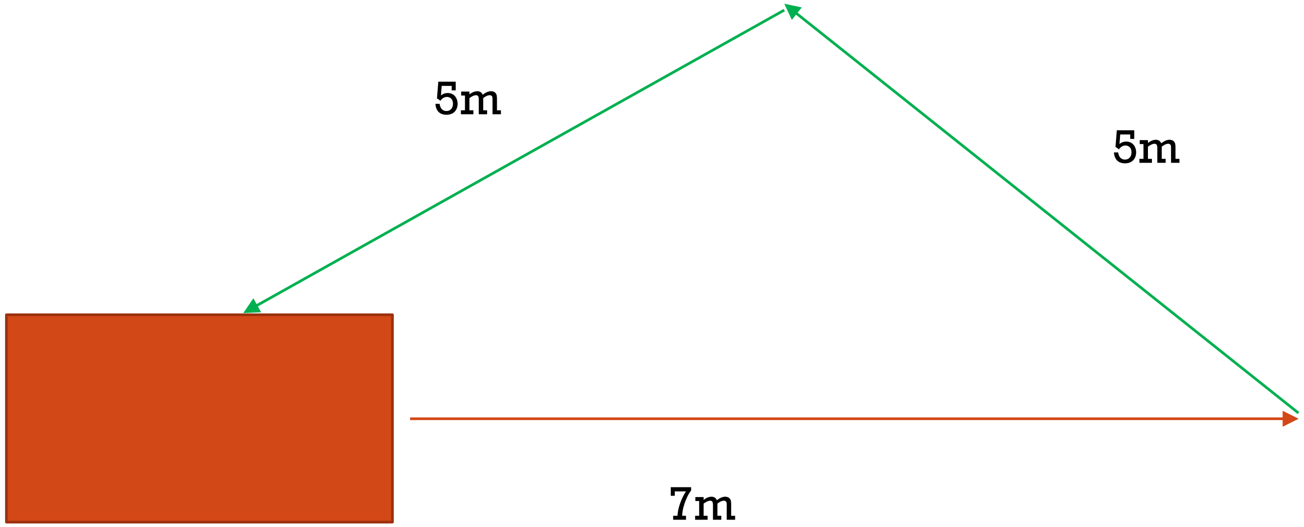
- Conservative force: القوة المحافظة مثل قوة الجاذبية وهي الوحيدة المطلوبة منا، وهناك غيرها مثل القوة الكهربائية أو قوة الزنبرك
  1) The work done by a conservative force does not depend on the path
  2) The work of a conservative force in a closed path = zero



6–5 Conservative and Non-Conservative Forces

- Non-conservative force:
  work depends on the pathway /
  work in a closed pathway ≠ 0

- You do more work on the green path because the distance is greater and, unlike the gravitational force, the pushing force is in the direction of motion at each point.



6–5 CONSERVATIVE AND NON-CONSERVATIVE FORCES

W_{f_k} along the path 1 $\neq$ W_{f_k} along path 2

↑ work done by friction.

W_{f_k} form 1 → 2 back to 1 is NOT 0.

When you raise an object up to a height h above the ground, you do work against gravity. This work is stored as gravitational potential energy in the system that consists of the object–earth system.

What is the evidence that energy is stored?

when you release the object it falls back to the groud.

In the figure above, work is done by the force $\vec{F}_p$ against the force of friction to move the box from point 1 → point 2. But this energy is NOT stored; it is lost as heat for example. The box does NOT move back from point 1 → point 2 when $\vec{F}_p$ is removed.

EXERCISE E An object acted on by a constant force F moves from point 1 to point 2 and back again. The work done by the force F in this round trip is 60 J. Can you determine from this information if F is a conservative or nonconservative force?

Work-Energy Extended

We can extend the **work-energy principle** (discussed in Section 6–3) to include potential energy. Suppose several forces act on an object which can undergo translational motion. And suppose only some of these forces are conservative. We write the total (net) work W_{net} as a sum of the work done by conservative forces, W_{C}, and the work done by nonconservative forces, W_{NC}:

$$W_{\text{net}} = W_{\text{C}} + W_{\text{NC}}.$$

Then, from the work-energy principle, Eq. 6–4, we have

$$\begin{aligned} W_{\text{net}} &= \Delta \text{KE} \\ W_{\text{C}} + W_{\text{NC}} &= \Delta \text{KE} \end{aligned}$$

where $\Delta \text{KE} = \text{KE}_2 - \text{KE}_1$. Then

$$W_{\text{NC}} = \Delta \text{KE} - W_{\text{C}}.$$

Work done by a conservative force can be written in terms of potential energy, as we saw in Eq. 6–7b for gravitational potential energy:

$$W_{\text{C}} = -\Delta \text{PE}.$$

We combine these last two equations:

$$W_{\text{NC}} = \Delta \text{KE} + \Delta \text{PE}. \qquad \textbf{(6–10)}$$

Thus, *the work* W_{NC} *done by the nonconservative forces acting on an object is equal to the total change in kinetic and potential energies*.

It must be emphasized that *all* the forces acting on an object must be included in Eq. 6–10, either in the potential energy term on the right (if it is a conservative force), or in the work term on the left (but not in both!).

6–6 MECHANICAL ENERGY AND ITS CONSERVATION

If we can ignore friction and other nonconservative forces, or if only conservative forces do work on a system, we arrive at a particularly simple and beautiful relation involving energy.

When no nonconservative forces do work, then $W_{NC} = 0$ in the general form of the work-energy principle (Eq. 6–10). Then we have

$$\Delta \text{KE} + \Delta \text{PE} = 0 \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–11a)}$$

or

$$(\text{KE}_2 - \text{KE}_1) + (\text{PE}_2 - \text{PE}_1) = 0. \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–11b)}$$

We now define a quantity E, called the **total mechanical energy** of our system, as the sum of the kinetic and potential energies at any moment:

$$E = \text{KE} + \text{PE}.$$

Now we can rewrite Eq. 6–11b as

CONSERVATION OF MECHANICAL ENERGY

$$\text{KE}_2 + \text{PE}_2 = \text{KE}_1 + \text{PE}_1 \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–12a)}$$

or

$$E_2 = E_1 = \text{constant.} \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–12b)}$$

Equations 6–12 express a useful and profound principle regarding the total mechanical energy of a system—namely, that it is a **conserved quantity**. The total mechanical energy E remains constant as long as no nonconservative forces do work: KE + PE at some initial time 1 is equal to the KE + PE at any later time 2.

If only conservative forces do work, the total mechanical energy of a system neither increases nor decreases in any process. It stays constant—it is conserved.

CONSERVATION OF MECHANICAL ENERGY

$$E = \text{KE} + \text{PE} = \tfrac{1}{2}mv^2 + mgy$$

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2. \qquad \text{[gravity only]}$$

6–6 MECHANICAL ENERGY AND ITS CONSERVATION

EXAMPLE 6–7 **Falling rock.** If the initial height of the rock in Fig. 6–17 is $y_1 = h = 3.0\,\text{m}$, calculate the rock's velocity when it has fallen to 1.0 m above the ground.

APPROACH We apply the principle of conservation of mechanical energy, Eq. 6–13, with only gravity acting on the rock. We choose the ground as our reference level $(y = 0)$.

SOLUTION At the moment of release (point 1) the rock's position is $y_1 = 3.0\,\text{m}$ and it is at rest: $v_1 = 0$. We want to find v_2 when the rock is at position $y_2 = 1.0\,\text{m}$. Equation 6–13 gives

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2.$$

The m's cancel out and $v_1 = 0$, so

$$gy_1 = \tfrac{1}{2}v_2^2 + gy_2.$$

Solving for v_2 we find

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.8\,\text{m/s}^2)[(3.0\,\text{m}) - (1.0\,\text{m})]} = 6.3\,\text{m/s}.$$

The rock's velocity 1.0 m above the ground is 6.3 m/s downward.

NOTE The velocity of the rock is independent of the rock's mass.

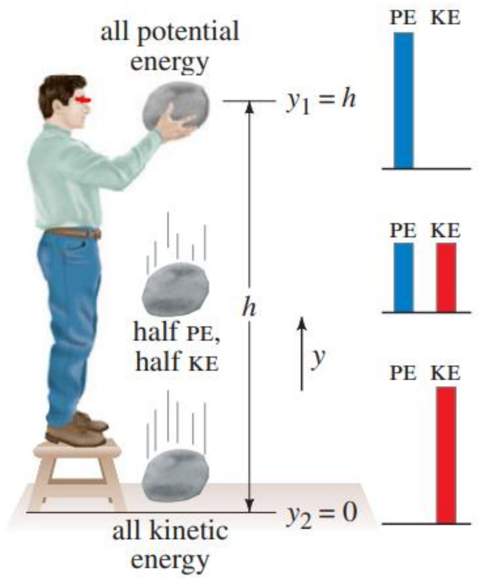


FIGURE 6–17 The rock's potential energy changes to kinetic energy as it falls. Note bar graphs representing potential energy PE and kinetic energy KE for the three different positions.

6–6 MECHANICAL ENERGY AND ITS CONSERVATION

EXAMPLE 6–8 Roller-coaster car speed using energy conservation. Assuming the height of the hill in Fig. 6–18 is 40 m, and the roller-coaster car starts from rest at the top, calculate (*a*) the speed of the roller-coaster car at the bottom of the hill, and (*b*) at what height it will have half this speed. Take $y = 0$ at the bottom of the hill.

APPROACH We use conservation of mechanical energy. We choose point 1 to be where the car starts from rest $(v_1 = 0)$ at the top of the hill $(y_1 = 40\text{ m})$. In part (*a*), point 2 is the bottom of the hill, which we choose as our reference level, so $y_2 = 0$. In part (*b*) we let y_2 be the unknown.

SOLUTION (*a*) We use Eq. 6–13 with $v_1 = 0$ and $y_2 = 0$, which gives

$$mgy_1 = \tfrac{1}{2}mv_2^2$$

or

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.8\text{ m/s}^2)(40\text{ m})} = 28\text{ m/s}.$$

(*b*) Now y_2 will be an unknown. We again use conservation of energy,

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2,$$

but now $v_2 = \frac{1}{2}(28\text{ m/s}) = 14\text{ m/s}$ and $v_1 = 0$. Solving for the unknown y_2 gives

$$y_2 = y_1 - \frac{v_2^2}{2g} = 40\text{ m} - \frac{(14\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 30\text{ m}.$$

That is, the car has a speed of 14 m/s when it is 30 *vertical* meters above the lowest point, both when descending the left-hand hill and when ascending the right-hand hill.

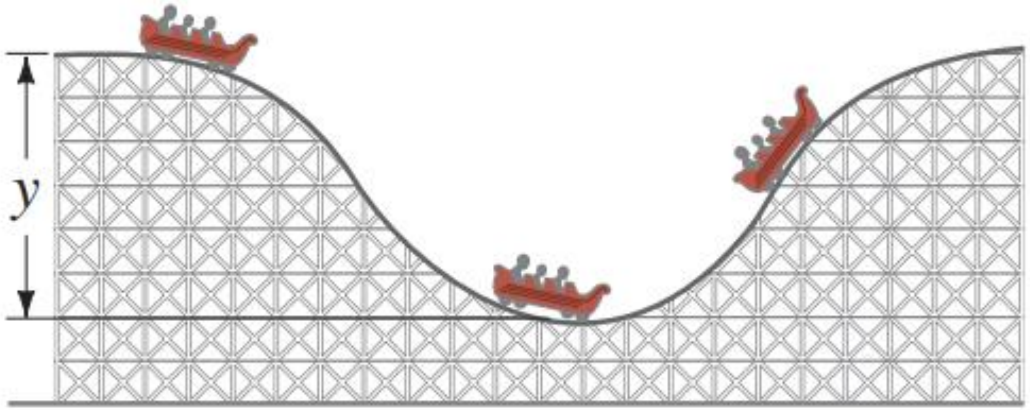


FIGURE 6–18 A roller-coaster car moving without friction illustrates the conservation of mechanical energy.

CONCEPTUAL EXAMPLE 6–9 **Speeds on two water slides.** Two water slides at a pool are shaped differently, but start at the same height h (Fig. 6–19). Two riders start from rest at the same time on different slides. (*a*) Which rider, Paul or Corinne, is traveling faster at the bottom? (*b*) Which rider makes it to the bottom first? Ignore friction and assume both slides have the same path length.

RESPONSE (*a*) Each rider's initial potential energy mgh gets transformed to kinetic energy, so the speed v at the bottom is obtained from $\frac{1}{2}mv^2 = mgh$. The mass cancels and so the speed will be the same, regardless of the mass of the rider. Since they descend the same vertical height, they will finish with the same speed. (*b*) Note that Corinne is consistently at a lower elevation than Paul at any instant, until the end. This means she has converted her potential energy to kinetic energy earlier. Consequently, she is traveling faster than Paul for the whole trip, and because the distance is the same, Corinne gets to the bottom first.

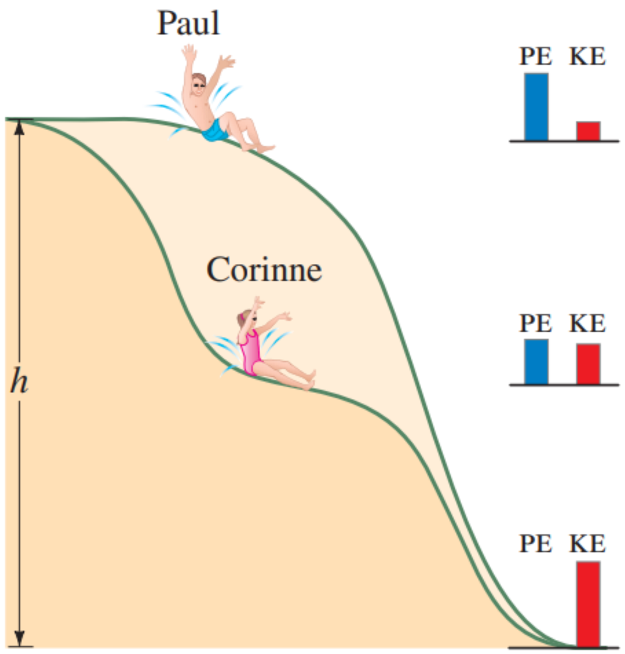
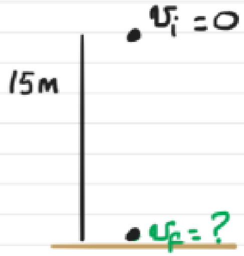


FIGURE 6–19 Example 6–9.

Example: An object of mass m is dropped from a height of 15 m above the earth's surface. Ignoring air resistance, Find:

i) Its speed just before hitting the ground.

Only the weight does work. It is a conservative force ⇒ total mechanical energy is conserved.



$\Delta K + \Delta U = 0$

For ΔU: If object rises (يرتفع) ⇒ $\Delta U = +mgh$ ↑h

If object descends (falls) ⇒ $\Delta U = -mgh$ ↓h

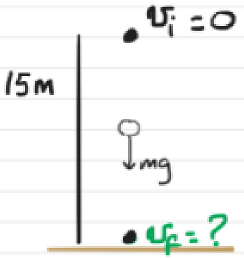
If object remains on the same level ⇒ $\Delta U = 0$

$\Delta K + \Delta U = 0$

$\frac{1}{2}m(v_f^2 - (0)^2) - mg(15) = 0$

$\frac{1}{2}v_f^2 = 15(g)$

$\therefore v_f = \sqrt{30g} \sim 17.1$ m/s

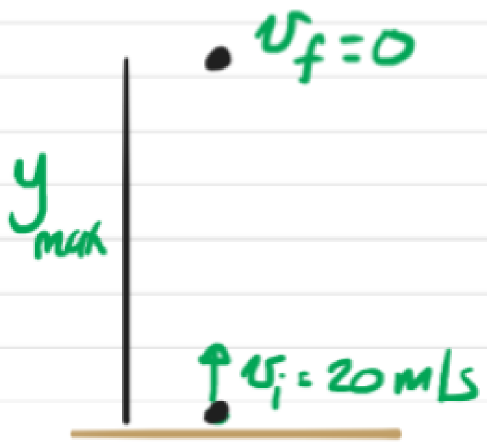


Example: A stone is projected vertically upwards from ground level with an initial speed of 20 m/s. Ignoring air resistance, find its maximum height.

Only the gravitational force acts $\Rightarrow$ $\Delta K + \Delta U = 0$

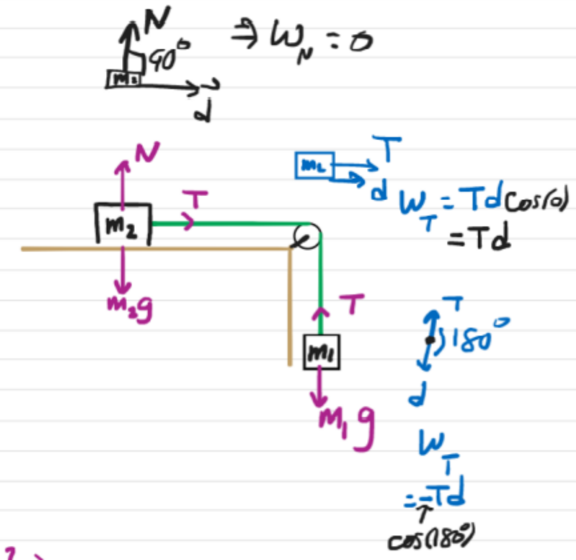
$$\frac{1}{2}m\left(0-(20)^2\right) + mgy_{max} = 0$$

$$\therefore \; y_{max} = \frac{(20)^2}{2g} = 20 \text{ m.}$$



Example:

Find the speed of the of m_1 when it has fallen a vertical distance of 2 m. All surfaces are smooth. Assume system started from rest.



$$\Delta K + \Delta U = 0$$

$$\underbrace{(\Delta K_1 + \Delta U_1)}_{\text{for mass 1}} + \underbrace{(\Delta K_2 + \Delta U_2)}_{\text{for mass 2}} = 0$$

$$\frac{1}{2}m_1(v_{1f}^2 - v_{1i}^2) - mg(2) + \frac{1}{2}m_2(v_{2f}^2 - v_{2i}^2) + 0 = 0$$

↑ since m_2 does NOT change height.

$$\frac{1}{2}m_1(v_{1f}^2 - 0) - 2m_1 g + \frac{1}{2}m_2 v_{2f}^2 = 0$$

m_1 and m_2 are connected by an inextensible string

$$\Rightarrow \quad v_{1f} = v_{2f} \equiv v_f$$

$$\frac{1}{2}(m_1 + m_2)v_f^2 = 2m_1 g$$

$$\therefore \quad v_f^2 = \frac{4m_1 g}{m_1 + m_2} \Rightarrow v_f = \sqrt{\frac{4m_1 g}{m_1 + m_2}}$$

6–9 ENERGY CONSERVATION WITH DISSIPATIVE FORCES: SOLVING PROBLEMS

$$W_{NC} = \Delta KE + \Delta PE,$$

where W_{NC} is the work done by nonconservative forces such as friction. Consider an object, such as a roller-coaster car, as a particle moving under gravity with nonconservative forces like friction acting on it. When the object moves from some point 1 to another point 2, then

$$W_{NC} = KE_2 - KE_1 + PE_2 - PE_1 .$$

We can rewrite this as

$$KE_1 + PE_1 + W_{NC} = KE_2 + PE_2 . \quad \textbf{(6–15)}$$

For the case of friction, $W_{NC} = -F_{fr}\,d$, where d is the distance over which the friction (assumed constant) acts as the object moves from point 1 to point 2. ($\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$ are in opposite directions, hence the minus sign from $\cos 180° = -1$ in Eq. 6–1.)

With $KE = \frac{1}{2}mv^2$ and $PE = mgy$, Eq. 6–15 with $W_{NC} = -F_{fr}\,d$ becomes

$$\tfrac{1}{2}mv_1^2 + mgy_1 - F_{fr}\,d = \tfrac{1}{2}mv_2^2 + mgy_2 . \quad \left[\begin{array}{c}\text{gravity and}\\ \text{friction acting}\end{array}\right] \quad \textbf{(6–16a)}$$

That is, the initial mechanical energy is reduced by the amount $F_{fr}\,d$. We could also write this equation as

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{fr}\,d$$

or

$$KE_1 + PE_1 = KE_2 + PE_2 + F_{fr}\,d, \quad \left[\begin{array}{c}\text{gravity and}\\ \text{friction}\\ \text{acting}\end{array}\right] \quad \textbf{(6–16b)}$$

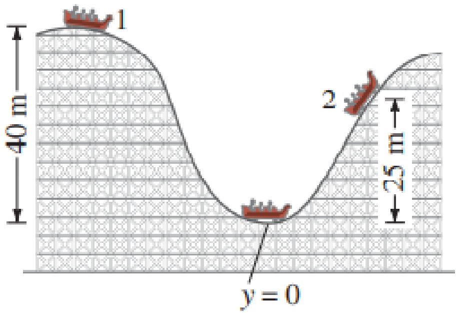


FIGURE 6–27 Example 6–12. Because of friction, a roller-coaster car does not reach the original height on the second hill. (Not to scale.)

EXAMPLE 6–12 ESTIMATE **Friction on the roller-coaster car.** The roller-coaster car in Example 6–8 reaches a vertical height of only 25 m on the second hill, where it slows to a momentary stop, Fig. 6–27. It traveled a total distance of 400 m. Determine the thermal energy produced and estimate the average friction force (assume it is roughly constant) on the car, whose mass is 1000 kg.

APPROACH We explicitly follow the Problem Solving Strategy above.

SOLUTION

1. **Draw a picture.** See Fig. 6–27.
2. **The system.** The system is the roller-coaster car and the Earth (which exerts the gravitational force). The forces acting on the car are gravity and friction. (The normal force also acts on the car, but does no work, so it does not affect the energy.) Gravity is accounted for as potential energy, and friction as a term $F_{fr}\,d$.
3. **Choose initial and final positions.** We take point 1 to be the instant when the car started coasting (at the top of the first hill), and point 2 to be the instant it stopped at a height of 25 m up the second hill.
4. **Choose a reference frame.** We choose the lowest point in the motion to be $y = 0$ for the gravitational potential energy.
5. **Is mechanical energy conserved?** No. Friction is present.
6. **Apply conservation of energy.** There is friction acting on the car, so we use conservation of energy in the form of Eq. 6–16b, with $v_1 = 0$, $y_1 = 40\,\text{m}$, $v_2 = 0$, $y_2 = 25\,\text{m}$, and $d = 400\,\text{m}$. Thus

$$0 + (1000\,\text{kg})(9.8\,\text{m/s}^2)(40\,\text{m}) = 0 + (1000\,\text{kg})(9.8\,\text{m/s}^2)(25\,\text{m}) + F_{fr}\,d.$$

7. **Solve.** We solve the above equation for $F_{fr}\,d$, the energy dissipated to thermal energy:

$$F_{fr}\,d = mg\,\Delta h = (1000\,\text{kg})(9.8\,\text{m/s}^2)(40\,\text{m} - 25\,\text{m}) = 147{,}000\,\text{J}.$$

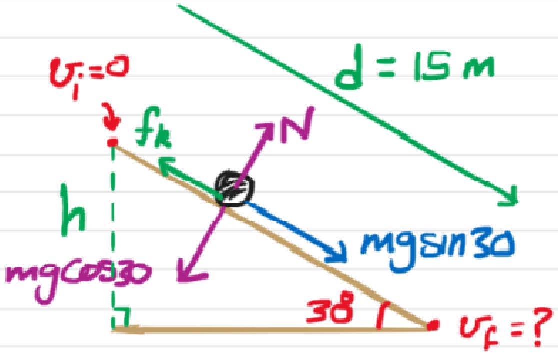
The friction force, which acts over a distance of 400 m, averages out to be

$$F_{fr} = (1.47 \times 10^5\,\text{J})/400\,\text{m} = 370\,\text{N}.$$

NOTE This result is only a rough average: the friction force at various points depends on the normal force, which varies with slope.

Example: Starting from rest a skier slides down a 30° inclined plane a distance of 15 m. If the coefficient of kinetic friction is 0.1 find his speed at the bottom of the slide.

Note N and mgcos30 do No work.



$$\Delta K + \Delta U = W_{nc}$$

$$\frac{1}{2}m(v_f^2 - 0) - mg\overbrace{(d\sin 30)}^{h} = W_{nc}$$

h: vertical distance desended by the skier

$$\frac{1}{2}m(v_f^2 - 0) - mg\overbrace{(d\sin 30)} = W_{nc}$$

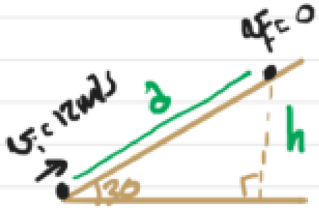
h: vertical distance desended by the skier

$$\frac{1}{2}\cancel{m}v_f^2 - \cancel{m}g(15\times\frac{1}{2}) = f_k\, d \cos 180°$$

$$= (\mu_k \cancel{m}g\cos 30)(15)(-1)$$

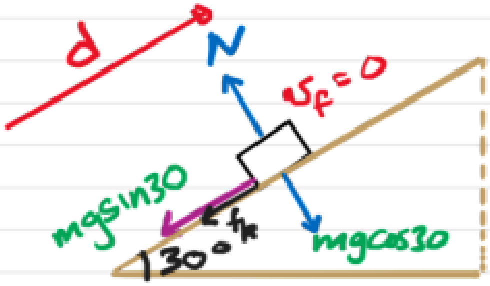
$$\therefore v_f^2 = \frac{159}{2} - (0.1)(9)(\frac{\sqrt{3}}{2})(15) \Rightarrow v_f \simeq 7.8 \text{ m/s}.$$

Example A box of mass m is given an initial speed of 12 m/s up a 30° inclined plane. If the coefficient of kinetic friction between the box and the plane is $\mu_k = 0.15$ find the maximum distance the box moves up the inclined plane.



$h = d \sin\theta$
$= d \sin 30$
$h = \frac{1}{2} d$

N and mg cos30 do No work.



$$\Delta K + \Delta U = W_{nc}$$

$$\frac{1}{2} m (v_f^2 - v_i^2) + mg(\underbrace{d \sin 30°}_{h}) = f_k d \cos 180°$$

$$\frac{1}{2} \cancel{m} (0 - (12)^2) + \cancel{m} g \left(\frac{d}{2}\right) = (\mu_k \underbrace{\cancel{m} g \cos 30}_{N})(d)(-1)$$

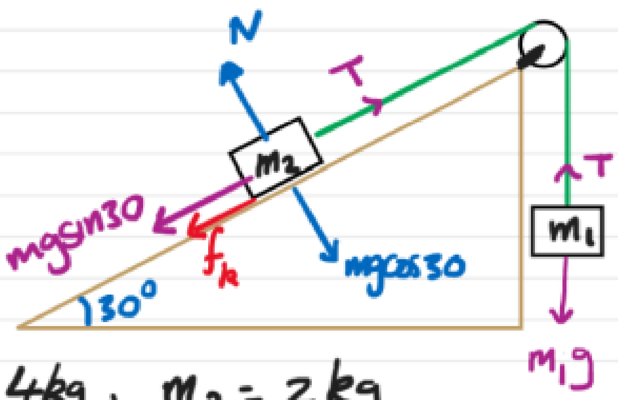
$$-72 = -d(\mu_k g \cos 30 + \tfrac{1}{2} g) \Rightarrow d = 11.7 \text{ m}.$$

$$\Delta K + \Delta U = W_{nc}$$

$$\left[\tfrac{1}{2}m_1(v_f^2 - v_i^2) - mg(1.5)\right] + \left[\tfrac{1}{2}m_2(v_f^2 - v_i^2) + mg(1.5\sin 30)\right] = f_k(1.5)\cos 180$$

Example

In the figure μ_k between m_2 and the inclined plane is 0.2. Find the speed of m_1 after it has fallen a distance of 1.5 m. Assume system started from rest. Also $m_1 = 4kg$, $m_2 = 2kg$



Note# since m_1 and m_2 are connected by an inextensible string ⇒ the have the same speed.

when determining ΔU we only need the vertical distance ascended (for m_2) or desended for (m_1)

ascended: $1.5\sin 30$; desended: 1.5 m

$$\tfrac{1}{2}(m_1+m_2)v_f^2 + 1.5g\left[m_2\sin 30 - m_1\right] = \mu_k m_2 g\cos 30 \times (-1)$$

$$3v_f^2 + [-34.02] = -3.46$$

$$\therefore v_f \sim 3.2 \text{ m/s}.$$

6–10 POWER

Power is defined as the *rate at which work is done*. Average power equals the work done divided by the time to do it. Power can also be defined as the *rate at which energy is transformed*. Thus

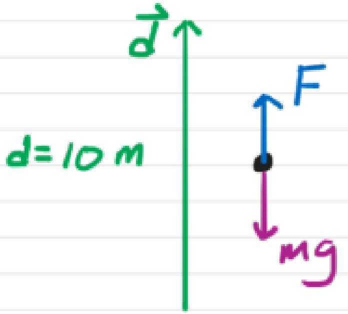
$$\overline{P} = \text{average power} = \frac{\text{work}}{\text{time}} = \frac{\text{energy transformed}}{\text{time}}. \qquad \textbf{(6–17)}$$

The power rating of an engine refers to how much chemical or electrical energy can be transformed into mechanical energy per unit time. In SI units, power is measured in joules per second, and this unit is given a special name, the **watt** (W): $1\ \text{W} = 1\ \text{J/s}$. We are most familiar with the watt for electrical devices, such as the rate at which an electric lightbulb or heater changes electric energy into light or thermal energy. But the watt is used for other types of energy transformations as well.

Example

A 60-kg firefighter climbs a 10 m vertical wire (rop[e]) in 10 seconds at constant speed. Find his average power output. ($\bar{P}$)

The firefighter exerts a force F as he climbs the rope.



$$\Rightarrow \bar{P} = \frac{W_F}{t} = \frac{Fd\cos(0)}{t} = \frac{Fd}{t}$$

How can we find F? He moves upwards at constant speed ($a_y = 0$).

$\bar{P} = F\left(\frac{d}{t}\right) = Fv$

Using Newton's second law

$\uparrow^{+}$ $\Sigma F_y = ma_y \Rightarrow F - mg = m(0)$

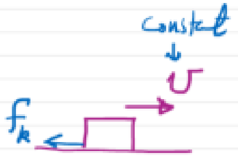
$\therefore F = mg$

$\bar{P} = \frac{W}{t}$

$W = \bar{P}t$

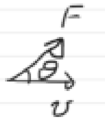
$$\therefore \bar{P} = \frac{\overbrace{mg}^{F}d}{t} = \frac{(60)(10)(10)}{10} = 600 \text{ W}.$$

NOTE: using $\bar{P} = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv$



power of f_k $\rightarrow \bar{P} = f_k v \cos 180° = -f_k v$

$\bar{P} = Fv\cos\theta$ ONLY wheen speed is constant.
↑ angle between $\vec{F}$ and $\vec{v}$.



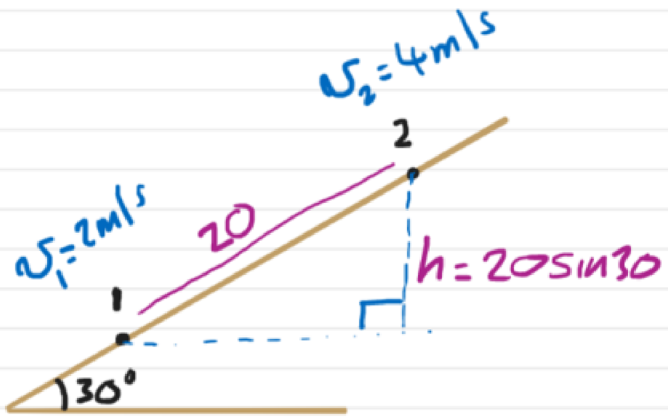
$\bar{p} = Fv\cos\theta$

$\bar{P} = mg\left(\frac{10}{10}\right) = mg(1) = 60 \times 10 \times 1 = 600$ J

As before.

Example

A runner (mass = 50 kg) is going up a 30° inclined plane. At point 1 his speed is 2 m/s while at point 2 his speed is 4 m/s. The distance between points 1 and 2 is 20 m. He takes 10 s to move from point 1 → point 2.



If his mass is 55 kg find his average power output.

$$\bar{P} = \frac{W}{t}$$

How to find W?

Remember, the force of the runner is a nonconservative force ⇒

$$W = W_{nc} = \Delta K + \Delta U$$
$$= \underbrace{\tfrac{1}{2}m(v_f^2 - v_i^2)}_{} + mg(20 \sin 30)$$

when running at constant speed this term = 0 as in the above example of the fire fighter.

∴ $W = W_{nc} = \frac{1}{2}(50)[16-4] + (50)(10)\underbrace{(20 \sin 30)}_{\text{vertical distance ascended}}$

$= 5300 \text{ J}$

$\Rightarrow \bar{P} = \frac{5300}{10} = 530$ Watt

i.e 530 J/s.

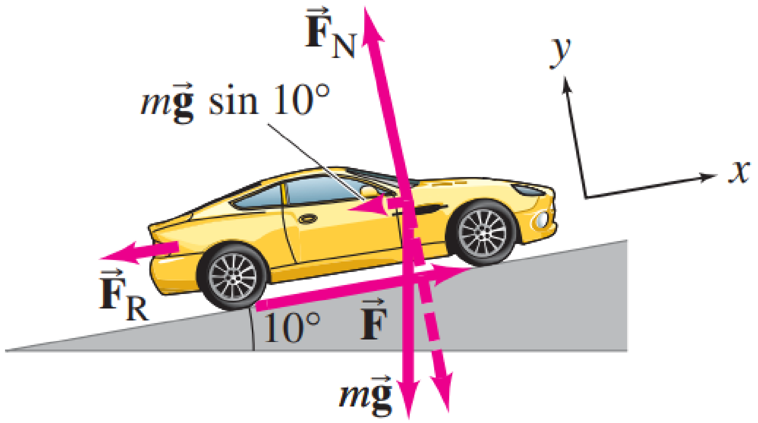


FIGURE 6–29 Example 6–14. Calculation of power needed for a car to climb a hill.

EXAMPLE 6–14 **Power needs of a car.** Calculate the power required of a 1400-kg car under the following circumstances: (*a*) the car climbs a 10° hill (a fairly steep hill) at a steady 80 km/h; and (*b*) the car accelerates along a level road from 90 to 110 km/h in 6.0 s to pass another car. Assume the average retarding force on the car is $F_R = 700\,\text{N}$ throughout. See Fig. 6–29.

APPROACH First we must be careful not to confuse $\vec{\mathbf{F}}_R$, which is due to air resistance and friction that retards the motion, with the force $\vec{\mathbf{F}}$ needed to accelerate the car, which is the frictional force exerted by the road on the tires—the reaction to the motor-driven tires pushing against the road. We must determine the magnitude of the force F before calculating the power.

We mentioned in Example 6–14 that only part of the energy output of a car engine reaches the wheels. Not only is some energy wasted in getting from the engine to the wheels, in the engine itself most of the input energy (from the burning of gasoline or other fuel) does not do useful work. An important characteristic of all engines is their overall **efficiency** e, defined as the ratio of the useful power output of the engine, P_{out}, to the power input, P_{in} (provided by burning of gasoline, for example):

$$e = \frac{P_{\text{out}}}{P_{\text{in}}}.$$

The efficiency is always less than 1.0 because no engine can create energy, and no engine can even transform energy from one form to another without some energy going to friction, thermal energy, and other nonuseful forms of energy. For example, an automobile engine converts chemical energy released in the burning of gasoline into mechanical energy that moves the pistons and eventually the wheels. But nearly 85% of the input energy is "wasted" as thermal energy that goes into the cooling system or out the exhaust pipe, plus friction in the moving parts. Thus car engines are roughly only about 15% efficient. We will discuss efficiency in more detail in Chapter 15.

SOLUTION (*a*) To move at a steady speed up the hill, the car must, by Newton's second law, exert a force F equal to the sum of the retarding force, 700 N, and the component of gravity parallel to the hill, $mg \sin 10°$, Fig. 6–29. Thus

$$\begin{aligned} F &= 700\,\text{N} + mg \sin 10° \\ &= 700\,\text{N} + (1400\,\text{kg})(9.80\,\text{m/s}^2)(0.174) = 3100\,\text{N}. \end{aligned}$$

Since $\bar{v} = 80\,\text{km/h} = 22\,\text{m/s}^{\dagger}$ and is parallel to $\vec{\mathbf{F}}$, then (Eq. 6–18) the power is

$$\overline{P} = F\bar{v} = (3100\,\text{N})(22\,\text{m/s}) = 6.8 \times 10^4\,\text{W} = 68\,\text{kW} = 91\,\text{hp}.$$

(*b*) The car accelerates from 25.0 m/s to 30.6 m/s (90 to 110 km/h) on the flat. The car must exert a force that overcomes the 700-N retarding force plus that required to give it the acceleration

$$\bar{a}_x = \frac{(30.6\,\text{m/s} - 25.0\,\text{m/s})}{6.0\,\text{s}} = 0.93\,\text{m/s}^2.$$

We apply Newton's second law with x being the horizontal direction of motion (no component of gravity):

$$ma_x = \Sigma F_x = F - F_{\text{R}}.$$

We solve for the force required, F:

$$\begin{aligned} F &= ma_x + F_{\text{R}} \\ &= (1400\,\text{kg})(0.93\,\text{m/s}^2) + 700\,\text{N} = 1300\,\text{N} + 700\,\text{N} = 2000\,\text{N}. \end{aligned}$$

Since $\overline{P} = F\bar{v}$, the required power increases with speed and the motor must be able to provide a maximum power output in this case of

$$\overline{P} = (2000\,\text{N})(30.6\,\text{m/s}) = 6.1 \times 10^4\,\text{W} = 61\,\text{kW} = 82\,\text{hp}.$$

NOTE Even taking into account the fact that only 60 to 80% of the engine's power output reaches the wheels, it is clear from these calculations that an engine of 75 to 100 kW (100 to 130 hp) is adequate from a practical point of view.