CHAPTER (6)/ WORK & ENERGY

Done by: Abdelhadi Okasha

CONCEPTS REQUIRED

- 6–1 Work Done by a Constant Force
- 6–3 Kinetic Energy, and the Work-Energy Principle

6–1 WORK DONE BY A CONSTANT FORCE

- **<u>Work</u>** done on an object by a constant force (constant in both magnitude and direction) is defined to be:
  the product of the magnitude of the displacement times the component of the force parallel to the displacement.
- $W = Fd\cos\theta$ (Theta is the angle between the directions of the force and the displacement)
- From the previous formula, Work can = zero even if there is force if: 1) there is no displacement, or 2) cos **θ** = zero
- Work is measured in newton-meters (N.M) A special name is given to this unit, the joule (J): 1J = 1 Nm

To find the **net work** done on the object, either (*a*) find the work done by each force and add the results algebraically; or (*b*) find the net force on the object, F_{net}, and then use it to find the net work done, which for constant net force is:

$$W_{\text{net}} = F_{\text{net}}\, d\cos\theta.$$

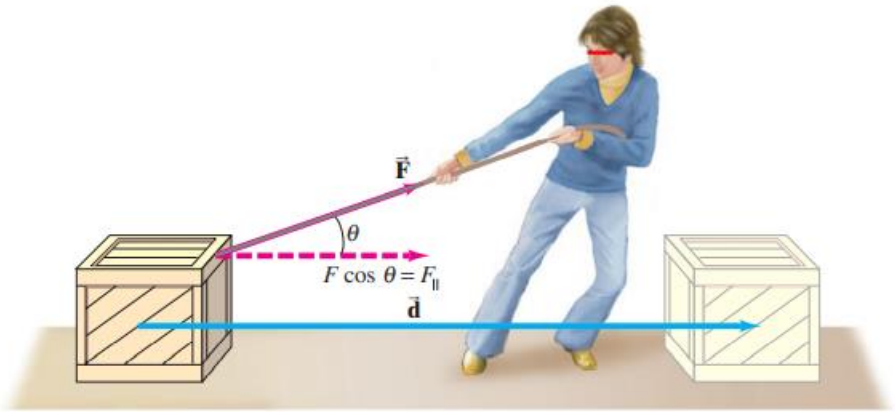


FIGURE 6–1 A person pulling a crate along the floor. The work done by the force $\vec{F}$ is $W = Fd\cos\theta$, where $\vec{d}$ is the displacement.

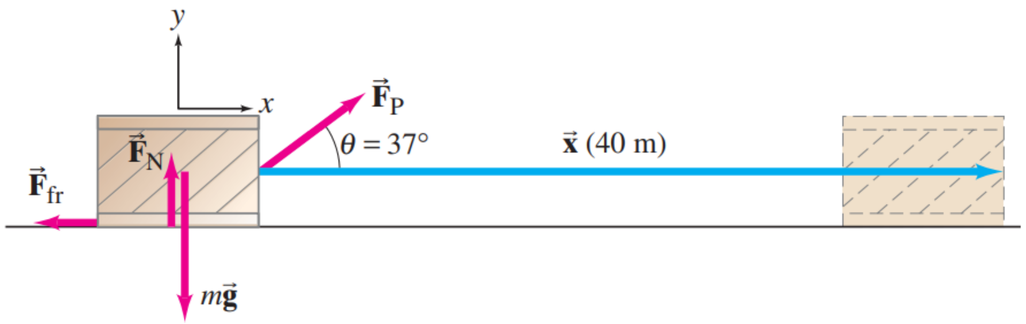


FIGURE 6–3 Example 6–1. A 50-kg crate is pulled along a floor.

EXAMPLE 6–1 **Work done on a crate.** A person pulls a 50-kg crate 40 m along a horizontal floor by a constant force $F_P = 100\text{ N}$, which acts at a 37° angle as shown in Fig. 6–3. The floor is rough and exerts a friction force $\vec{\mathbf{F}}_{fr} = 50\text{ N}$. Determine (*a*) the work done by each force acting on the crate, and (*b*) the net work done on the crate.

SOLUTION (*a*) The work done by the gravitational force $(\vec{\mathbf{F}}_G)$ and by the normal force $(\vec{\mathbf{F}}_N)$ is zero, because they are perpendicular to the displacement $\vec{\mathbf{x}}$ $(\theta = 90°$ in Eq. 6–1):

$$W_G = mgx\cos 90° = 0$$
$$W_N = F_N x\cos 90° = 0.$$

The work done by $\vec{\mathbf{F}}_P$ is

$$W_P = F_P x\cos\theta = (100\text{ N})(40\text{ m})\cos 37° = 3200\text{ J}.$$

The work done by the friction force is

$$W_{fr} = F_{fr} x\cos 180° = (50\text{ N})(40\text{ m})(-1) = -2000\text{ J}.$$

The angle between the displacement $\vec{\mathbf{x}}$ and $\vec{\mathbf{F}}_{fr}$ is 180° because they point in opposite directions. Since the force of friction is opposing the motion (and $\cos 180° = -1$), the work done by friction on the crate is *negative*.

(*b*) The net work can be calculated in two equivalent ways.

(1) The net work done on an object is the algebraic sum of the work done by each force, since work is a scalar:

$$\begin{aligned} W_{net} &= W_G + W_N + W_P + W_{fr} \\ &= 0 + 0 + 3200\text{ J} - 2000\text{ J} = 1200\text{ J}. \end{aligned}$$

(2) The net work can also be calculated by first determining the net force on the object and then taking the component of this net force along the displacement: $(F_{net})_x = F_P\cos\theta - F_{fr}$. Then the net work is

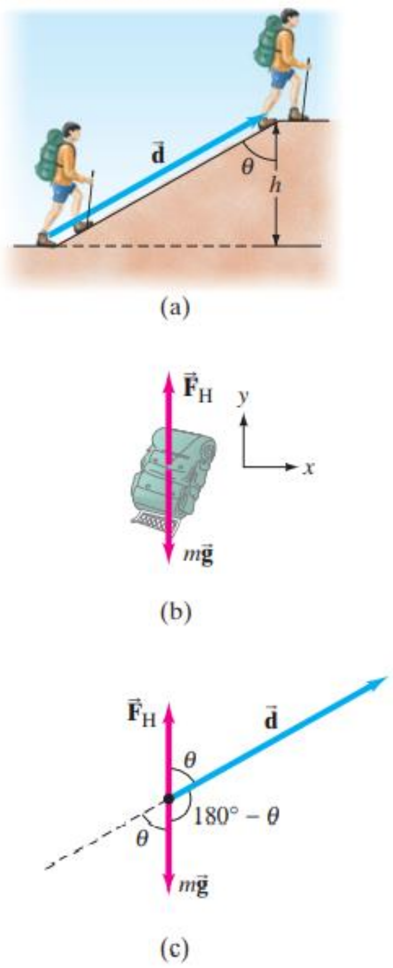
$$\begin{aligned} W_{net} &= (F_{net})_x x = (F_P\cos\theta - F_{fr})x \\ &= (100\text{ N}\cos 37° - 50\text{ N})(40\text{ m}) = 1200\text{ J}. \end{aligned}$$

In the vertical (y) direction, there is no displacement and no work done.

EXAMPLE 6–2 **Work on a backpack.** (*a*) Determine the work a hiker must do on a 15.0-kg backpack to carry it up a hill of height $h = 10.0\,\text{m}$, as shown in Fig. 6–4a. Determine also (*b*) the work done by gravity on the backpack, and (*c*) the net work done on the backpack. For simplicity, assume the motion is smooth and at constant velocity (i.e., acceleration is zero).

APPROACH We explicitly follow the steps of the Problem Solving Strategy above.

FIGURE 6–4 Example 6–2.



What does the sign of the work done on an object mean?

$W_{net} > 0$ (i.e positive) $\Rightarrow$ object accelerates

$W_{net} < 0$ (i.e negative) $\Rightarrow$ object decelerates

$W_{net} = 0$ $\Rightarrow$ object moves at constant speed.

PROBLEM SOLVING

Work done by gravity depends on height of hill (not on angle)

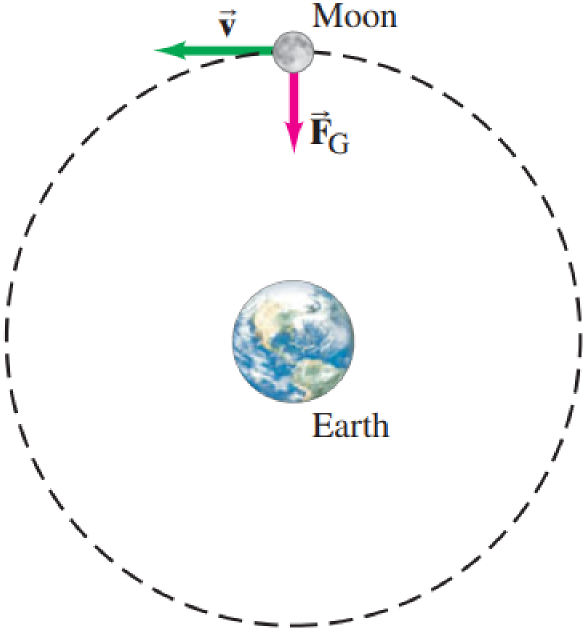


FIGURE 6–5 Example 6–3.

CONCEPTUAL EXAMPLE 6–3 **Does the Earth do work on the Moon?** The Moon revolves around the Earth in a nearly circular orbit, kept there by the gravitational force exerted by the Earth. Does gravity do (*a*) positive work, (*b*) negative work, or (*c*) no work on the Moon?

RESPONSE The gravitational force $\vec{\mathbf{F}}_G$ exerted by the Earth on the Moon (Fig. 6–5) acts toward the Earth and provides its centripetal acceleration, inward along the radius of the Moon's orbit. The Moon's displacement at any moment is tangent to the circle, in the direction of its velocity, perpendicular to the radius and perpendicular to the force of gravity. Hence the angle θ between the force $\vec{\mathbf{F}}_G$ and the instantaneous displacement of the Moon is 90°, and the work done by gravity is therefore zero $(\cos 90° = 0)$. This is why the Moon, as well as artificial satellites, can stay in orbit without expenditure of fuel: no work needs to be done against the force of gravity.

6–3 KINETIC ENERGY, AND THE WORK-ENERGY PRINCIPLE

- For the purposes of this Chapter, we can define energy in the traditional way as **"the ability to do work."** This simple definition is not always applicable,† but it is valid for mechanical energy

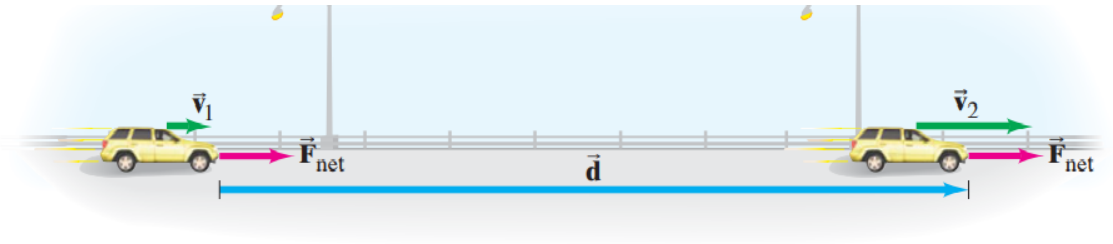


FIGURE 6–7 A constant net force F_{net} accelerates a car from speed v_1 to speed v_2 over a displacement d. The net work done is $W_{net} = F_{net}d$.

To obtain a quantitative definition for kinetic energy, let us consider a simple rigid object of mass m (treated as a particle) that is moving in a straight line with an initial speed v_1. To accelerate it uniformly to a speed v_2, a constant net force F_{net} is exerted on it parallel to its motion over a displacement d, Fig. 6–7. Then the net work done on the object is $W_{net} = F_{net}d$. We apply Newton's second law, $F_{net} = ma$, and use Eq. 2–11c $(v_2^2 = v_1^2 + 2ad)$, which we rewrite as

$$a = \frac{v_2^2 - v_1^2}{2d},$$

where v_1 is the initial speed and v_2 is the final speed. Substituting this into $F_{net} = ma$, we determine the work done:

$$W_{net} = F_{net}d = mad = m\left(\frac{v_2^2 - v_1^2}{2d}\right)d = m\left(\frac{v_2^2 - v_1^2}{2}\right)$$

or

$$W_{net} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \qquad \textbf{(6–2)}$$

We *define* the quantity $\frac{1}{2}mv^2$ to be the **translational kinetic energy (KE)** of the object:

$$\text{KE} = \tfrac{1}{2}mv^2. \qquad \textbf{(6–3)}$$

Kinetic energy (defined)

$$W_{net} = \text{KE}_2 - \text{KE}_1$$

or

$$W_{net} = \Delta\text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \qquad \textbf{(6–4)}$$

WORK-ENERGY PRINCIPLE

Equation 6–4 is a useful result known as the **work-energy principle**. It can be stated in words:

The net work done on an object is equal to the change in the object's kinetic energy.

WORK-ENERGY PRINCIPLE

Can kinetic energy ever be negative?
Of course no

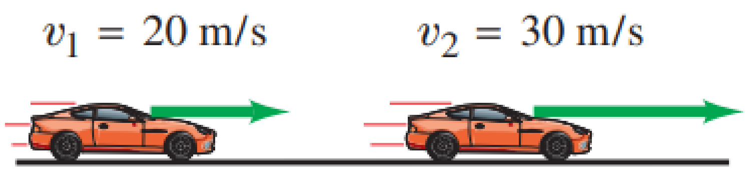


FIGURE 6–9 Example 6–4.

EXAMPLE 6–4 ESTIMATE Work on a car, to increase its kinetic energy. How much net work is required to accelerate a 1000-kg car from 20 m/s to 30 m/s (Fig. 6–9)?

- We can solve all questions about free fall by using the law ($W_{net} = \Delta K$) except if we want to know the time
- Example: An object of mass m is projected vertically upwards from the earths surface with an initial speed of 20m/s
  a) Find it's maximum height
  b) Find the speed of the object when it's at a height of 15m
  c) Find the speed of the object just before hitting the ground

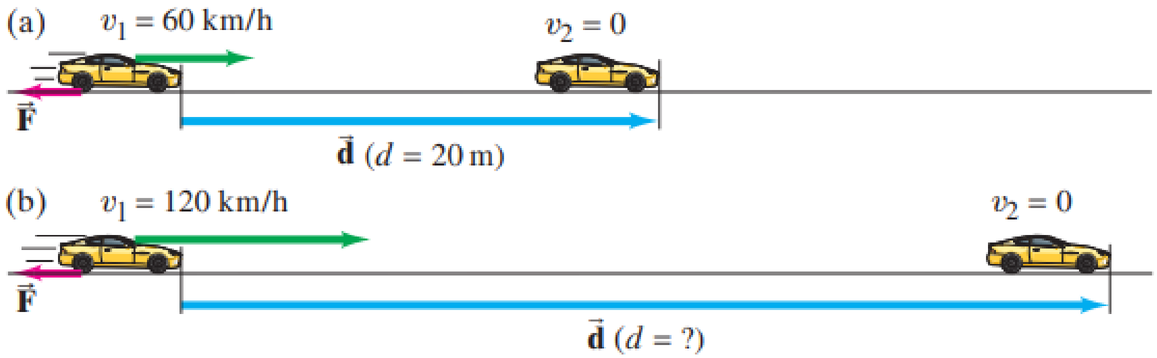


FIGURE 6–10 Example 6–5. A moving car comes to a stop. Initial velocity is (a) 60 km/h, (b) 120 km/h.

CONCEPTUAL EXAMPLE 6–5 **Work to stop a car.** A car traveling 60 km/h can brake to a stop in a distance d of 20 m (Fig. 6–10a). If the car is going twice as fast, 120 km/h, what is its stopping distance (Fig. 6–10b)? Assume the maximum braking force is approximately independent of speed.