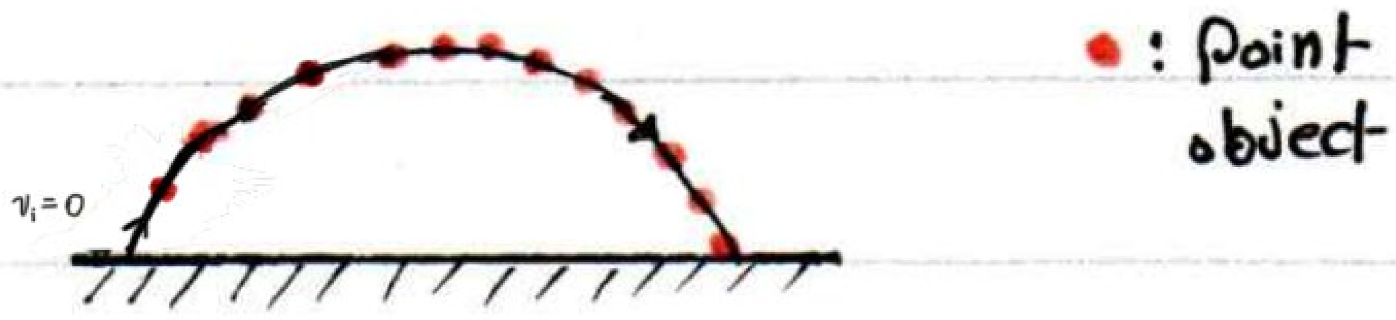
7-8 Center of Mass (CM):

If we want to study a specific concept such as, momentum, we deal with two types of objects :-

a) A point particle:

- undergoes translational motion only. (all parts of the object follow the same path)

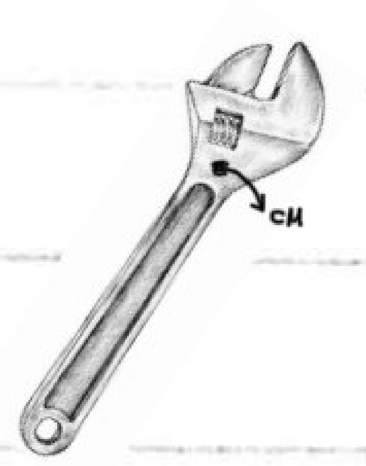
e.g. If you through a small metal sphere with an initial velocity makes $\theta > 0$ with the horizontal, it follows this trajectory:-



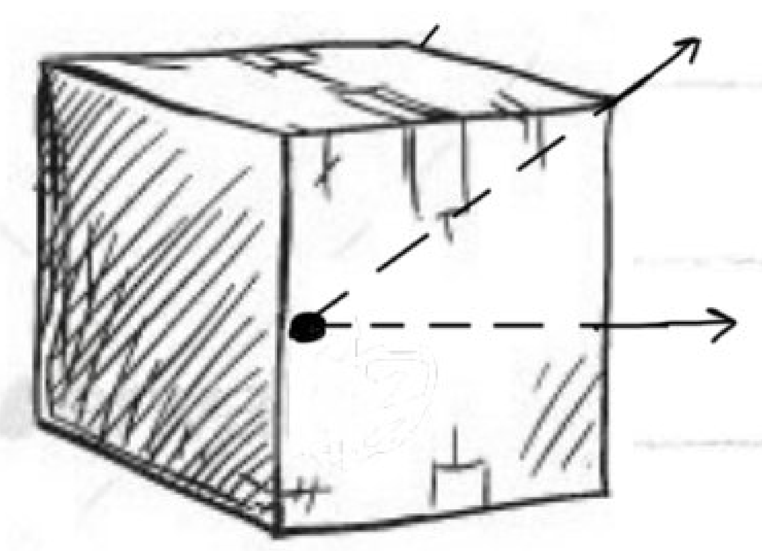
b) An extended object:

- an object that has size (dimensions).
- Undergoes both translational and rotational motion. (general motion).

e.g. If you through a wrench there is only one point that moves in the same path, it is the point of the Center of mass (CM).
The wrench behaves as if (not really) all its mass is concentrated at the point of (CM)

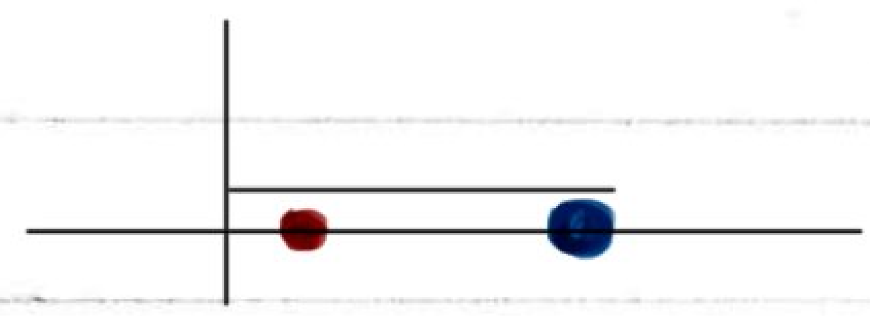


* In the previous chapters we used to draw a free-body diagram where forces act on objects. For an extended object like this box, where should the force act?



↑ We assume a force to act on the (CM).

* Because of the important properties of the (CM), we are going to learn the definition of the (CM) as the position of x_{CM}:-



$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_a + m_B} \quad , \quad \text{where } M = M_A + m_B$$

$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

* This definition is used for:-

a) A system of two particles in one dimension.

b) A system of more than two particles in on dimension:

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C \dots\dots}{M}$$

c) System particles in two dimensions.

→ For examples

Example (1) (a)

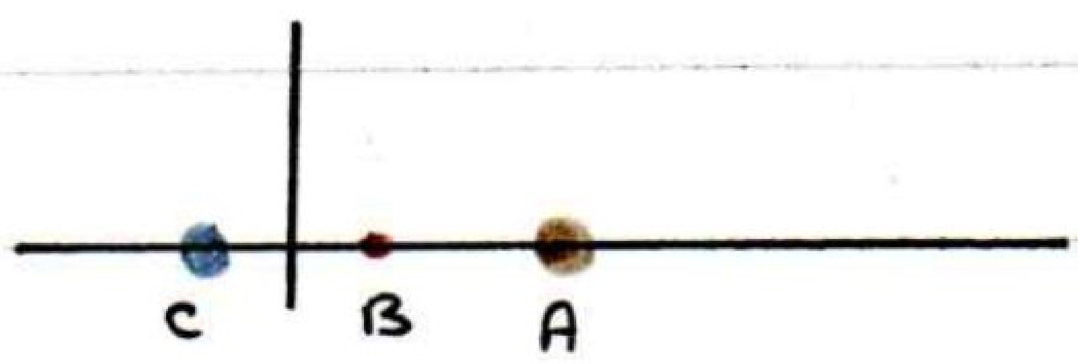
$m_A = 6\ Kg$ $\quad x_A = 2m$

$m_B = 2Kg$ $\quad x_B = 4m$

* Find the (CM) with resped to the origin

$$x_{CM} = \frac{(6)(2) + (2)(4)}{6+2} = \frac{20}{8} = 2.5\,m$$

Example (2) (b)



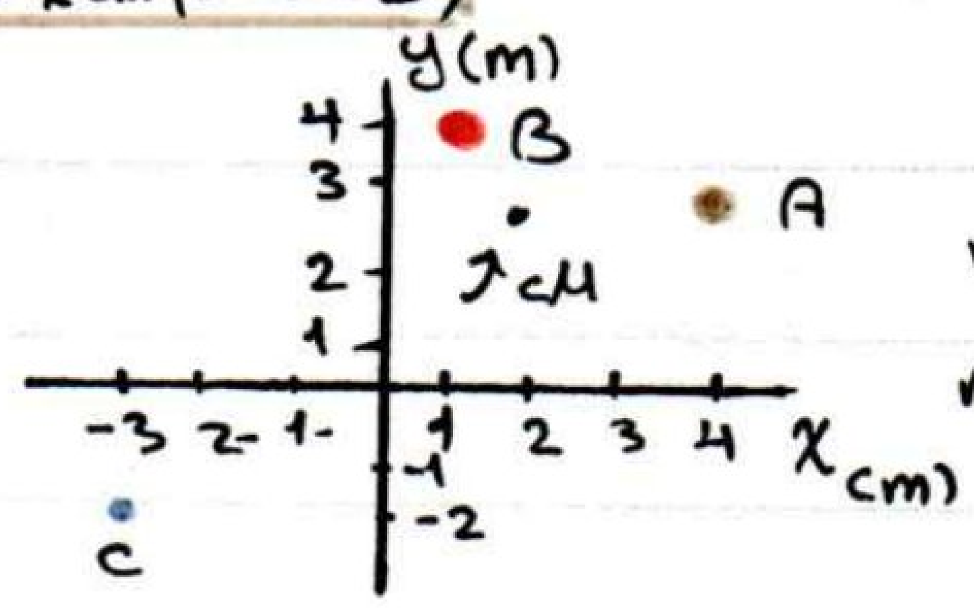
$m_A = 5Kg$ $\quad x_A = 3m$

$m_B = 1\ Kg$ $\quad x_B = 2m$

$m_c = 4\ Kg$ $\quad x_c = -2m$

$$x_{CM} = \frac{(5)(3) + (1)(2) + (4)(-2)}{5+1+4} = 0.9\ m$$

Example (3) (c)



$m_A = 4\ Kg$

$m_B = 3Kg$

$m_c = 1Kg$

$A \to \overset{x\ \ y}{(4,3)}$ $\quad B \to \overset{x\ \ y}{(1,4)}$ $\quad C \to \overset{x\ \ y}{(-3,-2)}$

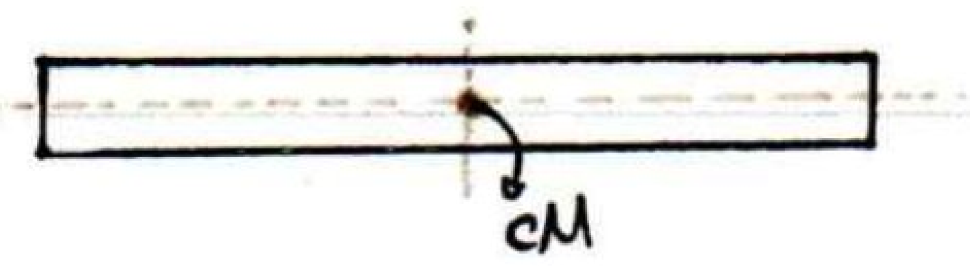
$$x_{CM} = \frac{(4)(4) + (3)(1) + (1)(-3)}{4+3+1} = 2\,m$$

$$y_{CM} = \frac{(4)(3) + (3)(4) + (1)(-2)}{4+3+1} = 2.75\,m$$

$$CM \to (\overset{x}{2}, \overset{y}{2.75})$$

*Note: The coordinates of the (CM) depend on the reference frame choosen, but the physical location of the (CM) is independent of the choice.

* For symmetrically shaped objects such as, uniform cylinders (wheels), spheres and rectangular solids. The (CM) is located at the geometric center.



Uniform ruler

Circular ring

* the CM may be outside the mass distribution of the object. like this ring unlike this col

which has no gap (Its (CM) inside its mass)

* We can determine the (CM) experimentally:-

(a) Suspend the object using a string tied at Point 1 and draw a vertical line passes through Point 1.

(b) Repeat the previous step but choose another Point call Point 2.