# CHAPTER 7

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# CONCEPTS

- 7–8 Center of Mass (CM)
- 7–9 CM for the Human Body

# 7–8 CENTER OF MASS (CM)

(a)

(b)

- Large objects that have dimensions are called extended objects, while small objects that have undetectable dimensions are called point objects
- For extended objects: there is one point that moves in the same path that a particle would move if subjected to the same net force. This point is called the center of mass (abbreviated CM), it acts as a point of mass.
- the diver in Fig.a undergoes only translational motion (all parts of the object follow the same path), whereas the diver in Fig. b undergoes both translational and rotational motion. We will refer to motion that is not pure translation as general motion
- Observations indicate that even if an object rotates, or several parts of a system of objects move relative to one another, there is one point that moves in the same path that a particle would move if subjected to the same net force. This point is called the center of mass (abbreviated CM). The general motion of an extended object (or system of objects) can be considered as the sum of the translational motion of the CM, plus rotational, vibrational, or other types of motion about the CM
- When we draw free body diagrams, they affect the CM

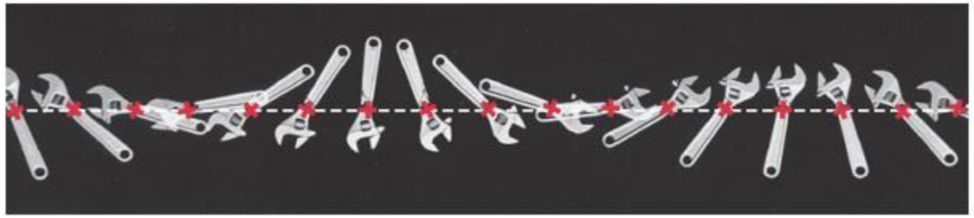
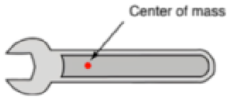


**FIGURE 7–21** Translation plus rotation: a wrench moving over a smooth horizontal surface. The CM, marked with a red cross, moves in a straight line because no net force acts on the wrench.




VS.

# 7–8 CENTER OF MASS (CM)

How to find the net point of masses?

1) Net point of masses in one dimension

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{m_A x_A + m_B x_B}{M},$$

where $M = m_A + m_B$ is the total mass of the system. The center of mass lies on the line joining $m_A$ and $m_B$. If the two masses are equal $(m_A = m_B = m)$, then $x_{CM}$ is midway between them, because in this case

$$x_{CM} = \frac{m(x_A + x_B)}{2m} = \frac{(x_A + x_B)}{2}.$$

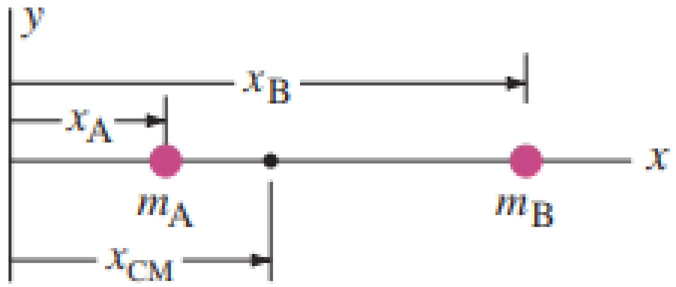
If one mass is greater than the other, then the CM is closer to the larger mass.

If there are more than two particles along a line, there will be additional terms:

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C + \cdots}{m_A + m_B + m_C + \cdots} = \frac{m_A x_A + m_B x_B + m_C x_C + \cdots}{M}, \quad \textbf{(7–9a)}$$

where $M$ is the total mass of all the particles.

**FIGURE 7–22** The center of mass of a two-particle system lies on the line joining the two masses. Here $m_A > m_B$, so the CM is closer to $m_A$ than to $m_B$.



**EXAMPLE 7–12** **CM of three guys on a raft.** On a lightweight (air-filled) "banana boat," three people of roughly equal mass $m$ sit along the $x$ axis at positions $x_A = 1.0\text{ m}$, $x_B = 5.0\text{ m}$, and $x_C = 6.0\text{ m}$, measured from the left-hand end as shown in Fig. 7–23. Find the position of the CM. Ignore the mass of the boat.

**FIGURE 7–23** Example 7–12.


**EXERCISE G** Calculate the CM of the three people in Example 7–12, taking the origin at the driver $(x_C = 0)$ on the right. Is the physical location of the CM the same?

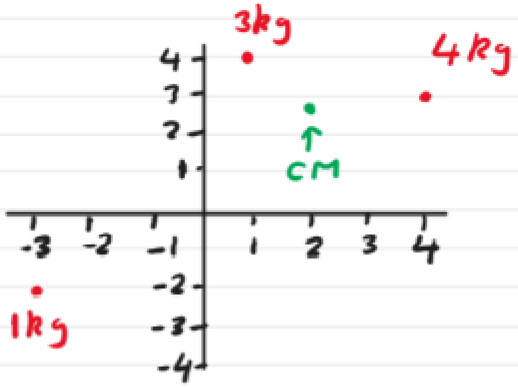
**G:** $x_{CM} = -2.0\text{ m}$; yes.

# 7–8 CENTER OF MASS (CM)

2) Net point of masses in two dimension

$$x_{CM} = \frac{1(-3) + 3(1) + 4(4)}{8} = \frac{16}{8} = 2\,m$$

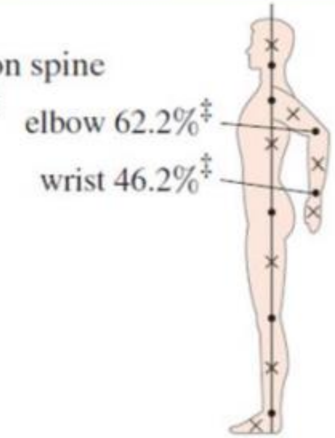
$$y_{CM} = \frac{1(-2) + 3(4) + 4(3)}{8} = \frac{22}{8} = 2.75\,m$$



# 7–9 CM for the Human Body

**TABLE 7–1 Center of Mass of Parts of Typical Human Body, given as %**
(full height and mass = 100 units)

| Distance of Hinge Points from Floor (%) | Hinge Points (•) (Joints) |
|---|---|
| 91.2% | Base of skull on spine |
| 81.2% | Shoulder joint |
| 52.1% | Hip joint |
| 28.5% | Knee joint |
| 4.0% | Ankle joint |




| | Center of Mass (×) (% Height Above Floor) | Percent Mass |
|---|---|---|
| Head | 93.5% | 6.9% |
| Trunk and neck | 71.1% | 46.1% |
| Upper arms | 71.7% | 6.6% |
| Lower arms | 55.3% | 4.2% |
| Hands | 43.1% | 1.7% |
| Upper legs (thighs) | 42.5% | 21.5% |
| Lower legs | 18.2% | 9.6% |
| Feet | 1.8% | 3.4% |
| Body CM = | 58.0% | 100.0% |

**EXAMPLE 7–13** **A leg's CM.** Determine the position of the CM of a whole leg (*a*) when stretched out, and (*b*) when bent at 90°. See Fig. 7–26. Assume the person is 1.70 m tall.

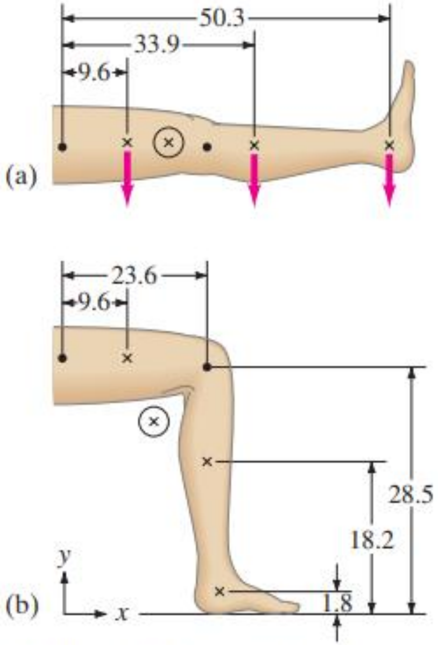



**FIGURE 7–26** Example 7–13: finding the CM of a leg in two different positions using percentages from Table 7–1. (⊗ represents the calculated CM.)

**FIGURE 7–27** A high jumper's CM may actually pass beneath the bar.

*46. (III) Billiard balls A and B, of equal mass, move at right angles and meet at the origin of an $xy$ coordinate system as shown in Fig. 7–36. Initially ball A is moving along the $y$ axis at $+2.0\,\text{m/s}$, and ball B is moving to the right along the $x$ axis with speed $+3.7\,\text{m/s}$. After the collision (assumed elastic), ball B is moving along the positive $y$ axis (Fig. 7–36) with velocity $v'_B$. What is the final direction of ball A, and what are the speeds of the two balls?

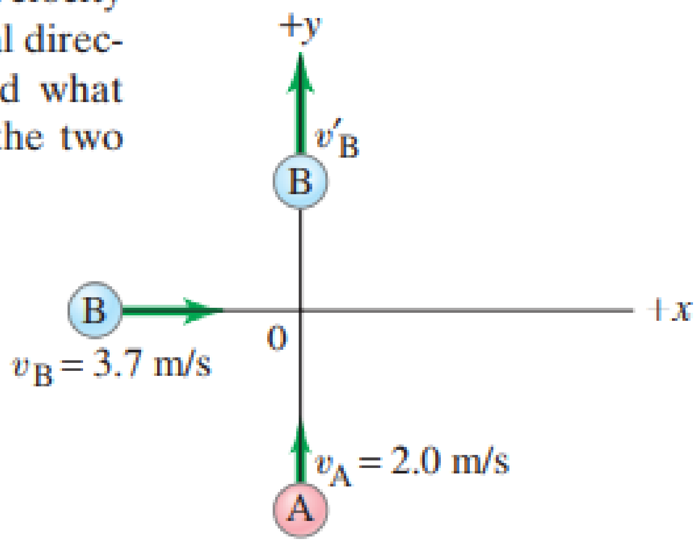



**FIGURE 7–36** Problem 46. (Ball A after the collision is not shown.)

46. Write momentum conservation in the $x$ and $y$ directions and KE conservation. Note that both masses are the same. We allow $\vec{v}_A$ to have both $x$ and $y$ components.

$$p_x: \; mv_B = mv'_{Ax} \;\rightarrow\; v_B = v'_{Ax}$$
$$p_y: \; mv_A = mv'_{Ay} + mv'_B \;\rightarrow\; v_A = v'_{Ay} + v'_B$$
$$\text{KE}: \; \tfrac{1}{2}mv_A^2 + \tfrac{1}{2}mv_B^2 = \tfrac{1}{2}mv_A'^2 + \tfrac{1}{2}mv_B'^2 \;\rightarrow\; v_A^2 + v_B^2 = v_A'^2 + v_B'^2$$

Substitute the results from the momentum equations into the KE equation.

$$(v'_{Ay} + v'_B)^2 + (v'_{Ax})^2 = v_A'^2 + v_B'^2 \;\rightarrow\; v_{Ay}'^2 + 2v_{Ay}'^2 v'_B + v_B'^2 + v_{Ay}'^2 = v_A'^2 + v_B'^2 \;\rightarrow$$
$$v_A'^2 + 2v_{Ay}'^2 v'_B + v_B'^2 = v_A'^2 + v_B'^2 \;\rightarrow\; 2v_{Ay}'^2 v'_B = 0 \;\rightarrow\; v'_{Ay} = 0 \text{ or } v'_B = 0$$

Since we are given that $v'_B \neq 0$, we must have $v'_{Ay} = 0$. This means that the final direction of A is the $\boxed{x \text{ direction}}$. Put this result into the momentum equations to find the final speeds.

$$v'_A = v'_{Ax} = v_B = \boxed{3.7\text{ m/s}} \qquad v'_B = v_A = \boxed{2.0\text{ m/s}}$$

**51.** (II) The CM of an empty 1250-kg car is 2.40 m behind the front of the car. How far from the front of the car will the CM be when two people sit in the front seat 2.80 m from the front of the car, and three people sit in the back seat 3.90 m from the front? Assume that each person has a mass of 65.0 kg.

**52.** (II) Three cubes, of side $\ell_0$, $2\ell_0$, and $3\ell_0$, are placed next to one another (in contact) with their centers along a straight line as shown in Fig. 7–38. What is the position, along this line, of the CM of this system? Assume the cubes are made of the same uniform material.

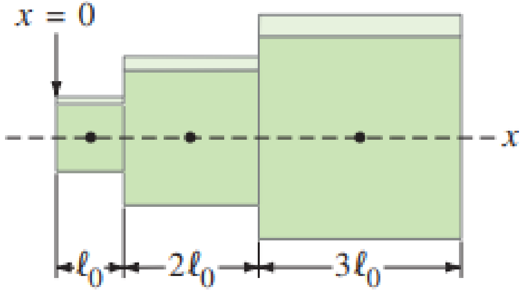



**FIGURE 7–38** Problem 52.

**53.** (II) A (lightweight) pallet has a load of ten identical cases of tomato paste (see Fig. 7–39), each of which is a cube of length $\ell$. Find the center of gravity in the horizontal plane, so that the crane operator can pick up the load without tipping it.

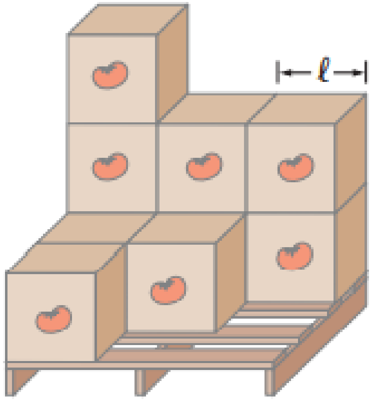



**FIGURE 7–39** Problem 53.

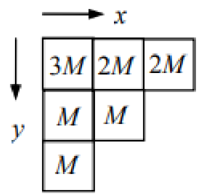
51. Find the CM relative to the front of the car. Use Eq. 7–9a.

$$x_{CM} = \frac{m_{car}x_{car} + m_{front}x_{front} + m_{back}x_{back}}{m_{car} + m_{front} + m_{back}}$$
$$= \frac{(1250\text{ kg})(2.40\text{ m}) + 2(65.0\text{ kg})(2.80\text{ m}) + 3(65.0\text{ kg})(3.90\text{ m})}{1250\text{ kg} + 5(65.0\text{ kg})} = \boxed{2.62\text{ m}}$$

52. By the symmetry of the problem, since the centers of the cubes are along a straight line, the vertical CM coordinate will be 0, and the depth CM coordinate will be 0. The only CM coordinate to calculate is the one along the straight line joining the centers. The mass of each cube will be the volume times the density, so $m_1 = \rho(\ell_0)^3$, $m_2 = \rho(2\ell_0)^3$, $m_3 = \rho(3\ell_0)^3$. Measuring from the left edge of the smallest block, the locations of the CMs of the individual cubes are $x_1 = \frac{1}{2}\ell_0$, $x_2 = 2\ell_0$, $x_3 = 4.5\ell_0$. Use Eq. 7–9a to calculate the CM of the system.

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{\rho\ell_0^3\left(\frac{1}{2}\ell_0\right) + 8\rho\ell_0^3(2\ell_0) + 27\rho\ell_0^3(4.5\ell_0)}{\rho\ell_0^3 + 8\rho\ell_0^3 + 27\rho\ell_0^3} = \frac{138}{36}\ell_0 = \frac{23}{6}\ell_0$$
$$= \boxed{3.8\ell_0 \text{ from the left edge of the smallest cube}}$$

53. Let each case have a mass $M$. A top view of the pallet is shown, with the total mass of each stack listed. Take the origin to be the back left corner of the pallet. Use Eqs. 7–9a and 7–9b.




$$x_{CM} = \frac{(5M)(\ell/2) + (3M)(3\ell/2) + (2M)(5\ell/2)}{10M} = \boxed{1.2\ell}$$
$$y_{CM} = \frac{(7M)(\ell/2) + (2M)(3\ell/2) + (1M)(5\ell/2)}{10M} = \boxed{0.9\ell}$$