Chapter (7)

Confidence intervals

Sheet (1)

Q1 The lifetime (age) in hours of a random sample of one of the batteries produced in Jordan gave the following summary:

Sample size	Sample average	Sample standard deviation
N=9	$\bar{X} = 95$	**S=3**

A 99% confidence interval for the population variance σ^2 is:

A) (1.09 , 17.86) B) (1.09 , 6.61) C) (3.28 , 53.57)
D) (1.89 , 6.61) E) (1.81 , 7.32)

$1 - \alpha = 0.99 \rightarrow \alpha = 0.01 \rightarrow \frac{\alpha}{2} = 0.005$, $1 - 0.05 = 0.995$

$X^2_{0.005}(8) = 21.955$ & $X^2_{0.995}(8) = 1.344$

$\left(\frac{(n-1)S^2}{X^2_{\alpha/2}}, \frac{(n-1)S^2}{X^2_{1-\alpha/2}}\right) = \left(\frac{8*9}{21.955}, \frac{8*9}{1.344}\right) = (3.28 , 53.57) \rightarrow C$

Q2 If the 95% confidence interval for the population mean μ is (54.3 , 57.7) , then the point estimate of μ *is* :

A) 54 B) 55 C) 56 D) 95 E) 1.7

The point estimation for μ is $\bar{X} = \frac{L+U}{2} = \frac{54.3 + 57.7}{2} = 56 \rightarrow C$

Q3 It is known that the standard deviation of weights of orange is 5 kgs. The smallest sample size that we can choose to estimate the population mean by 95% C.I with the interval length of 0.8 is :

A) 601 B) 600 C) 25 D) 13 E) None

$L = 2*E \rightarrow E = \frac{0.8}{2} = 0.4$

$Z^*_{0.95} = 1.96$

$n = \left(Z_{\frac{\alpha}{2}} * \frac{\sigma}{E}\right)^2 = \left(\frac{1.96*5}{0.4}\right)^2 = 600.25 = 601 \rightarrow A$

Q4 The lifetime (age) in hours of a random sample of one of the batteries produced in Jordan gave the following: n = 9 , $\bar{X}$ = 90 & S = 3. A 98% C.I for the mean is :

A) (87.18 , 92.82) **B) (87.1 , 92.9)** **C) (6.1 , 11.9)**

σ unknown & n < 30 → use t-dis tables

$1 - \alpha = 0.98 \rightarrow \alpha = 0.02 \rightarrow \frac{\alpha}{2} = 0.01 \rightarrow t_{0.01}(8) = 2.9$

$(\bar{X} - t_{\frac{\alpha}{2}} * \frac{S}{\sqrt{n}} , \bar{X} + t_{\frac{\alpha}{2}} * \frac{S}{\sqrt{n}}) = (90 - 2.9 * \frac{3}{\sqrt{9}} , 90 + 2.9 * \frac{3}{\sqrt{9}})$

= (87.1, 92.9) → B

Q5 If the 90% C.I for the mean is (36, 44), one of the following could be 97% C.I. computed from the same data:

A) (36 , 41) **B) (39 , 41)** **C) (38 , 45)** **D) (35 , 45)** **E) (39 , 43)**

As the confidence level increase, the error increase, and the C.I become wider → D

Q6 The lower limit of a confidence interval at the 95% level of confidence for the population proportion if a sample of size 100 had 40 successes is :

A) 0.3898 **B) 0.2102** **C) 0.304** **D) 0.2959** **E) 0.4001**

$\hat{P} = \frac{X}{n} = \frac{40}{100} = 0.40$ & $Z^*_{0.95} = 1.96$

$L = \hat{P} - Z_{\frac{\alpha}{2}} * \sqrt{\frac{\hat{P}(1-\hat{P})}{n}} = 0.40 - 1.96 * \sqrt{\frac{0.40*0.60}{100}} = 0.304 \rightarrow C$

Q7 Suppose that a population is normally distributed with mean 30 . A random sample of size 13 is chosen and showed a standard deviation of 5.862, a 99% C.I for the standard deviation is:

$1 - \alpha = 0.99 \rightarrow \alpha = 0.01 \rightarrow \frac{\alpha}{2} = 0.005$, $1 - 0.005 = 0.995$ "df = 12"

$X^2_{0.005}(12) = 28.3$, $X^2_{0.995}(12) = 3.07$

$$\left(\sqrt{\frac{(n-1)S^2}{X^2_{\alpha/2}}}, \sqrt{\frac{(n-1)S^2}{X^2_{1-\alpha/2}}}\right) = \left(\sqrt{\frac{12*5.862^2}{28.3}}, \sqrt{\frac{12*5.862^2}{3.07}}\right) = (3.82 , 11.59)$$

Q8 Suppose the height of university trees (in meters) is normally distributed with mean μ and standard deviation σ. if the mean height of a random sample of 16 trees is (9.16) , it is known that the population std is (2.5) meters and we want to construct a 98% C.I for the mean heights , the standard error equal to

A) 8.26 B) 7.83 C) 6.33 D) 13.67 E) 0.625

$SE = \frac{\sigma}{\sqrt{n}} = \frac{2.5}{\sqrt{16}} = 0.625 \rightarrow E$

Q9 A random sample with size 6 and variance 2 is drawn from a normally distributed population with variance σ^2. Then a 90% C.I. for σ^2 is…….

A) (0.903, 8.734) B) (1.355, 12.1) C) (1.81, 17.47) D)(2.26, 21)

$1 - \alpha = 0.90 \rightarrow \alpha = 0.1 \rightarrow \frac{\alpha}{2} = 0.05$ and $1 - \frac{\alpha}{2} = 0.95$ with d.f = 5

$X^2_{0.05}(5) = 11.07$ & $X^2_{0.95}(5) = 1.145$

$$\left(\frac{(n-1)S^2}{X^2_{\alpha/2}}, \frac{(n-1)S^2}{X^2_{1-\alpha/2}}\right) = \left(\frac{5*2}{11.07}, \frac{5*2}{1.145}\right) = (0.903 , 8.734) \rightarrow A$$

Q10 From a normally distributed population, a random sample of size 16 is chosen and showed mean 51 and variance 25. A 90% C.I. for the population mean is:

A) 51 ± 2.63 **B) 51 ± 3.07** **C) 51 ± 1.27**

D) 51 ± 3.51 **E) 51 ± 2.19**

$n = 16 \quad , \quad \overline{X} = 51 \quad , \quad S^2 = 25$

$90\%\ \text{C.I for}\ \mu \rightarrow \overline{X} \pm t_{a/2} \frac{S}{\sqrt{n}}$

$1 - \alpha = 0.90 \rightarrow \alpha = 0.10 \rightarrow \frac{\alpha}{2} = 0.05 \rightarrow t_{0.05}^{(15)} = 1.753$

$51 \pm 1.753 \frac{(5)}{\sqrt{16}} \rightarrow 51 \pm 2.19 \rightarrow E$

Q11 Using t-tables, Report the t-table for the 80% confidence interval with d.f. = 10

A) 2.797 **B) 2.060** **C) 1.316** **D) 1.372** **E) None**

$1 - \alpha = 0.80 \rightarrow \alpha = 0.2 \rightarrow \frac{\alpha}{2} = 0.1 \rightarrow t_{10}^{(10)} = 1.372 \rightarrow D$

Q12 In a random sample of 200 males, 180 are smokers. A 80% C.I for P the proportion of smokers in the sampled population equals to:

A) (0.53 ,0.67) **B) (0.511 ,0.689)** **C) (0.873 , 0.927)** **D) (0.556 ,0.644)**

$\widehat{P} = \frac{x}{n} = \frac{180}{200} = 0.9\ \text{and}\ Z_{80\%} = 1.282$

$\widehat{P} \mp Z_{\alpha/2} \cdot \sqrt{\frac{\widehat{P}(1 - \widehat{P})}{n}} = 0.9 \pm 1.282 \sqrt{\frac{0.9(0.1)}{200}} = (0.873\ , 0.927) \rightarrow C$

Q14 If it is known that the population variance is 25 and we want to estimate the population mean by a 95% C.I with margin of error 0.3 , then the smallest suitable sample size is ….:

A) 32.66 B) 32 C) 1067.11 D) 1068 E) 1067

$E = 0.3 \quad , \quad \sigma^2 = 25 \quad , \quad 95\% = \text{C.I for } \mu \rightarrow Z_{\alpha/2} = 1.96$

$$n = \left(Z_{\alpha/2} \cdot \frac{\sigma}{E}\right)^2 = \left(\frac{1.96\,(5)}{0.3}\right)^2 = 1067.11 = 1068 \rightarrow D$$

Q15 If (0.36, 0.86) is a confidence interval for the population mean, Then the point estimator in this confidence interval is:

A) 0.35 B) 0.61 C) 0.3 D) 0.5 E) 0.15

$$\overline{X} = \frac{\mathcal{L} + \mu}{2} = \frac{0.36 + 0.86}{2} = 0.61 \rightarrow B$$

Sheet (2)

Q1 the following table present the weights of new born babies before and after taking a certain vitamin, $\bar{d}$ = -1 and s.d = 1 . construct a 90% C.I for μ_d.

Baby no.	Before	After
1	4	6
2	5	5
3	3	4

$1 - \alpha = 0.90 \rightarrow \alpha = 0.1 \rightarrow \frac{\alpha}{2} = 0.05$ with df = 2 $\rightarrow$ $t_{0.05}(2) = 2.92$

$(\bar{d} - t_{\alpha/2} * \frac{s.d}{\sqrt{n}} , \bar{d} + t_{\alpha/2} * \frac{s.d}{\sqrt{n}}) = (-1 - 2.92*\frac{1}{\sqrt{3}} , -1 + 2.92*\frac{1}{\sqrt{3}}) = (-2.69 , 0.69)$

Q2 sample (1) of size n_1 = 8 has mean $\bar{X}_1$ = 40 and variance S_1^2 = 20 is chosen from population (1) with mean μ_1.

Sample (2) of size n_2 = 14 has mean $\bar{X}_2$ = 46 and variance S_2^2 = 25 is chosen from population (2) with mean μ_2.

Assuming that the two populations are normally distributed with equal population variances and the two samples are independent. a 90% C.I for the difference between the two means $\mu_1 - \mu_2$ is:

A) -6 ∓ 3.59 B) -6 ∓ 3.39 C) -6 ∓ 3.29 D) -6 ∓ 3.69

σ_1 & σ_2 are unknown , n < 30 & m < 30 $\rightarrow$ use t-dis tables

$S.P = \sqrt{\frac{S_1^2(n-1)+S_2^2(m-1)}{n+m-2}} = \sqrt{\frac{7*20+13*25}{8+14-2}} = 4.82$

$1 - \alpha = 0.90 \rightarrow \alpha = 0.1 \rightarrow \frac{\alpha}{2} = 0.05$ with df = 8+14-2 = 20

$t_{0.05}(20) = 1.725$

$(\bar{X} - \bar{Y}) \mp t_{\alpha/2} * S.P * \sqrt{\frac{1}{n} + \frac{1}{m}} = (40 - 46) \mp 1.725*4.82*\sqrt{\frac{1}{8} + \frac{1}{12}}$

$= -6 \mp 3.69 \rightarrow$ D

Q3 The given table represents information about the grades of two groups of students. Assuming that the two populations are normally distributed with equal variances, use S^2p=25 to calculate a 80% confidence interval (CI) for the difference between the two population means $\mu_m - \mu_f$:

	Males (M)	Females (F)
Sample size (n)	**12**	**10**
Sample mean ($\bar{x}$)	**50**	**60**

A) (-16.09, -3.91) B) (-14.466, -5.534) C) (-13.692, -6.307) D) (-12.837, -7.163)

$1 - \alpha = 0.80 \rightarrow \alpha = 0.2 \rightarrow \frac{\alpha}{2} = 0.1$ with d.f = 12+10-2 = 20

$t_{0.1}(20) = 1.325$

$(\bar{X} - \bar{Y}) \mp t_{\alpha/2} * S.P * \sqrt{\frac{1}{n} + \frac{1}{m}} = (50-60) \mp 1.325 * \sqrt{25} * \sqrt{\frac{1}{12} + \frac{1}{10}} =$ (-12.837 , -7.163) → D

Q4 To compare the mean finishing times of male and female participants in a 10K race, you randomly select several finishing times from both sexes. The results are shown at the table below. Construct an 80% confidence interval for the difference in mean finishing times of male and female participants in the race.

Males	Females
$\bar{X}_1$ = 0.73 h **S_1 = 0.17 h** **n_1=20**	**$\bar{X}_2$ = 0.75 h** **S_2 = 0.04 h** **n_2=12**

σ_1 & σ_2 are unknown & $n_1, n_2 < 30 \rightarrow t$

$$S.P = \sqrt{\frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2}} = \sqrt{\frac{(0.17)^{2(19)}+(0.04)^2(11)}{20+12-2}} = 0.1374$$

$$df = 20 + 12 - 2 = 30 \rightarrow t_{80\%}^{(30)} = 1.310$$

$$\overline{x_1} - \overline{x_2} \pm t_{\alpha/2} SP \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 0.73 - 0.75 \pm 1.310(0.137)\sqrt{\frac{1}{20} + \frac{1}{12}}$$
$$= (-0.0767, 0.0367)$$

Q5 A sample of size 3 has a standard deviation of 9, and another sample of size 4 has a standard deviation of 5. Assuming the two populations are distributed normal, then the pooled standard deviation is closest to

A) 6.173 B) 6.885 C) 47.4 D) 5.944 E) None

$$S.P = \sqrt{\frac{S_1^2(n-1)+S_2^2(m-1)}{n+m-2}} = \sqrt{\frac{(3-1)*9^2+(4-1)5^2}{3+4-2}} = 6.885 \rightarrow$$

Q6 Suppose the cholesterol levels of 6 patients before taking a medicine (the first number) and after taking a medicine (the second number) are (225,194), (222,205), (202,203), (222,207), (224,205), (217,205), then the 99% confidence interval for the difference between the two cholesterol levels is:

A) (-34.2018 , 65.20) B) (-1.60, 32.60)
C) (-57.01 , 88.01) D) (2.04 , 28.96)

$Before$	225	222	202	222	224	217
$After$	194	205	203	207	205	205
d_i	31	17	-1	15	19	12
d_i^2	961	281	1	225	361	144

$$\sum d_i = 93, \sum d_i^2 = 98$$

$$\bar{d} = \frac{\sum d_i}{n} = \frac{93}{6} = 15.5$$

$$S^2 = \frac{\sum d_i^2}{n-1} - \frac{(\sum d_i)^2}{n(n-1)} = \frac{1981}{5} - \frac{(93)^2}{6(5)} = 107.9 \rightarrow S = \sqrt{var} = \sqrt{107.9} = 10.39$$

$$t_{99\%}^{(5)} = 4.032$$

$$\bar{d} \pm t_{\alpha/2}\frac{S}{\sqrt{n}} = 15.5 \pm 4.032\frac{(10.39)}{\sqrt{6}} = (-1.6, 32.6) \rightarrow B$$

Q7 To compare the mean number of days spent waiting to see a family doctor for two large cities, you randomly select several people in each city who have had an appointment with a family doctor. The results are shown at the table below. Find the error of estimation that could be used to construct a 90% confidence interval for the difference in mean number of days spent waiting to see a family doctor for the two cities.

Miami	**Seattle**
$\bar{X}_1 = 28$ **days** $S_1 = 39.7$ **days** $n_1=20$	$\bar{X}_2 = 26$ **days** $S_2 = 42.4$ **days** $n_2=17$

σ_1 and σ_2 are unknown & $n_1, n_2 < 30 \to t$

$$S.P = \sqrt{\frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2}} = \sqrt{\frac{(39.7)^2(19)+(42.4)^2(16)}{17+20-2}} = 40.956$$

$$df = 20+17-2 = 35 \to t_{90\%}^{(35)} = 1.69$$

$$\text{Error}=t_{\alpha/2}SP\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 1.69(40.956)\sqrt{\frac{1}{20}+\frac{1}{17}} = 22.83$$

Q8 Consider the following information about two independent samples where $\widehat{P}_1$ is the sample proportion for the first sample and $\widehat{P}_2$ is the sample proportion for the second sample. Find the error of estimation and then construct a 90% confidence interval for the difference between two population.

Sample statistics: $x_1 = 471$, $n_1 = 785$ and $x_2 = 372$, $n_2 = 465$

Sample (1)	Sample (2)
$n_1 = 785$	$n_2 = 465$
$x_1 = 471$	$x_2 = 372$

$$\widehat{p_1} = \frac{x_1}{n_1} = \frac{471}{785} = 0.6 \text{ and } \widehat{p_2} = \frac{x_2}{n_2} = \frac{372}{465} = 0.8$$

$$E = Z_{\alpha/2}\sqrt{\frac{\widehat{p_1}(1-\widehat{p_1})}{n_1}+\frac{\widehat{p_2}(1-\widehat{p_2})}{n_2}} = 1.645\sqrt{\frac{0.6(0.4)}{785}+\frac{0.8(0.2)}{465}} = 0.0149$$

For C.I $\to \widehat{p_1} - \widehat{p_2} + E = 0.6 - 0.8 \pm 0.0149 = (-0.02149, -0.1851)$