To Summer up:

- Random variable:

1) Discrete when it has a finite or countable number of possible outcomes that can be listed. قيم معدودة

2) Continuous when it has an uncountable number of possible outcomes, represented by an interval on a number line.قيم غير معدودة

⚠ **Note:** In any probability distribution we have $\sum \mathbf{P}(\mathbf{x_i}) = \mathbf{1}$

- **Probability density function (P.D.F):**

A function $f(x) = P(X=x_i)$ is called a p.d.f if:

1. $f(x) \geq 0$, for all x → يعني اي احتمالية لازم تكون أكبر من صفر
2. $\sum f(x_i) = 1$ → 1 = مجموع الاحتمالات كلها

اول خطوة بالحل هي تحويل الاقتران الى جدول يحتوي قيم اكس و احتمالاتهم.

⚠ **Note:** The CDF of random variable X is defined as: $F(a) = P(X \leq a)$

- **The expected value:**

The weighted mean of all possible values of the random variable X, denoted by E(X) or μ.

$$\mu = E(X) = \sum X * P(X = x_i)$$

بضرب كل اكس باحتمالاها و بعدها بجمّعهم.
عادي يكون سالب او موجب او صفر.

- The variance:

Measures the distance from the mean.

$Var(x) = \sigma^2 = E(X - \mu)^2$ OR $Var(x) = E(X^2) - (E(X))^2$

Standard deviation= $\sigma = \sqrt{variance}$

ممنوع يكون سالب فقط موجب او صفر.

⚠ **Note**: $E(X^2) = Var(X) + (E(X))^2$ "Extremely important"

Properties of the expected value:	Properties of the variance:
$E(a) = a$	$Var(a) = zero$
$E(aX) = a\,E(X)$	$Var(aX) = a^2\,Var(X)$
$E(X \pm Y) = E(X) \pm E(Y)$	$Var(aX + b) = a^2\,Var(X)$
$E(g(x)) = \sum g(x) * P(X = x_i)$	$VAR(X \pm Y) = VAR(X) \pm Var(Y)$ Only if X and Y are independent.

- **Special discrete distributions:**

Binomial distribution $X \sim Bin(n, p)$	Poisson distribution $X \sim poi(\mu)$
Conditions: 1. **Two outcomes.** 2. **Independent trials** 3. **Prob of success constant each time.**	**Conditions:** 1. **Occurrences independent** 2. **Occurrences proportional to the length of time.** 3. **Prob. Of an event is the same for each period.**
n: number of trials p: prob. of success	**μ : Average of occurrences**
$\mathbf{p.d.f}: P(X = x_i) = \binom{n}{x_i} p^{x_i} q^{n-x_i}$	$\mathbf{p.d.f}: P(X = xi) = \frac{e^{-\mu} * \mu^x}{x!}$
$\mathbf{P(X \le K)}$ use tables	$\mathbf{P(X \le K)}$ use tables
$\mathbf{E(X)} = \mu = n * p$	$\mathbf{E(X)} = \mu$
$\mathbf{Var(X)} = \sigma^2 = n * p * q$	$\mathbf{Var(X)} = \sigma^2 = \mu$
$\mathbf{Std(X)} = \sigma = \sqrt{variance}$	$\mathbf{Std(X)} = \sigma = \sqrt{\mu}$

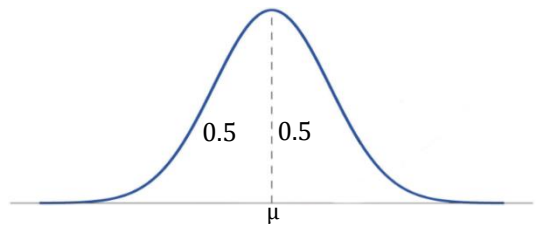
- **Binomial → poisson :**

  If $n \ge 30$ and $P < 0.1$ we approximate Binomial to Poisson by:

  $X \sim Bin(n, p) \rightarrow X \sim Poi(n \times P)$

Normal Distribution:

$X \sim N(\mu, \sigma^2)$

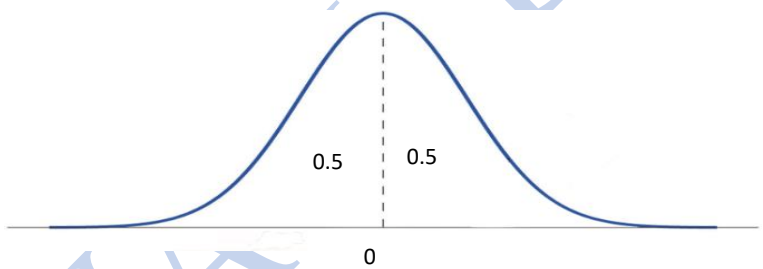


⚠ **A normal distribution has the following properties:**

1) Symmetrical ≡ bell-shaped
2) Mean = Median = Mode and are all located at the canter
3) Probability = area
4) Total area under the curve = 1

- **The standard normal:**

- It is a normal distribution with a mean of zero and variance of 1.
- $X \sim N(\mu, \sigma^2) \rightarrow Z \sim N(0, 1)$



1. $P(Z \leq K) \rightarrow$ *we use tables*
2. $P(Z > K) = 1 - P(Z \leq K)$
3. $P(Z > k) = P(Z < -K)$ "لأنه متماثل فبقدر اعكس اشارة الداخل و اقلب المتباينة"
4. $P(Z = k) =$ zero
5. $P(a < Z < b) = P(Z < b) - P(Z < a)$

✅ **Note:** $P(Z < a) =$ area

if area > 0.5 → a = +

if area < 0.5 → a = -

if area = 0.5 → a = 0

Note:

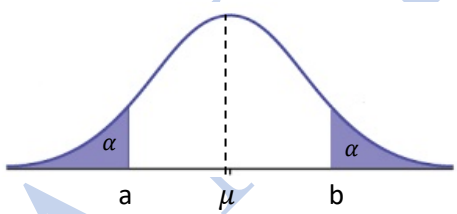
I. Any number more than 3.5, it's probability = 1
II. Any number less than -3.5, it's probability = zero

_ Transformation formulas:

$X \sim N(M, \sigma^2)$ we should transform X to Z	$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ we should transform X to Z , but how?
$Z = \frac{X - \mu}{\sigma} \rightarrow Z \sim N(0,1)$	$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \rightarrow Z \sim N(0,1)$

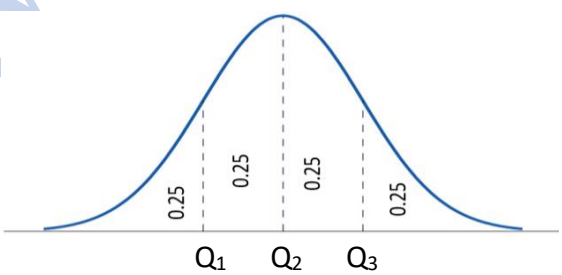
Very important notes:

[1]



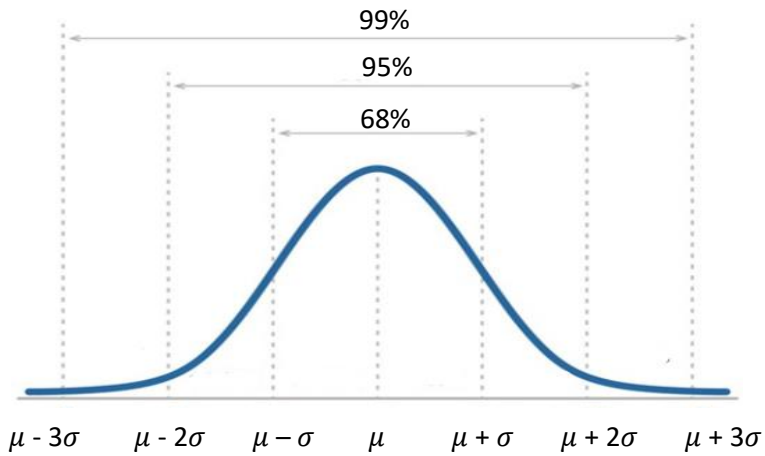
$$\therefore P(X < a) = P(X > b) \rightarrow \mu = \frac{a + b}{2}$$

[2]



- $P(X \leq Q1) = 0.25$ "المساحة المحصورة تحت الربع الاول"
- $P(X \leq Q2) = 0.5$ "المساحة المحصورة تحت الربع الثاني"
- $P(X \leq Q3) = 0.75$ "المساحة المحصورة تحت الربع الثالث"
- $P(X \leq Pk) = k\%$ "المساحة المحصورة تحت نسبة معينة"

[3]



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- The probability that X lies within k standard deviation:

  $\therefore P(\mu - K.\sigma < X < \mu + K.\ \sigma)$

- **The distribution of sample proportion:**

$\widehat{P} \sim N\left(P, \frac{P(1-p)}{n}\right)$, so the mean is p and the variance is $\frac{P(1-p)}{n}$

$\therefore\ \widehat{P}$ is distributed as Normal with $\mu = P$ & $\sigma^2 = \frac{P(1-p)}{n}$

- How to convert $\widehat{P}$ to Z?

$$Z = \frac{\widehat{P} - P}{\sqrt{\frac{P(1-P)}{n}}} \sim N(0,1)$$