Section 8.4: Torque $\vec{\tau}$

- So far, we have seen that a force $\vec{F}$ can cause changes in linear "translational" motion, as described by Newton's 2^{nd} law. However, we have not addressed an analogous question: What is the cause of changes in rotational motion?
- In dealing with a rotating object, PHY 105 is greatly simplified by assuming the object is "rigid - جاسِئ". A rigid object is one that is <u>nondeformable</u>: the relative positions of all particles composing it remain fixed "constant".
- Suppose you are using a wrench to loosen a bolt (or as a dentist using an "elevator" to loosen a tooth!):

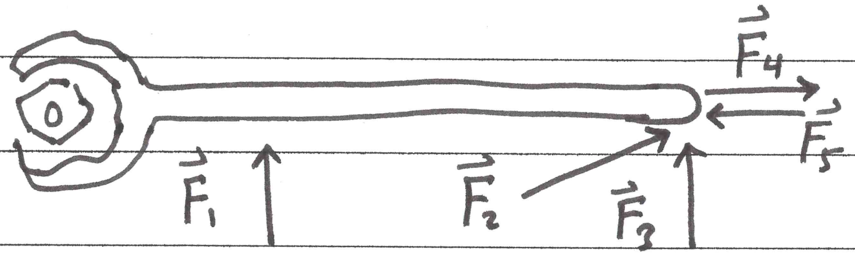


Fig-1

Your hand is acting by different forces at different angles: $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, $\vec{F}_4$ and $\vec{F}_5$.

It is clear, "presumably", that it will be easiest to turn the bolt in case $\vec{F}_3$, quite a bit harder in case $\vec{F}_1$, and impossible with cases $\vec{F}_4$ and $\vec{F}_5$.

- This example shows that the magnitude of the force is not the only relevant quantity in extracting a tooth!
- Let's consider the wrench-bolt system once again with some geometry added in Fig-2:

- $\vec{r}$: is defined as the position vector from the point O at the axis of rotation to the point of application of $\vec{F}$ i.e. the point where $\vec{F}$ acts.

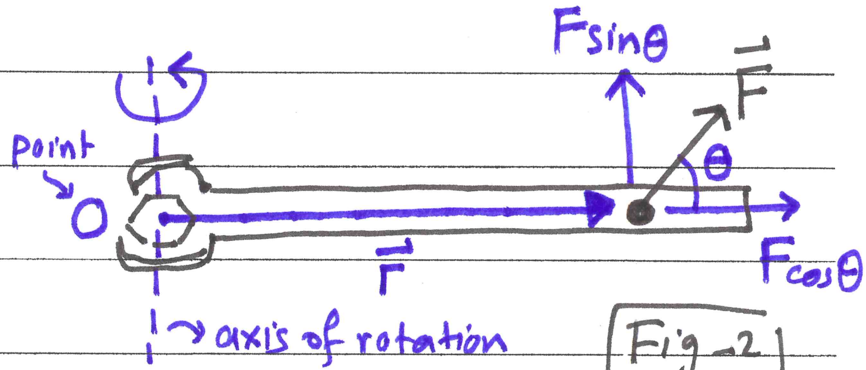


Fig-2

- θ : is defined as the angle between $\vec{F}$ and $\vec{r}$: the angle at which the force is applied relative to $\vec{r}$.

- Fig-2 shows that the only component of $\vec{F}$ that tends to cause rotation of the wrench around the axis through O is $F\sin\theta$; the component perpendicular to $\vec{r}$.

- The horizontal component $F\cos\theta$ has no tendency to produce rotation about the axis passing through O.

$\Rightarrow$ These considerations are quantified by the concept of torque, $\vec{\tau}$. Torque is the vector product of the force vector and the position vector:

$$|\vec{\tau}| = |\vec{F}| * \left[|\vec{F}| * \sin\theta\right] \quad \text{——} (1)$$

- Consider the wrench-bolt one more time!

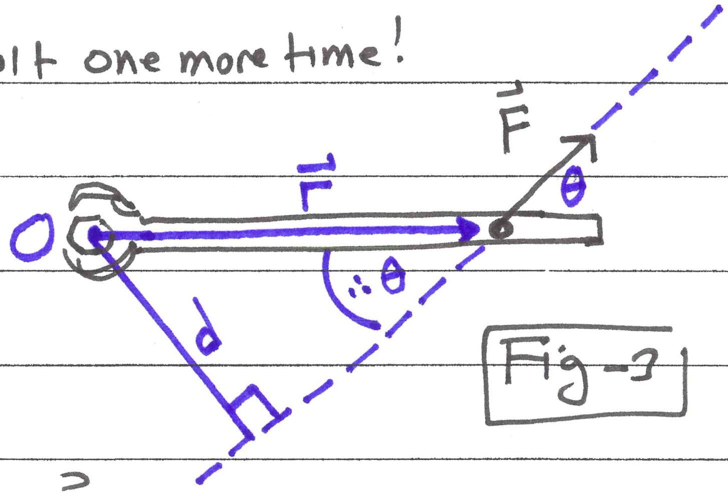


Fig-3

- d $\Rightarrow$ The pictorial "imaginary" dashed line extending out both ends (tail and head) of the force is called the line of action of $\vec{F}$.
- d: is defined as the perpendicular "distance" between the rotation axis and the line of action of $\vec{F}$.

From Fig-3, one can see that $d = |\vec{F}|\sin\theta$.

This "quantity" d is called the lever arm or the moment arm of $\vec{F}$.

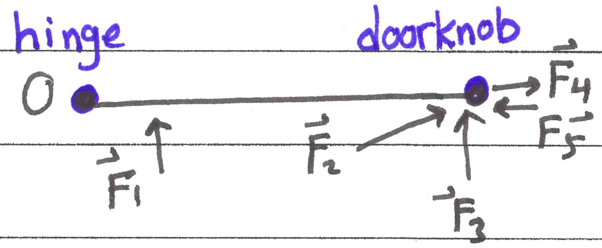
- Using the value d, eq (1) reads:

$$|\vec{\tau}| = |\vec{r}| * \left[|\vec{F}| * \sin\theta \right] = \left[|\vec{r}| * \sin\theta \right] * |\vec{F}| = d * |\vec{F}| \quad (2)$$

You can use any form of $|\vec{\tau}|$: eq (1) or eq (2), whichever is easiest !!

⇒ Back to Fig-1, and lets sum up what we have learned:-

From the definition of τ in eq (1) or (2), we have realized that the lever (moment) arm is <u>also</u> important. In addition, the angle θ at which the force is applied, relative to the lever arm, <u>matters</u> as well.



The torque increases as $|\vec{F}|$ increases and as d increases.

- This explains why it is easier to rotate a door if we push at the doorknob rather than at a point close to a hinge: $\tau_3 > \tau_1$
- This also explains why we apply our push as closely

perpendicular to the door as we can so that θ is close to 90°, which maximizes the lever arm $\therefore$ $\tau_3 > \tau_2$.

- Pushing sideways on the doorknob ($\theta = 0^\circ$) ($\theta = 180^\circ$) will not cause the door to rotate: $\tau_4 = \tau_5 = 0$.
- We said that torque is the vector product of $\vec{F}$ and $\vec{r}$. The torque points in a direction perpendicular to the plane spanned by the force and position vectors. In other words, $\vec{\tau}$ is perpendicular to both $\vec{F}$ and $\vec{r}$.
- Torque around any axis of rotation can be clockwise or counterclockwise. We use the convention that the sign of the torque is +ve for counterclockwise (CCW) and −ve for clockwise (CW).

$\therefore$ An angle θ of 270° would be just as effective as an angle θ of 90°, but then the force would act in the opposite direction.

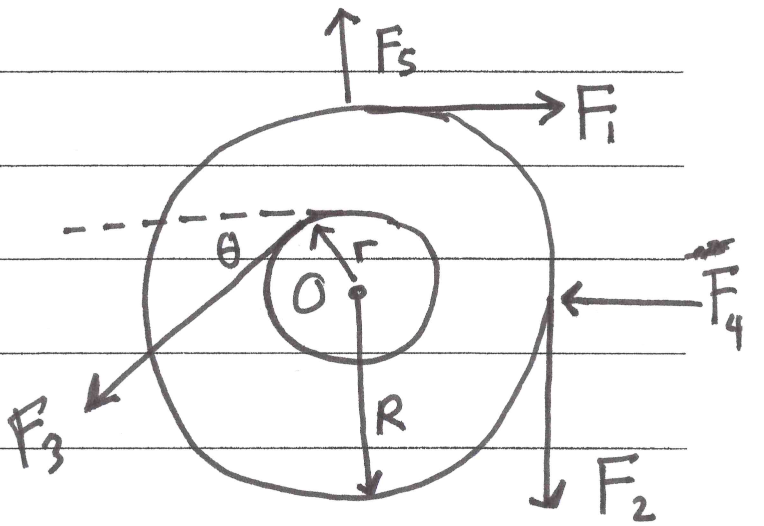
$\therefore$ The net torque about an axis of rotation through O is:

$$\vec{\tau}_{net} = \underbrace{\sum_{CCW} \vec{\tau}}_{+ve} \underbrace{\oplus}_{\text{vector sum}} \underbrace{\sum_{CW} \vec{\tau}}_{-ve} \quad \text{——— (3)}$$

- The SI unit of torque is (N.m), not to be confused with the unit of energy, which is the joule (J = Nm). Thus, torque should be reported in these units. Do not confuse torque $\vec{\tau}$ and work W, which have the same units but are very different concepts.

⇒ As a dentist, you will have come to know that the torque is the decisive factor in determining how easy or hard it is to loosen (or tighten) a tooth!!

- Example: The wheel has inner rim of radius r and outer rim of radius R. The forces shown are applied to both rims. Find the net torque on the wheel about the axel through O:



Recall of (3)

$\vec{\tau}_{net} = \sum_{CCW} \vec{\tau} + \sum_{CW} \vec{\tau}$

$\vec{\tau}_1 = -RF_1$ N.m

$\vec{\tau}_2 = -RF_2$ N.m

$\vec{\tau}_3 = +rF_3$ N.m

$\vec{\tau}_4 = 0 \Rightarrow \theta = 180^\circ$

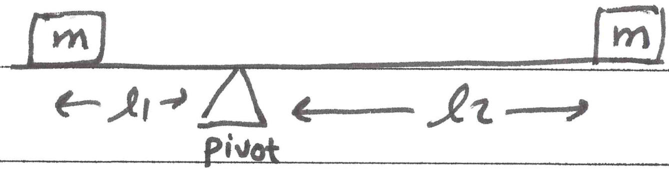
$\vec{\tau}_5 = 0 \Rightarrow \theta = 0^\circ$

- Do problem 8.25
- Do problem 8.29

Problem 8.27 (how to balance a seesaw) and chapter 9:

We conclude this lecture with this problem as a motivation for the theme of Chapter 9; the static equilibrium. Check example 9.4 !

The massless board serves as a seesaw for the two blocks. Calculate the net torque about the pivot.



$$\vec{\tau}_{net} = \vec{\tau}_{m_{left}} + \vec{\tau}_{m_{right}} = +mg\ell_1 + -mg\ell_2$$

$$= mg(\ell_2 - \ell_1) \ \underline{\underline{CW}}.$$

⇒ To balance the seesaw, the two lever arms, ℓ_1 and ℓ_2, must be equal → then $\vec{\tau}_{net} = 0$.