Dr. Mohammad Hussein PHY 105 - Fall 2022 Lecture: 11* Date: 27/11/2022

1. In Section 8.4, we have assumed objects remain **rigid** when external forces act on them. In reality, all objects are somewhat **elastic** (deformable) to some extent, even though they do not appear to be. That is, it is possible to change the shape or the size (or both) of an object by applying external forces.

2. If a rigid object is deformed a small amount, it will return to its original size and shape when the deforming force is removed. If a rigid object is deformed past a point called its **elastic limit**, it will not return to its original size and shape but will remain permanently deformed. If a rigid object is deformed too far beyond its elastic limit, it will **break**, or **fracture**, as we will see in **item 9** (Section 9.6).

3. Deformations of solids are usually classified into three types:
stretching (or pulling) - (Figure 9.20), compression (or pushing) - (Figure 9.21), or shearing (or twisting) - (Figure 9.23). A comparison between these three patterns is displayed in Figure 9.22. What these three deformations have in common is that a **stress**, or deforming force per unit area, produces a **strain**, or unit deformation.

4. **<u>Recall the causality of Newton's 2nd law: stress is applied to the object by external agents, whereas strain is the object's response to the stress</u>**; strain is a measure of how much the object has been deformed. Stretching, or tension, is associated with tensile stress. Compression can be produced by hydrostatic stress. Shear is produced by shearing stress (by scissors).

5. Although stress and strain take different forms for the three types of deformation, they are related **linearly** through a constant called the **modulus of elasticity**

stress = [modulus of elasticity] x [strain]

This **empirical** relationship applies as long as the **elastic limit** of the material is not exceeded. In this lecture we consider these three types of deformation and define an elastic modulus for each:
1. **Young's modulus** measures the resistance of a solid to a change in its length.
2. **Bulk modulus** measures the resistance of solids or fluids (liquids and gases - chapter 10) to changes in their volume.
3. **Shear modulus** measures the resistance to motion of the planes within a solid parallel to each other.

Insight: it's possible to establish a sort of analogy between the general empirical equation given above and Newton's 2^{nd} law. The modulus, being a measure of the **resistance** to deformation, is analogous to the mass (**inertia**) in Newton's 2^{nd} law. By the same token, stress and strain are analogous to force and acceleration respectively. That being said, we will learn in item 6 that strain is a pure number (dimensionless), while the modulus has the dimension of force per area.

6. Changing the length of a solid

In the case of tension, a force F is applied to opposite ends of a solid rod of length L and the rod stretches to a new length, $L + \Delta L$, as shown in the **Figure 1** below. **<u>The stress for stretching is defined as the force, *F*, per unit area, *A*, applied to the end of an object</u>** [you will come to know in chapter 10 that the pressure is force per unit area]. **<u>The strain is defined as the fractional change in length of the object, $\Delta L/L$.</u>** Thus, stain is a pure number (dimensionless). The relationship between stress and strain up to the elastic limit is then

$$\frac{F}{A} = \Delta P = E\ \frac{\Delta L}{L} \quad \text{Equation 9.5,}$$

where E (the constant of proportionality) is called the **elastic modulus**, or **Young's modulus**, named for the English physicist Thomas Young (1773–1829). One can rewrite Equation 9.5 to get Equation 9.4:

$$\Delta L = \frac{1}{E}\frac{F}{A}\ L = \frac{1}{E}\Delta P\ L \quad \text{, Equation 9.4.}$$

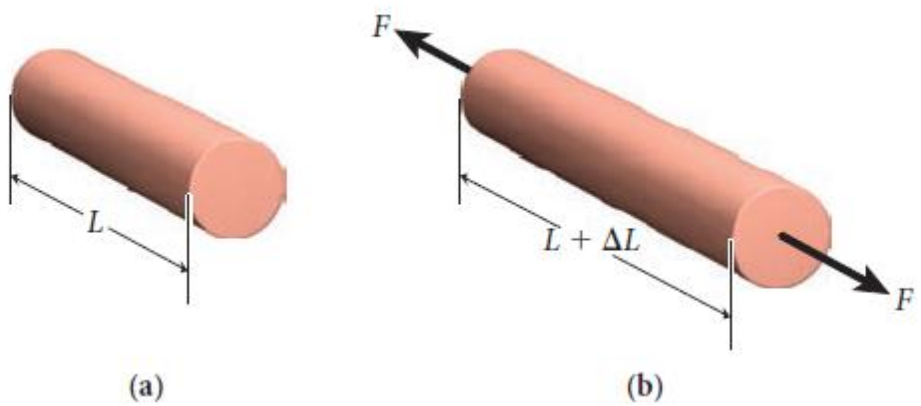


Figure 1: (a) Object before force is applied. (b) Object after force is applied to its to opposite ends by a **pulling** force. Note: Tension can also be applied by **pushing**, with a resulting **negative** change in length (not shown).

Equation 9.4 says that the stretch ΔL increases by an amount proportional to the force, $\Delta L \propto F$. The stretch ΔL is also proportional to the initial length of the rod, L. Finally, the amount of stretch for a given force F is inversely proportional to the cross-sectional area A of the rod. For example, a rod with a cross-sectional area $2A$ is like two rods of cross-sectional area A placed side by side. Thus, applying a force F to a rod of area $2A$ is equivalent to applying a force $F/2$ to two rods of area A. The result is half the stretch when the area is doubled; that is, $\Delta L \propto 1/A$. This is the theme of **Exercise E** - page 242 (answer is b).

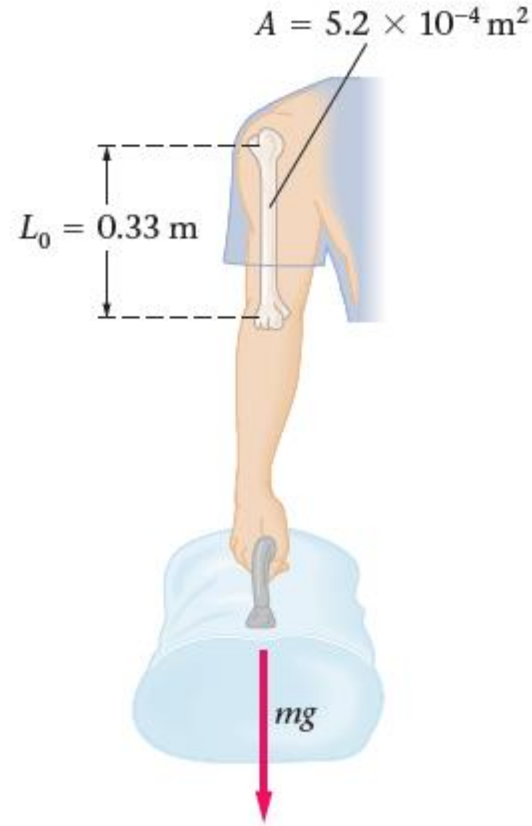
Comparing the two sides of Equation 9.5, we see that Young's modulus has the units of force per area (N/m^2). Notice that Young's modulus depends only on the type of material and not on its size or shape: **It is a material property not an object property**. Some typical values of Young's modulus are given in **Table 9.1**. Notice that the values vary from material to material, but are all rather large.
This means that a large force is required to cause even a small stretch in a solid **(*E* is in the denominator in Equation 9.4).**
For your information (especially for the girls): Diamond Young's modulus is about 1000 – 1200 10^9 N/m^2.

Linear compression can be treated in a manner similar to stretching for most materials, within the elastic limits. One can say that **compression is the exact opposite of stretching** (read the caption of Figure 1). **Thus Equations 9.5 and 9.4 apply equally well to a compression and stretch.** However, some materials have a slightly different Young's modulus for compression and stretching. **For example, human bones under tension (stretching) have a Young's modulus of 15 x 10^9 N/m^2, while bones under compression have a slightly smaller Young's modulus of 9.4 x 10^9 N/m^2.** Example 1 addresses this issue.

Example 1:

A PHY-105 student carries a 21-kg duffel bag in one hand.
i) Assuming the humerus (the upper arm bone) supports the entire weight of the bag, determine the amount by which the bone stretches. (The humerus may be assumed to be 33 cm in length and to have an effective cross-sectional area of 5.2 x 10^{-4} m^2.)
The force applied to the bone is simply the weight of the duffel bag, $F = mg$, with = 21 kg. (We ignore the relatively small weight of the forearm and hand.) Simply, recall Equation 9.4 and substitute the given numerical values into it:

$$\Delta L = \frac{1}{E}\frac{F}{A}L, \quad \Delta L = 8.9 \times 10^{-6} \text{ m.}$$



We find that the amount of stretch is imperceptibly small. The reason for this, of course, is that Young's modulus is such a large number.

ii) If the bone had been compressed rather than stretched by the same applied force, its change in length, though still minuscule, would have been greater by a factor of …? Fill in the gap.
The factor is 15/9.4.

iii) Please do Example 9.10 of your 'tedious' text! Enjoy.

7. Changing the volume of a solid

The stress related to volume compression is caused by a force per unit area **applied to the entire surface area of an object** for example, one submerged in a liquid (**Figure 2**). The resulting **strain is the fractional change in the volume of the object, Δ*V*/*V*.** The modulus of elasticity in this case is the **bulk modulus**, *B*.

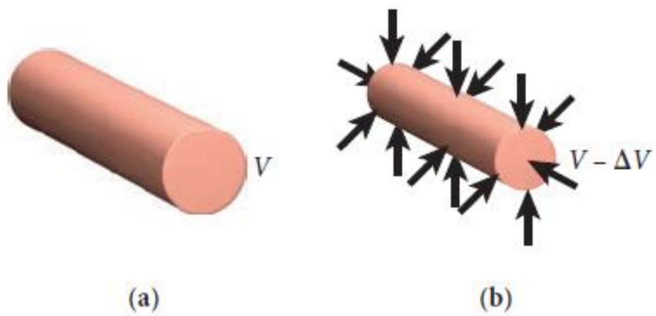


Figure 2: Compression of an object by fluid pressure; (a) object before compression; (b) object after compression. Notice that the force applies only to the ends of the object in **Figure 1**, while here the force applies over the entire object.

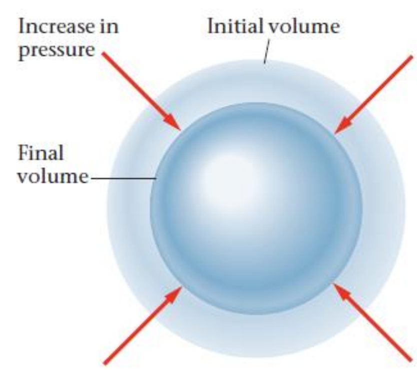


Figure 3

If a piece of Styrofoam (used for making food containers) is taken deep into the ocean, the tremendous pressure of the water causes it to shrink to a fraction of its original volume. Styrofoam has a very small bulk modulus, which means that even a relatively small **increase** in pressure can cause a large **decrease** in volume. This is an extreme example of the volume change that occurs in all solids when the pressure of their surroundings is changed.

The general situation is illustrated in **Figure 3**, where we show a spherical solid whose volume **decreases** by the amount Δ*V* when the pressure acting on it **increases** by the amount Δ*P*. **Experiments show that the pressure difference required to cause a given change in volume, Δ*V*, is proportional to Δ*V* and inversely proportional to the initial volume of the object, *V*.** Therefore, we can write Δ*P* as follows:

$$\frac{F}{A} = \Delta P = -B\left[\frac{\Delta V}{V}\right] \quad \text{Equation 9.7, or}$$

$$\Delta V = -\frac{1}{B}\frac{F}{A}\,V = -\frac{1}{B}\Delta P\,V.$$

Notice that the expressions (Equation 9.5) and (Equation 9.7) are similar in structure, and follow the general equation given in item 5 on page 1.

Like Young's modulus, the bulk modulus is a **positive** quantity with units of N/m^2. The **negative** sign given in Equation 9.7 means that the volume **decreases** with an increase in pressure. Some typical values of the bulk modulus are given in Table 9.1. Since liquids and gases don't have a fixed shape, only the bulk modulus (not the Young's or shear moduli) applies to them. Note the **extremely large jump** in the bulk modulus from air, which is a gas and can be compressed rather easily, to liquids such as mercury and water. Solids such as granite and metals have values for the bulk modulus that are higher than those of liquids by a factor between 20 and 80, indicating that even small volume changes require large changes in pressure.

<u>Example 2</u>:

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is 1.0×10^5 N/m^2; the normal atmospheric pressure (as will be discussed in chapter 10). The sphere is lowered into the ocean to a depth where the pressure is 2.0×10^7 N/m^2. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

The pressure squeezes the sphere and reduces its volume. Recall Equation 9.7 and substitute the given numerical values into it (Table 9.1 lists that the bulk modulus for brass is 80×10^9 N/m^2): -1.2×10^{-4} m^3. The negative sign indicates that the volume of the sphere decreases.

8. Changing the shape of a solid

Another type of deformation, referred to as a **shear** deformation, changes the **shape** of a solid. Consider a book of thickness L_0 resting on a table, as shown in **Figure 4**. A force F is applied to the right on the top cover of the book, and static friction applies a force *F* to the left on the bottom cover of the book. The result is that the book remains at rest but becomes **slanted** by the amount Δx. **The force required to cause a given amount of slant is proportional to Δx, <u>inversely</u> proportional to the <u>thickness</u> of the book L_0, and proportional to the surface area *A* of the book's cover; that is, $F \propto A\, \Delta x / L_0$.** Writing this as an equality, we have

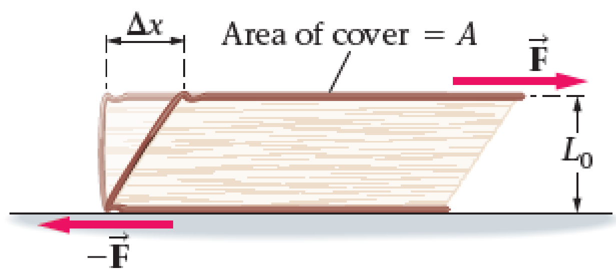


Figure 4

$$\frac{F}{A} = \Delta P = G\,\frac{\Delta x}{L_0}, \qquad \text{Equation 9.6 or}$$

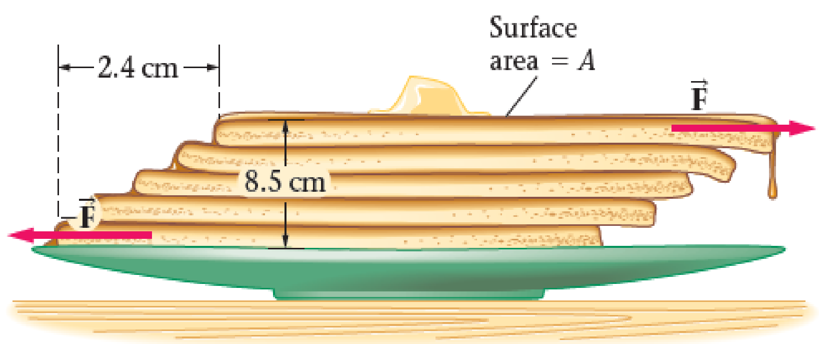
$$\Delta x = \frac{1}{G}\frac{F}{A}\,L_0 = \frac{1}{G}\Delta P\,L_0\,.$$

The constant of proportionality in this case is the **shear modulus**, *G*. Like Young's modulus and the bulk modulus, the shear modulus is a positive quantity and has the units N/m^2. Typical values of the shear modulus are collected in Table 9.1. As with the other two moduli, the shear modulus is large in magnitude, meaning that most solids require a large force to cause even a small amount of shear. From the values listed in Table 9.1, one can infer that *G* is generally one-half to one-third *E* for all the solids **but the human bone! For human bones, the *G* value is about 5 times larger than the *E* value, meaning that the resistance to fracture (crack) in human bones is greater in shear than in tension!** In item 6 we knew that bones under compression have a slightly smaller *E* than under stretching. **Orthopedic surgeons test and analyze measurements of human bone fatigue under shear, tension, and compression.**

The expressions $\frac{F}{A} = \Delta P = E\ \frac{\Delta L}{L}$ (Equation 9.5) and $\frac{F}{A} = \Delta P = G\ \frac{\Delta x}{L0}$ (Equation 9.6) **are similar in structure, but it's important to be aware of their differences as well.** For example, the term *L* in the Young's modulus equation refers to the **length** of a solid measured in the direction of the applied force (**Figure 1**). In contrast, L_0 in the shear modulus equation refers to the **thickness** of the solid as measured in a direction perpendicular to the applied force (**Figure 4**). Similarly, the **cross-sectional area** of the solid *A* in Equation 9.5 is **perpendicular** to the applied force. In contrast, the **area** *A* in Equation 9.6 is the area of the solid in the **plane** of the applied force, i.e. **parallel** to the applied force.

Example 3:

A horizontal force of 1.3 N is applied to the top of a stack of pancakes 16 cm in diameter and 8.5 cm high. The result is a shear deformation of 2.4 cm.



i) What is the shear modulus for these pancakes?

In our sketch, we see the stack of pancakes deformed by a force of magnitude F = 1.3 N to the right at the top of the stack and a force of equal magnitude to the left at the bottom of the stack. The result is a shear deformation of 2.4 cm. Recall Equation 9.6 and solve for *G*, where $A = \pi d^2/4$: $G = 230\ N/m^2$.

Notice the small value of the pancakes' shear modulus, especially when compared to the shear modulus of a typical solid. This is a reflection of the fact that the pancake stack is easily deformed (or say **eaten**!).

ii) Suppose the stack of pancakes is doubled in height, but everything else in the system remains the same. By what factor does the shear deformation change?
The shear deformation doubles.

9. Section 9.6: Fracture

As noted earlier (item 2), the stress applied to an object is proportional to the strain as long as the **elastic limit** of the material is not exceeded. **Figure 5** (or **Figure 9.19** in your text) shows a typical stress-strain diagram for a **ductile** (easily drawn into a wire) metal under tension. Up to the **proportional limit**, the ductile metal responds linearly to stress; **meaning that Equations 9.5, 9.6, and 9.7 do still hold. If the stress (the cause) is removed, the material will return to its original length (the deformation-response vanishes). When a deformation is reversible, we say that it is an elastic deformation.** If stress is applied past the proportional limit, the material will continue to lengthen until it reaches its **yield point** (**elastic limit point** as depicted in Figure 9.19). If stress is applied between the proportional limit and the **fracture point** (**breaking point** as depicted in Figure 9.19) and then is removed, the material will **not return** to its original length but will be permanently deformed (it enters the so-called **plastic region** as depicted in Figure 9.19). The yield point is the point where the stress causes sudden deformation without any increase in force as can be seen from the flattening of the curve (see **Figure 5**). Additional stress will continue to stretch the material until it reaches its **fracture point**, where it **breaks** or tears apart (see **Figure 9.24**). This **breaking stress** (at which the maximum elongation is reached) is also called the **ultimate stress**.

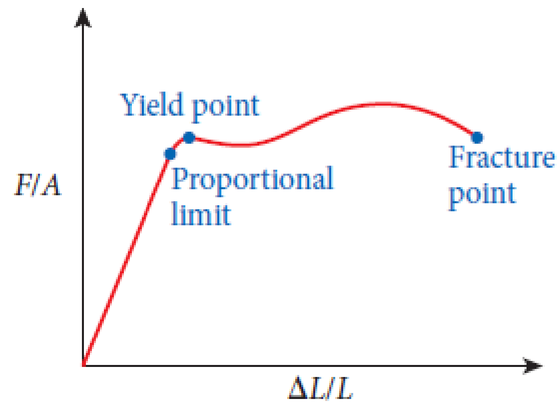


Figure 5: A typical stress-strain diagram for a ductile metal under tension showing the proportional limit, the yield point, and the fracture point. **Figure 9.19** in your text displays the applied force (F) versus elongation (ΔL) though.

Some approximate breaking stresses for tension, compression, and shear are given in **Table 9.2** (called **ultimate strength**). Notice that in the case of tension, the ultimate strength is also called **tensile strength**. Because these values give the maximum stress that a material can **withstand before it breaks**, it is therefore necessary to maintain a **safety factor** of from 3 to perhaps 10 or more, i.e. the actual stress on a material should not exceed 1/10 to 1/3 of the values given in the Table. You may encounter tables of allowable stresses in which appropriate safety factors have been included.

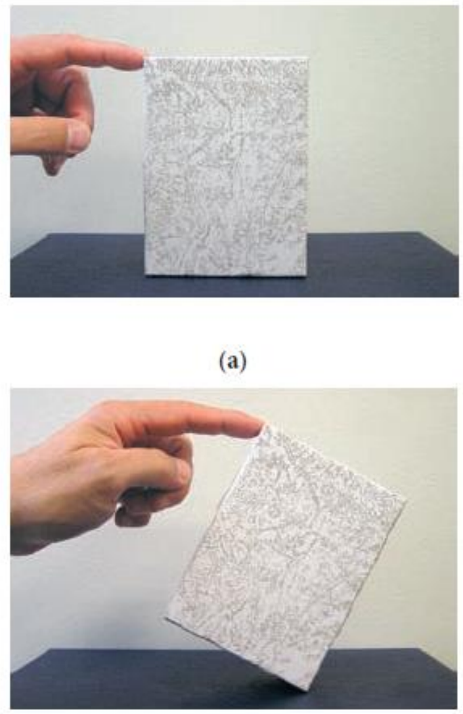
Many materials have different breaking points for stretching and compression. The most notable example is concrete, which resists compression much better than stretching (please ponder the values listed in Table 9.2 for concrete). Can you now explain why concrete can be used as vertical columns placed under compression (Figure 9.21) while can't be used as beams? On the other hand, can you explain why cables of steel are used in suspension bridges (like Wadi Abdoun bridge; the only cable-stayed bridge in Jordan!). The steel cables have significant forces pulling on them from either end. As a result, they are under huge tension, yet we safely use the bridge because we know that the steel can withstand such tensile strength. Relevant to this issue is the 'reinforced' concrete, in which steel rods

are added to it in places where greater tolerance of stretching is required. A steel rod resists stretching much better than compression (500-2500 vs 500 in Table 9.2), under which it can buckle.
For your information (especially for the girls): Gold is the most ductile of all metals. For example, one gram of gold can be drawn into a wire 2.40km long!

10. Section 9.4: Stability and balance

For a skyscraper or a bridge, architects and civil engineers need to worry about the ability of the structure to remain standing under the influence of external forces.

Let's try to quantify the concept of **stability** by looking at **Figure 6 a**, which shows a box in static equilibrium, resting on a horizontal surface. Our experience tells us that if we use a finger to push with a small force in the way shown in the figure (pushing against the **upper edge** of a box), the box remains in the same position. The small force we exert on the box is exactly balanced by the force of static friction between the box and the supporting surface. The net force is zero, and there is no motion. If we steadily increase the magnitude of the force we apply, there are two possible outcomes: If the static friction force is not sufficient to counterbalance the force exerted by the finger, the box begins to **slide** to the right. Or, if the torque due to the weight of the box acting at its center of gravity is less than the torque due the applied force and the friction force, the box starts to **tilt** as shown in **Figure 6 b**. **Thus, the static equilibrium of the box is stable with respect to small external forces, but a sufficiently large external force destroys the equilibrium.**



(a)

(b)

Figure 6

This simple example illustrates the characteristic of stability. Engineers need to be able to calculate the maximum external forces and torques that can be present without undermining the stability of a structure.

11. Quantitative condition for stability

In order to be able to quantify the stability of an equilibrium situation, we need to establish a relationship between potential energy and force (say in one dimension). This means we need **calculus** to get the 2nd gradient derivative of the potential energy function U(x) with respect to the position vector x. And depending on the sign of the 2nd derivative, we can distinguish three different cases of stability. Therefore, and because PHY 105 is not a calculus-based course (hmm…!), here you are the three cases as a **rule of thumb** (cheer up!).

Case 1 Stable Equilibrium (**Figure 7a**)

A small deviation from the equilibrium position creates a **restoring force** that drives the system back to the equilibrium point. This situation is illustrated in **Figure 7a**: If the red dot is moved away from its equilibrium position at x_0 in either the positive or the negative direction and released, it will return to the equilibrium position.

Case 2 Unstable Equilibrium (**Figure 7b**)

A small deviation from the equilibrium position creates a force that drives the system away from the equilibrium point. This situation is illustrated in **Figure 7b**: If the red dot is moved even slightly away from its equilibrium position at x_0 in either the positive or the negative direction and released, it will move away from the equilibrium position.

Case 3 Neutral Equilibrium (**Figure 7c**)

This situation is illustrated in **Figure 7c**: If the red dot is displaced by a small amount, it will neither return to nor move away from its original equilibrium position. Instead, it will simply stay in the new position, which is also an equilibrium position.

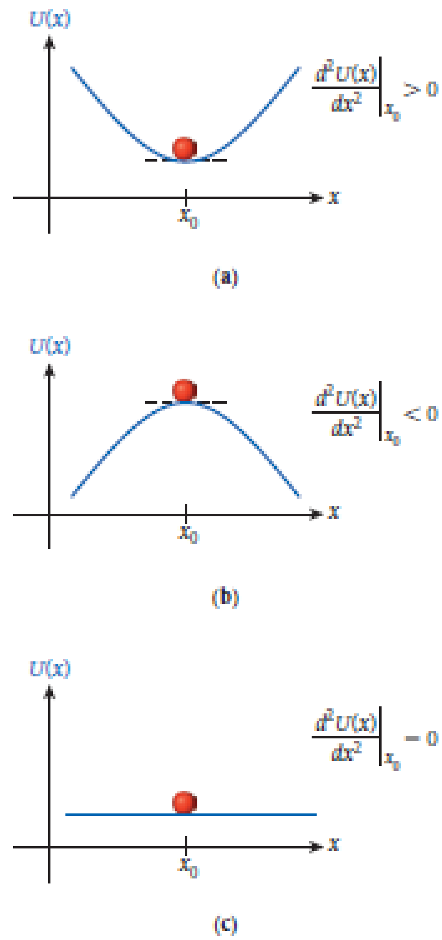


Figure 7

12. Problems:

1. To stretch a relaxed biceps muscle 2.5 cm requires a force of 25 N.
Find the Young's modulus for the muscle tissue, assuming it to be a uniform cylinder of length 0.24 m and cross-sectional area 47 cm^2.

When a force of 25 N is applied to a relaxed bicep muscle, it stretches by 2.5 cm. Solve Equation 9.5 (or 9.4) for Young's modulus of the bicep:

$$\frac{F}{A} = \Delta P = E\,\frac{\Delta L}{L}$$

$E = 5.1 \times 10^4$ N/m^2.

Insight: Young's Modulus for the bicep is five orders of magnitude smaller than the moduli given in Table 9.1. The force required to stretch a steel rod by the same distance as the bicep would be about a million times stronger than the force on the bicep. What force would you need (as orthopedic surgeon to stretch a human bone by the same distance as the bicep?

2. The deepest place in all the oceans is the Marianas Trench, where the depth is 10.9 km and the pressure is 1.10×10^8 N/m^2. If a steel ball 15.0 cm in diameter is taken to the bottom of the trench, by how much does its volume decrease? Take the pressure at the sea level 1.10×10^5 N/m^2.

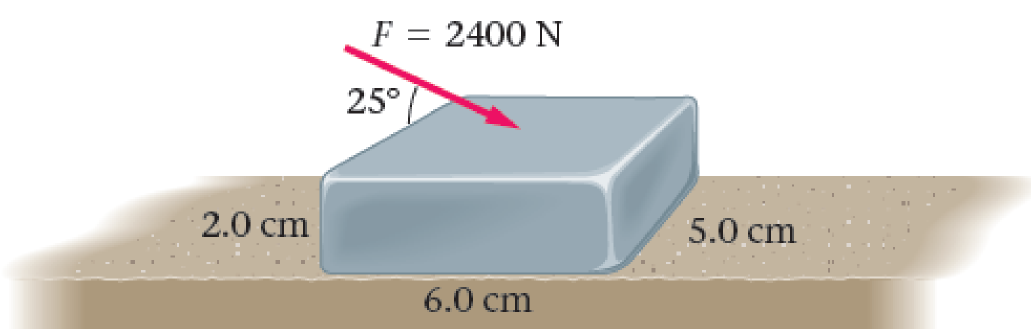
A steel sphere is submerged to the bottom of the trench. The increased pressure compresses the ball. Solve Equation 9.7 for the change in volume:

$$\frac{F}{A} = \Delta P = -B\left[\frac{\Delta V}{V}\right] \quad \text{---------->} \quad \Delta V = -\frac{1}{B}\Delta P\, V$$

$$\Delta V = -\ 1.4 \times 10^{-6}\ \text{m}^3$$

Insight: The volume decreases by 0.08%, which is not a noticeable change. At the bottom of the trench, the diameter of the ball is 14.996 cm, a change of 4/100 of a millimeter.

3. A lead brick with the dimensions shown in the figure rests on a rough solid surface. A force of 2400 N is applied as indicated. Knowing the compression and shear moduli for lead are 16×10^9 N/m^2 and 5.4×10^9 N/m^2 respectively, find (a) the change in height of the brick and (b) the amount of shear deformation.



A force diagonally applied to the top of a lead brick creates both a compressive and a shear deformation of the brick. Use Equation 9.5 to calculate the compression and Equation 9.6 to calculate the shear deformation. **The compression force is the vertical component of the applied force and the shear force is the horizontal component.** Notice that the length of the brick is 6.0 cm, the width is 5.0 cm, and the thickness (height) is 2.0 cm.

$$\Delta L = \frac{1}{E}\frac{F\,\sin\theta}{A}\ L = \frac{(2400\ N)\ (\sin 25°)\ (0.02m)}{(16 \times 10^9\ N/m^2)\ (0.06m)\ (0.05m)} = 4.2\ \times 10^{-7}\text{m}$$

$$\Delta x = \frac{1}{G}\frac{F\,\cos\theta}{A}\ L_0 = \frac{(2400\ N)\ (\cos 25°)\ (0.02m)}{(5.4 \times 10^9\ N/m^2)\ (0.06m)\ (0.05m)} = 2.7\ \times 10^{-6}\text{m}$$

Insight: The small shear modulus (compared to the Young's modulus) of lead results in a larger shear deformation than compressive deformation.