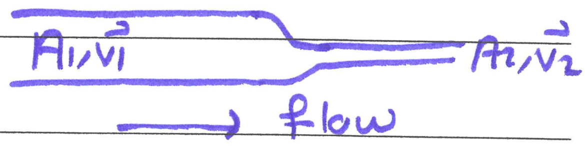
Fluid Dynamics : Sections 10.8, 10.9, and 10.10 .

- Consider ideal fluid flow through a tube of nonuniform cross-sectional area as illustrated in Fig 10.19 ≡ Fig 10.29 ; the Venturi tube :



The fluid moves with steady flow through the tube.

- Let's define the mass flow rate ≡ $\frac{\Delta m}{\Delta t}$ ≡ as the amount of fluid passing through a portion of the tube during Δt.

$$\frac{\Delta m}{\Delta t} = \rho \frac{\Delta V}{\Delta t} = \rho A \frac{\Delta x}{\Delta t} = \rho A v .$$

If there were no leakage in the tube, the mass flow rate must be constant "conserved".

$$\Rightarrow (\rho A v)_1 = (\rho A v)_2 = \text{constant} .$$

Because the fluid is incompressible "ρ doesn't change with pressure", then $A_1 v_1 = A_2 v_2$ —— (1).

This expression is called the eq of Continuity for fluids.

- The product of Av, which has the dimensions of $m^2 \times m/sec$ = volume per unit time, is called either the volume flux or the volume flow rate.

Thus, the condition $AV =$ constant is equivalent to the statement that the volume of fluid that enters one end of a tube in a given time interval equals the volume leaving the other end of the tube in the same time interval if no leaks are present.

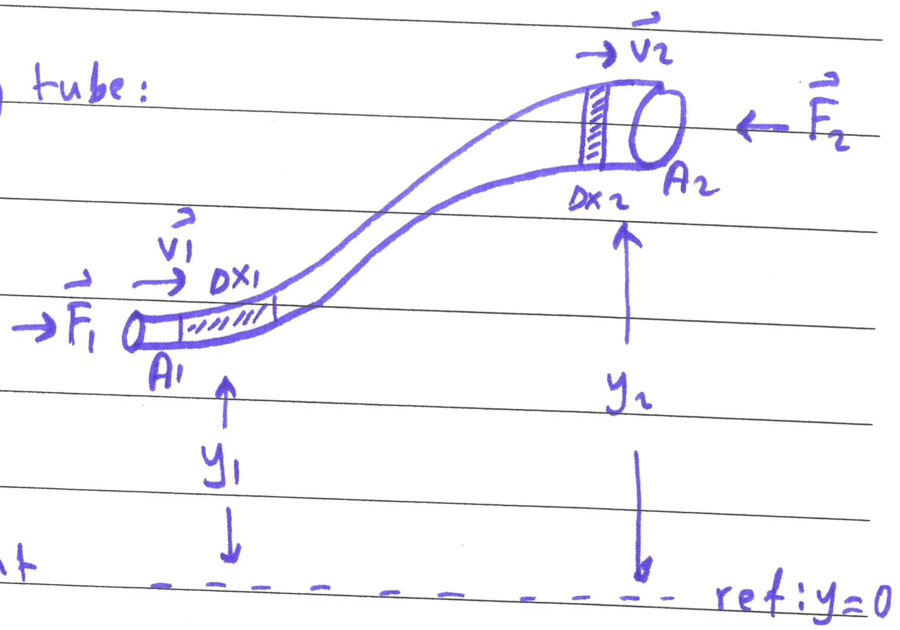
⇒ The continuity equation has three forms:

- $\frac{\Delta m}{\Delta t} =$ constant
- $AV =$ constant
- $\frac{\Delta V}{\Delta t} = AV =$ constant. (2)

- Eq (1): you demonstrate this eq each time you water your garden with your thumb over the end of a garden hose. By partially blocking the opening with your thumb, you reduce the cross-sectional area through which the water passes. As a result, the speed of the water increases as it exits the hose, and the water can be sprayed over a long distance.
- Quiz: Question 12 - Fig 10.43 - page 284: a stream of water from a faucet becomes narrower as it falls....?

* Bernoulli's Equation :-

Consider the following tube:



⇒ $\vec{F}$ exerted on the left side of the tube $\vec{F_1} = P_1 A_1$ to the right.

⇒ $\vec{F}$ exerted on the right side of the tube $\vec{F_2} = P_2 A_2$ to the left.

⇒ Work done by the ~~fluid~~ these forces on the whole tube:

$$W = +F_1 DX_1 + (-) F_2 DX_2 = P_1 A_1 DX_1 - P_2 A_2 DX_2 .$$

However, the volumes of the left portion and the right portion are equal because the fluid is incompressible.

$$\therefore \quad W = (P_1 - P_2) V .$$

Part of this work goes into changing the kinetic energy of the tube of fluid, and part goes into changing the gravitational potential energy of the tube of fluid.

$$W = (P_1 - P_2) V = DKE + DPE$$

$$(P_1 - P_2)V = \frac{m}{2}(v_2^2 - v_1^2) + mg(y_2 - y_1).$$

If we divide each term by the portion volume V and recall that $\rho = m/V$, this expression reduces to

$$P_1 - P_2 = \frac{\rho}{2}(v_2^2 - v_1^2) + \rho g(y_2 - y_1).$$

Rearranging terms gives

$$P_1 + \frac{\rho}{2}v_1^2 + \rho g y_1 = P_2 + \frac{\rho}{2}v_2^2 + \rho g y_2,$$ which is Bernoulli's equation as applied to an ideal fluid.

This equation is often expressed as

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant} \quad \text{—— (3).}$$

Eq (3) shows that:

i) the pressure of a fluid decreases as the speed of the fluid increases.

ii) the pressure decreases as the elevation increases: this point explains why water pressure from faucets on the upper floors of a tall building is weak.

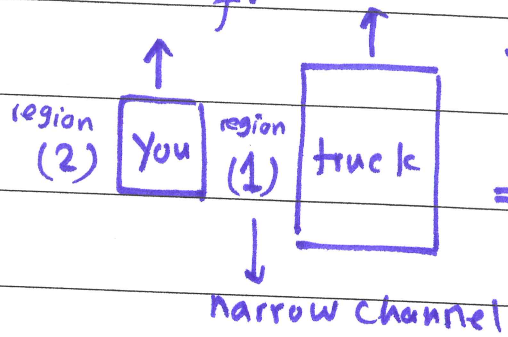
iii) When the fluid is at rest, $v_1 = v_2 = 0$, and eq (3) becomes $P_1 + \rho g y_1 = P_2 + \rho g y_2 \Rightarrow$

$$(P_1 - P_2) = \rho g(y_2 - y_1) = \rho g h.$$

This result is in agreement with Eq (5) - lecture 14 of fluid statics.

- You have probably experienced driving on a highway and having a large truck pass you at high speed. In this situation, you may have had the frightening feeling that your car was being pulled in toward the truck as it passed. The origin of this effect is explained by Bernoulli's eq.

As air passes between you and the truck, it must pass through a relatively narrow channel.



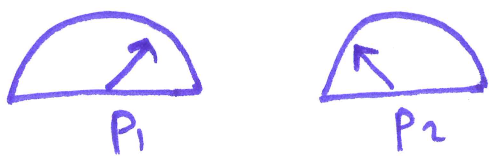
$V_1 > V_2$

$P_1 < P_2$

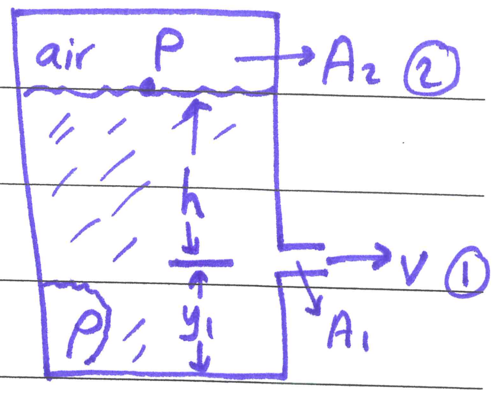
$\Rightarrow \vec{F}_{net}$

According to the continuity eq (1), the speed of the air is higher in region (1) than the slow~~er~~-moving air on the other side of your car - region (2). According to the Bernoulli effect - eq (3), this higher speed air exerts less pressure on your car - region (1) - than region (2). Therefore, there is a net force pushing you toward the truck!

- Do Exercise G - page 277.
- Ponder Figure 10-29 - page 278 and notice that $P_1 > P_2$ as depicted in the Venturi meter:



1 Torricelli's Law: Fig 10.27



An enclosed tank containing a liquid of density ρ has a hole in its side at a distance y_1 from the tank's bottom. The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air trapped above the liquid is maintained at a pressure P. Determine the speed of the liquid as it leaves the hole when the liquid's level is a distance h above the hole.

- Because $A_2 \gg A_1$, the liquid is approximately at rest at the top of the tank, where the pressure is P. At the hole, the pressure is equal to P_{atm}. Apply Bernoulli's eq (3) between points ① and ②:

$$P + \rho g(h+y_1) + 0 = P_{atm} + \rho g y_1 + \rho\frac{v^2}{2}$$

$$P - P_{atm} + \rho g h = \rho\frac{v^2}{2}$$

$$v = \sqrt{\frac{2}{\rho}\left[P - P_{atm}\right] + 2gh} \quad \text{———} \quad (*)$$

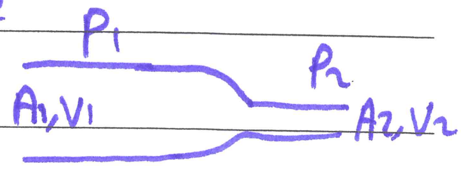
i) Imagine that the tank is a fire extinguisher. When the hole is opened, liquid leaves the hole with a certain speed, v. If the pressure P at the top is increased, the liquid leaves with a higher speed. If the pressure P falls too low, the liquid leaves with a low speed and the extinguisher must be replaced.

ii) When P is much greater than Patm (so the term 2gh in eq (*) can be neglected), the exit speed v is mainly a function of P → $v = \sqrt{\frac{2P}{\rho}}$.

iii) If the tank is open to the atmosphere, then P = Patm and eq *) reads $\boxed{v = \sqrt{2gh}}$ [Eq 10.6 - page 276].

In other words, for an open tank, the speed of the liquid leaving a hole a distance h below the surface is equal to that acquired by an object falling freely through a vertical distance h. This phenomenon is known as Torricelli's law. Although it is seen to be a special case of Bernoulli's eq, it was discovered a century earlier by Torricelli.

[2] Problem 10.50: The Venturi Tube



$$P_1 + \rho g y_1 + \frac{\rho v_1^2}{2} = P_2 + \rho g y_2 + \frac{\rho v_2^2}{2}.$$

Noting that $y_1 = y_2$ because the pipe is horizontal:

$$P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho v_2^2}{2} \Rightarrow \text{one equation with two unknowns.}$$

Solve the equation of continuity for $v_2 = \frac{A_1}{A_2} v_1$.

$$P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho}{2}\left(\frac{A_1}{A_2}\right)^2 v_1^2.$$ Solve for v_1. ✓

$\Rightarrow$ Do Problem 10.45! volume flux $= A_1 v_1 = A_2 v_2$.

[3] A gardener uses a water hose 2.5 cm in diameter to fill a 30-L bucket. The gardener notes that it takes 1 min to fill the bucket. A nozzle with an opening of cross-sectional area 0.5 cm^2 is then attached to the hose. Find the speed with which the water exits the nozzle.

$(Av)_{hose} = (Av)_{nozzle}$.

The volume flow rate is $(Av)_{hose} = 30\frac{L}{min} = \frac{30\,10^3\,cm^3}{60\,s} = 500\frac{cm^3}{s}$

$\Rightarrow$ The speed of the water in the hose $= \frac{500\,cm^3/sec}{\pi\left[\frac{(2.5\,cm)^2}{4}\right]} = 102\frac{cm}{s} = 1.02\,m/s$

$\therefore$ The speed with which the water exits the nozzle is

$$V = \frac{4.91\,cm^2}{0.5\,cm^2} * 1.02\frac{m}{s} = 10\frac{m}{s}.$$

right off: $\frac{500\,cm^3/sec}{0.5\,cm^2} = 10\frac{m}{sec}$!

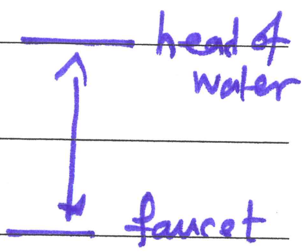
Do Ex 10.13 + Ex 10.14

[4] Fig 10.43 shows a stream of water in steady flow from a kitchen faucet. At the faucet, the diameter of the stream is 0.960 cm. The stream fills a 125 cm³ container in 16.3 sec. Find the diameter of the stream 13.0 cm below the opening of the faucet.

$Av = \frac{125\,cm^3}{16.3\,s} \Rightarrow \boxed{v = \sqrt{2gh},} \therefore (Av)_h = \frac{\pi d^2}{4}\sqrt{2gh}.$

Solve for the diameter, $d = 0.247$ cm.

[5] Problem 10.41: recall Ex 10.3: "pressure head" has a unit of length not a unit of pressure.



⇒ In Problem 10.41, the "head" of water is 12 m at the faucet, i.e., the pressure head is the initial height for the water with speed of zero.

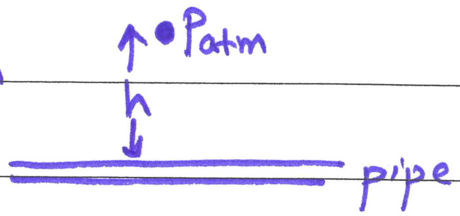
$\Rightarrow$ volume rate $= Av = \frac{\pi d^2}{4} \times \boxed{\sqrt{2gh}}.$

⇒ Do Problem 10.73.

[6] Problem 10.43: this is Problem [11] - Lecture 14 - page 31!

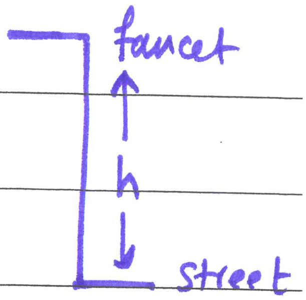
$P_h + \rho g h + 0 = P_{pipe} + 0 + 0, \; P_h = P_{atm}$

$\therefore P_{pipe} - P_{atm} = P_{gauge} = \rho g h.$



⇒ Please revisit problem [10] - lecture 14 - page 31.

$\boxed{7}$ Problem 10.48:



- $(vA)_{street} = (vA)_{faucet} \rightarrow v_{faucet} = 2.49\ m/s$, larger than v_{street} as expected.
- $\left[P_{absolute} + \rho g(0) + \rho \frac{v^2}{2}\right]_{street} = \left[P_{absolute} + \rho g h + \rho \frac{v^2}{2}\right]_{faucet}$

$P_{absolute} = P_{gauge} + P_{atm} \Rightarrow$

$P_{gauge-faucet} = P_{gauge-street} - \rho g h + \frac{\rho}{2}\left(v_{street}^2 - v_{faucet}^2\right)$.

$P_{gauge-faucet} = 2.2\ atm < P_{gauge-street}$ as it is expected.

$\boxed{8}$ Problem 10.49:

$Power = F * v$, $F = DP * A \Rightarrow Power = DP * (Av) = DP * Q$.

- Hint: Problem 10.51 needs the concept of linear momentum, $\vec{p} = m\vec{v}$, $\vec{F} = \frac{D\vec{P}}{Dt}$, eq 7.1 and eq 7.2 of your text.

Thus, problem 10.51 is excluded. Cheer up!

$\boxed{9}$ Problem 10.79: we just employ the answer of problem $\boxed{8}$:

each heartbeat $\Rightarrow 70\ cm^3$ of blood, with rate of 70 beats/min

$\Rightarrow Av = Q = 70\,10^{-6}\ m^3 * \frac{70}{60\,sec}$

$\Rightarrow Power = DP * Q$

$= (105 * 133) * 70\,10^{-6} * \frac{7}{6} = 1.14\ W.$

10 Problem 10.38 or Example 10.12:
Enjoy while reading about your circulatory system!
V of the arteries $\simeq$ 90.4 cm/sec.

11 • For the sake of knowledge, PHY 105 is expected to be eager to read about the Transient Ischemic Attack, TIA on page 278 - Figure 10.28. • More interesting is to ponder eq 10.9 on page 280 and the R^4 dependence: If the radius of arteries is reduced by half - as a result of cholesterol buildup for instance, the heart would have to increase the pressure by a factor of $2^4 = 16$ in order to maintain the same blood-flow rate!!

• Skimming Sec 10.14 is the least that PHY 105 should do! You have been already asked to do Problem 10.66 on page 15/33 - Lecture 14.