Dr. Mohammad Hussein PHY 105 - Fall 2022 Lecture: 16 Date: 1/1/2023

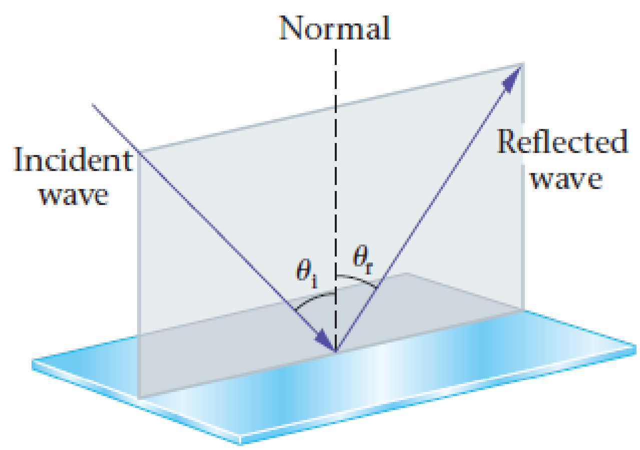
Chapter 23: Geometric optics

Section 23.1: Introduction

The Ray Mode: light travels in straight-line paths called light rays, see **Figure 23.1**.
We will use this model to describe some phenomena of light such as reflection (section 23.2), refraction (sections 23.4 & 23.5), total internal reflection (section 23.6), and the formation of images by lenses (sections 23.7 & 23.8). Because these explanations involve straight-line rays at various angles, this subject is referred to as geometric optics.

Section 23.2: Reflection

The figure shows a beam of light strikes a flat surface. We define the angle of incidence, **θ**i, to be the angle an incident ray (or wave) makes with the normal (perpendicular) to the surface, and the angle of reflection, **θ**r, to be the angle the reflected ray makes with the normal. The relationship between these two angles is **very simple**–they are equal:
Law of reflection: **θ**r = **θ**i
Notice, in addition, that the incident ray, the normal, and the reflected ray all lie in the same plane, as is also clear from the figure.



Section 23.4: Refraction

When a wave propagates from a medium in which its speed is v_1 to another medium in which it has a different speed, $v_2 \neq v_1$, it will, in general, change its direction of motion. This phenomenon is called **refraction**.

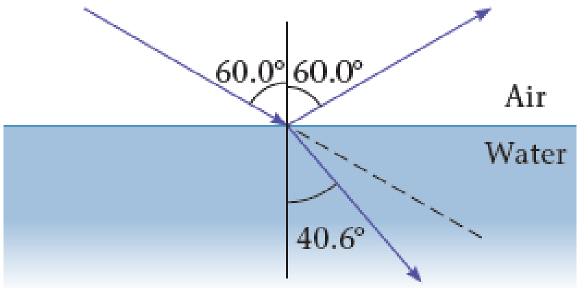
In general, the speed of light depends on the medium through which it travels. For example, we know that in a vacuum the speed of light is $c = 3.00 \times 10^8$ **m/s**. When light propagates through water, however, its speed is reduced by a factor of 1.33. In general, the speed of light in a given medium, v, is determined by the medium's **index of refraction**, n, defined as follows: $v = \frac{c}{n}$.
Values of the index of refraction for a variety of media are given in **Table 23-1**.
Quiz: The index of refraction is never less than 1: [True or False]. Answer: True.
Quiz: How much time does it take for light to travel 1.20 m in water? Answer: 5.32×10^{-9} s.

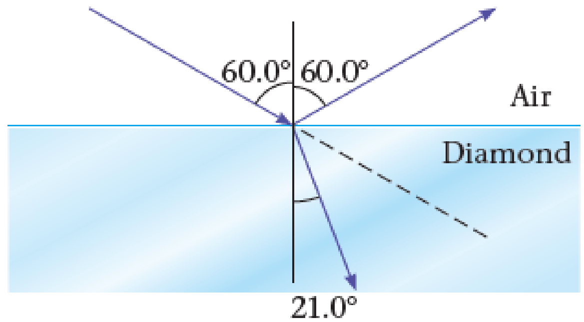
Section 23.5: Snell's Law

Returning to the direction of propagation, let's suppose light has the speed $v_1 = c/n_1$ in one medium and the speed $v_2 = c/n_2$ in a second medium. The direction of propagation in these two media is related through a relationship, known as **Snell's law**: $n_1 \sin\theta_1 = n_2 \sin\theta_2$, where θ_1 is the angle of incidence and θ_2 is the angle of refraction. A typical application of Snell's law is given in the following example.

EXAMPLE 1

A beam of light in air enters (a) water (n = 1.33) or (b) diamond (n = 2.42) at an angle of 60.0° relative to the normal. Find the angle of refraction for each case.

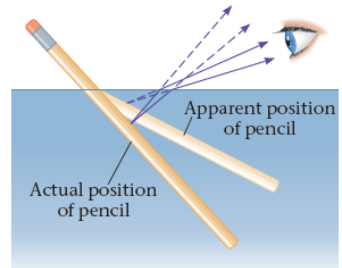




Because the beam starts in air, we refer to Table 23-1 and set n_1 = 1.000293, or simply n_1 = = 1.00 to three significant figures. We also solve Snell's law for θ_2 , giving $\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$.
(a) With n_2 = 1.33 we find θ_2 = 40.6°.
(b) With n_2 = 2.42 we find θ_2 = 21.0°.
Notice that the angle of reflection in both cases is 60° as required by the law of reflection.

Insight: From the preceding example we can see that the greater the difference in the index of refraction between two different materials, the greater the difference in the direction of propagation. In addition, light is bent closer to the normal in the medium where its speed is reduced (the beam enters a medium where the speed of light is less and the index of refraction is greater). Of course, the opposite is true when light passes into a medium in which its speed is greater, as can be seen by reversing the incident and refracted rays. The qualitative features of refraction are as follows:

• When a ray of light enters a medium where the index of refraction is increased, and hence the speed of the light is decreased, the ray is bent toward the normal.
• When a ray of light enters a medium where the index of refraction is decreased, and hence the speed of the light is increased, the ray is bent away from the normal.
• The greater the change in the index of refraction, the greater the change in the propagation direction. If there is no change in the index of refraction, there is no change in the direction of propagation.
• If a ray of light goes from one medium into another along the normal, it is undeflected, regardless of the index of refraction of each medium. This follows directly from Snell's law: If θ_1 is zero, then 0 = $n_2 \sin\theta_2$, which means that θ_2 = 0. Refraction is explored further in **Example 23.8**.



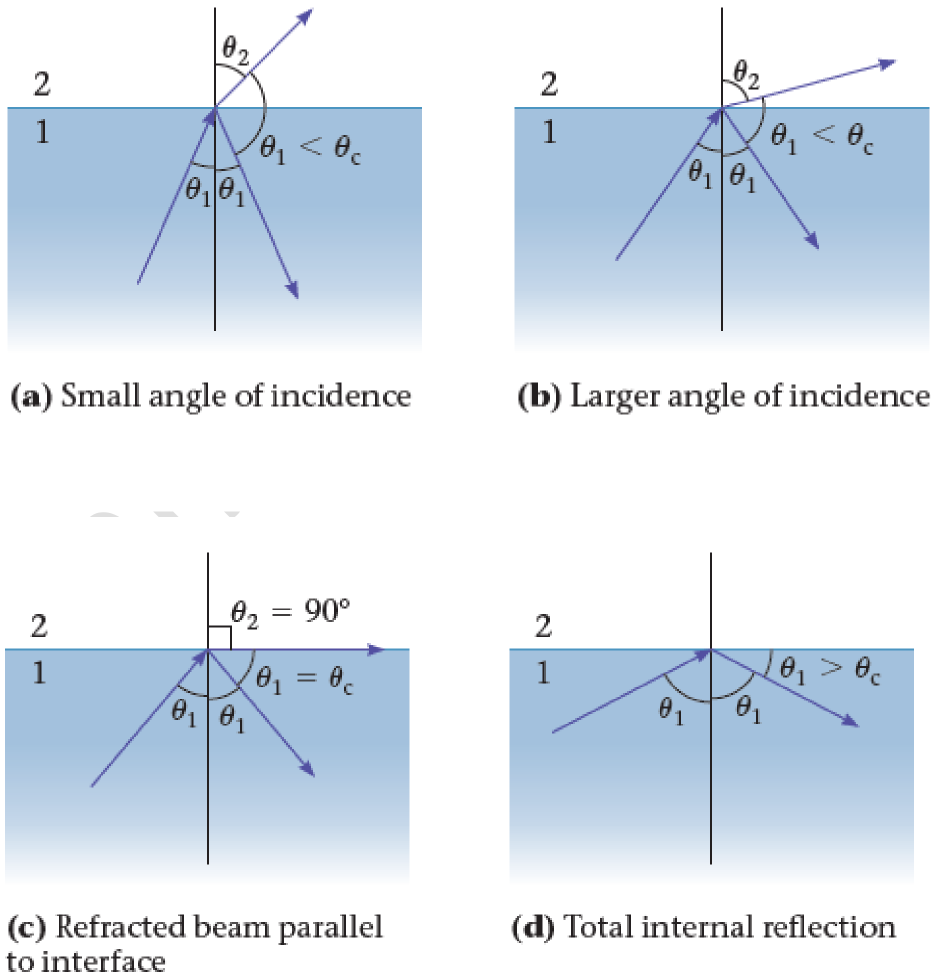
Refraction is responsible for a number of common "**optical illusions**." For example, we all know that a pencil placed in a glass of water appears to be bent, though it is still perfectly straight. The cause of this illusion is shown in the figure, where we see that rays leaving the water bend away from the normal and make the pencil appear to be above its actual position (see also **Figure 23.23**). This is an example of what is known as **apparent depth**, in which an object appears to be closer to the water's surface than it really is (see **Example 23.9**). Similarly, refraction can cause a **mirage**, as when hot, dry ground in the distance appears to be covered with water.

We will use the results of refraction in sections 23.7 & 23.8 when we investigate the behavior of lenses. For now, we turn to another phenomenon associated with refraction, namely the total internal reflection.

<u>Section 23.6: Total Internal Reflection</u>

Sometimes refraction can "**trap**" a ray of light and prevent it from leaving a material.

For example, Figure (a) shows a ray of light in water encountering a water–air interface. In such a case, it is observed that part of the light is reflected back into the water at the interface, while the rest emerges into the air along a direction that is bent away from the normal according to Snell's law (opposite to the case of Example 1). If the angle of incidence is increased, as in Figure (b), the angle of refraction increases as well. At some **critical angle** of incidence, θ_c , the refracted beam no longer enters the air but instead is directed parallel to the water–air interface [Figure (c)]. In this case, the angle of refraction is 90°. For angles of incidence **greater** than the critical angle, as in Figure (d), it is observed that **all** of the light is reflected back into the water—it's **effectively trapped in the water**.



(a) Small angle of incidence

(b) Larger angle of incidence

(c) Refracted beam parallel to interface

(d) Total internal reflection

This phenomenon is referred to as **total internal reflection** (see **Figure 23.26**). We can find the critical angle for total internal reflection by setting θ_2 = 90° and applying Snell's law: $n_1 \sin\theta_c = n_2 \sin 90° = n_2$. Therefore, the critical angle is given by the following relationship: $\sin\theta_c = \frac{n_2}{n_1}$. Because $\sin\theta$ is always less than or equal to 1, the index of refraction, n_1, must be larger than the index of refraction, n_2, if the above equation is to give a physical solution. Thus, total internal reflection can occur only when light in one medium encounters an interface with another medium in which the speed of light is greater.

For example, light moving from water to air can undergo total internal reflection, as was explained in the Figure above, but light moving from air to water cannot.

EXAMPLE 2

Consider a sample of glass whose index of refraction is *n* = 1.65. Find the critical angle for total internal reflection for light traveling from this glass to (a) air (*n* = 1.00) and (b) water (*n* = 1.33).

It is straightforward to obtain θ_c by simply substituting the appropriate indices of refraction for each case in the relationship $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$. Answers: (a): θ_c = 37.3° . (b): θ_c = 53.7°

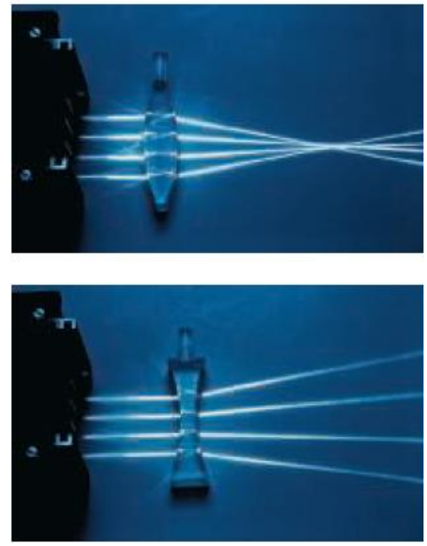
Insight: In the case of water and glass, the two indices of refraction are close in value; hence, light **escapes** from glass to water over a wider range of incident angles (0° to 53.7°) than from glass to air (0° to 37.3°). In general, a large change in the index of refraction, such as from diamond to air, means that very little light escapes (the majority internally reflects), which is why **diamonds sparkle** more than glass.

Practice Problem: Suppose the incident ray is in a different type of glass, with a glass–air critical angle of 40.0°. Is the index of refraction of this glass greater than or less than 1.65? [Answer: The critical angle is larger for this glass, which means that its index of refraction must be less than 1.65. Thus *n* = 1.56.]

Total internal reflection is frequently put to practical use. For example, many binoculars contain a set of prisms—referred to as Porro prisms—that use total internal reflection to “fold” a relatively long light path into the short length of the binoculars, as shown in **Figure 23.28**. **Optical fibers** are another important and common application of total internal reflection. These thin fibers are generally composed of a glass or plastic core with a high index of refraction surrounded by an outer coating, or cladding, with a low index of refraction. Light is introduced into the core of the fiber at one end. It then propagates along the fiber in a zigzag path, undergoing one total internal reflection after another, as indicated in **Figure 23.29**. The core is so transparent that even after light propagates through a 1-km length of fiber, the amount of absorption is roughly the same as if the light had simply passed through a glass window. In addition, the total internal reflections allow the fiber to go around corners, and even to be tied into knots, and still deliver the light to the other end.

The ability of optical fibers to convey light along curved paths has been put to good use in various fields of **medicine**. In particular, devices known as **endoscopes** allow physician **(a JU PHY-105 former student!)** to examine the interior of the body by snaking a flexible tube containing optical fibers into the part of the body to be examined. For example, a type of endoscope called the bronchoscope (**Figure 23.30**) can be inserted into the nose or throat, threaded through the bronchial tubes, and eventually placed in the lungs. There, the bronchoscope delivers light through one set of fibers and returns an image to the physician through another set of fibers. In some cases, the bronchoscope can even be used to retrieve small samples from the lung for further analysis. Similarly, the **colonoscope** can be used to examine the colon, making it one of the most important weapons in the fight against colon cancer.

Section 23.7: Ray Tracing for Lenses



As we've seen, a ray of light can be redirected as it passes from one medium to another. A device that takes advantage of this effect, and uses it to focus light and form images, is a **lens**. Typically, a lens is a thin piece of glass or other transparent substance that can be characterized by the effect it has on light. In particular, **converging lenses** take parallel rays of light and bring them together at a focus; **diverging lenses** cause parallel rays to spread out as if diverging from a point source. Examples are shown in the figure next: the paths of light rays through a **convex** (converging) lens (top) and a **concave** (diverging) lens (bottom). A variety of converging and diverging lenses are illustrated in **Figure 23.31 (a, b)**, though we consider only the most basic types here—namely, the double concave (or simply concave) and the double convex (or simply convex). Notice, in general, that converging lenses are thicker in the middle, and diverging lenses are thinner in the middle.

Let's start by considering a convex lens, as shown in **Figure 23.33.** Convex lenses are shaped so that they bring parallel light to a focus at a **focal point**, F, along the center line, or axis, of the lens, as indicated in the figure. In the case of concave lenses (**Figure 23.36**), parallel rays are bent away from the axis of the lens. When the diverging rays from a concave lens are extended back, they appear to originate at a focal point F on the axis of the lens.

Ray Tracing: To determine the type of image formed by a convex or concave lens, we can use ray tracing. The three principal rays for lenses are shown below in **Figure 1** and **Figure 2**. Their properties are as follows:

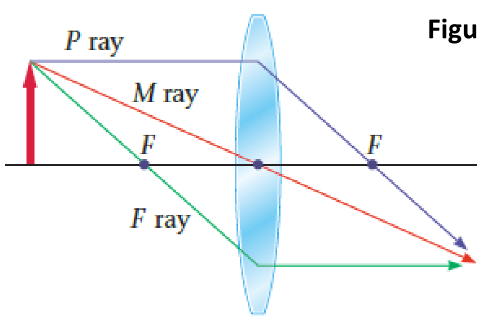
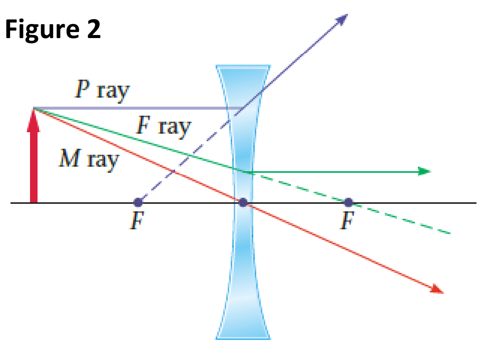


Figure 1

Figure 2



• The P ray—or parallel ray—approaches the lens parallel to its axis. The P ray is bent so that it passes through the focal point of a convex lens (Figure 1), or extrapolates back to the focal point on the same side of a concave lens (Figure 2).
• The F ray (focal-point ray) on a convex lens is drawn through the focal point and on to the lens, as pictured in Figure 1. The lens bends the ray parallel to the axis— basically the reverse of a P ray. For a concave lens, the F ray is drawn toward the focal point on the other side of the lens, as in Figure 2. Before it gets there, however, it passes through the lens and is bent parallel to the axis.
• The midpoint ray (M ray) goes through the middle of the lens, which is basically like a thin slab of glass

(recall Example 23.8). For ideal lenses, which are infinitely thin, the M ray continues in its original direction with negligible displacement after passing through the lens.

To illustrate the use of ray tracing, we start with the concave lens shown in **Figure 3** (**Figure 23.39**). Notice that the three rays originating from the top of the object extend backward to a single point on the left side of the lens—to an observer on the right side of the lens this point is the top of the image. Our ray diagram also shows that the **image is upright and reduced in size. In addition, the image is virtual**, because it is on the same side of the lens as the object. These are general features of the image formed by a concave lens.

Figure 3

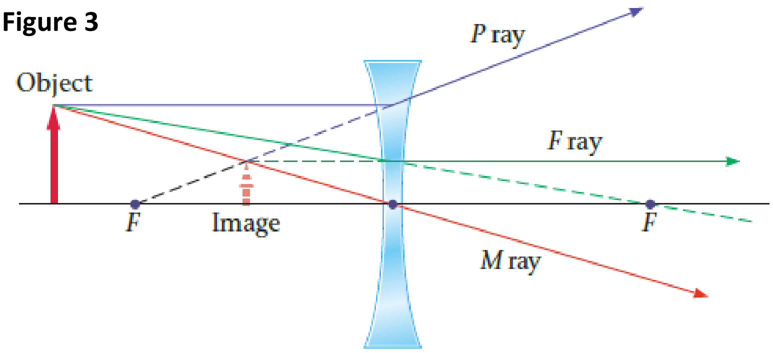


Figure 4 (a)

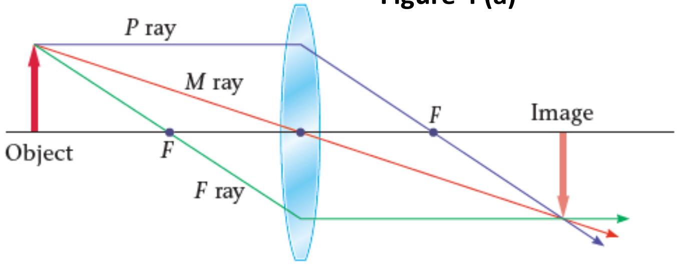
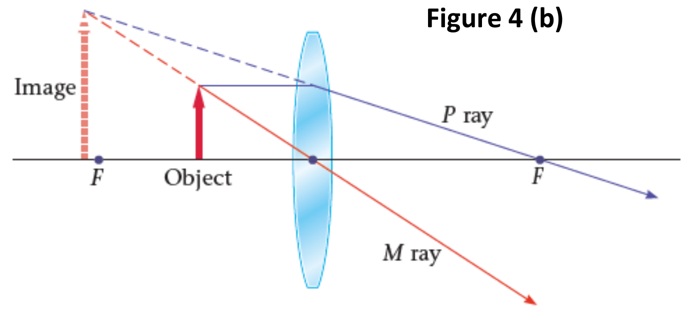


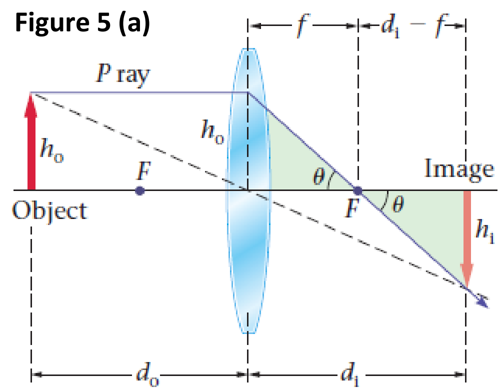
Figure 4 (b)



The behavior of a convex lens is more interesting in that the type of image it forms depends on the location of the object. For example, if the object is placed beyond the focal point, as in **Figure 4 (a)** (**Figure 23.37**), the image is **inverted**, on the opposite side of the lens, and light passes through it—it is a **real** image (**it might be reduced or enlarged in size -** see Example 3). If the object is placed between the lens and the focal point, as in **Figure 4 (b)**, the result is an image that is **virtual** (on the same side as the object), **upright, and enlarged in size**.

Section 23.8: The Thin-Lens Equation

Figure 5 (a)



We now derive an equation that relates the image distance to the object distance and the focal length of a thin lens. This thin-lens equation will make the determination of image position quicker and more accurate than doing ray tracing. This equation can be derived by referring to **Figure 5** (**Figure 23.40**), which shows the image produced by a convex lens, along with the P and M rays that locate the image.

First, notice that the P ray creates two similar blue-shaded triangles on the right side of the lens in **Figure 5 (a)**. Because the triangles are similar, it follows that

$$\frac{h_o}{f} = \frac{-h_i}{d_i - f}$$

In this expression, f is the **focal length**—that is, the distance from the lens to the focal point, F—and we use $-h_i$ on the right side of the equation, because h_i is negative for an inverted image. Next, the M ray forms another pair of similar triangles, shown with pink shading in **<u>Figure 5 (b)</u>**, from which we obtain the following:

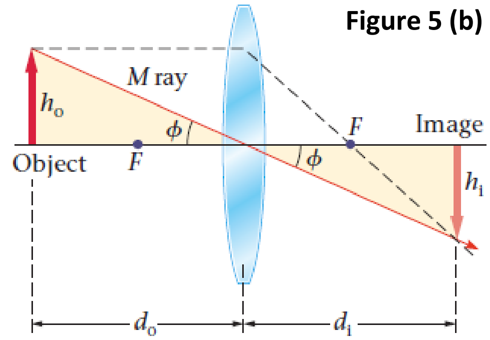


Figure 5 (b)

$$\frac{h_o}{d_o} = \frac{-h_i}{d_i} \text{ --- [\#]}$$

Combining these two relationships, we obtain a result known as the **thin-lens equation**:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Finally, the **magnification**, *m*, of the image is defined as the ratio of the image height to object height:

$$m = \frac{h_i}{h_o}$$

Rearranging the Equation ---[#] given above, we find that $h_i = \frac{-d_i}{d_o} h_o$. Therefore, the magnification for a lens is:

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

As before, the sign of the magnification indicates the orientation of the image, and the magnitude gives the amount by which its size is enlarged or reduced compared with the object. The thin-lens equation (though was derived for converging lens) <u>will be valid for both converging and diverging lenses</u>, and for all situations, **<u>if</u>** we use the following **sign conventions**:

Focal Length

f is positive for converging (convex) lenses. f is negative for diverging (concave) lenses.

Magnification

m is positive for upright images (same orientation as object).
m is negative for inverted images (opposite orientation of object).

Image Distance

d_i is positive for real images (images on the opposite side of the lens from the object).
d_i is negative for virtual images (images on the same side of the lens as the object).

Object Distance

d_o is positive for real objects (from which light diverges).
d_o is negative for virtual objects (toward which light converges).

We now apply the thin-lens equation and the definition of magnification to typical lens system.

EXAMPLE 3

A lens produces a real image that is twice as large as the object, inverted, and located 15 cm from the lens. Find (a) the object distance and (b) the focal length of the lens.

Answer: Because the image is real, the lens must be convex, and the object must be outside the focal point, as we indicate in our sketch. [Compare with **Figure 4 (a)**.] In addition, the image is inverted (negative magnification), which means the magnification is m = –2. Finally, the distance to the real image is given as d_i= 15 cm. To find both d_o and f requires two independent relationships. One is provided by the magnification, the other by the thin-lens equation. Thus: d_o = 7.5 cm and f = 5.0 cm.

Insight: As expected for a convex lens, the focal length is positive. In addition, notice that the object distance is greater than the focal length, in agreement with Figure 4 (a). Finally, the magnification produced by this lens is not always -2. In fact, it depends on the precise location of the object, as we see in the following Practice Problem.

Practice Problem: Suppose we would like to have a magnification of -3 using this same lens. (a) Should the object be moved closer to the lens or farther from it? Explain. (b) Find the object and image distances for this case. [Answer: (a) The object should be moved closer to the lens. This moves the image farther from the lens and makes it larger. (b) We find d_o = 6.67 cm (which is less than 7.5 cm, as expected) and d_i = 3d_o = 20.0 cm.]

Digression: Ophthalmologists and optometrists, instead of using the focal length, use the reciprocal of the focal length to specify the strength of eyeglass (or contact) lenses. This is called the **power**, *P*, of a lens:

$$P = \frac{1}{f}$$

The unit for lens power is the **diopter** (D), which is an inverse meter: 1 D = 1 m^{-1}. Therefore, the lens in this example (with 5-cm-focal-length) has a power *P* = 1/(0.05m) = 20.0 D. From the sign convention about the focal length mentioned above, it follows that the power of a converging lens, in diopters, is positive, whereas the power of a diverging lens is negative. A converging lens is sometimes referred to as a positive lens, and a diverging lens as a negative lens. We will mainly use the focal length—the power of a lens is usually used when discussing eyeglass lenses (chapter 25). That being said, I highly recommend you read section 25.2.

EXAMPLE 4

An object is placed 12 cm in front of a diverging lens with a focal length of -7.9 cm. Find (a) the image distance and (b) the magnification.

Answer: Given the focal length and object distance, we can use the thin-lens equation to find the image distance. Once the image and object distances are known, we can use them to find the magnification. Thus: d_i = -4.8 cm and m = 0.40.

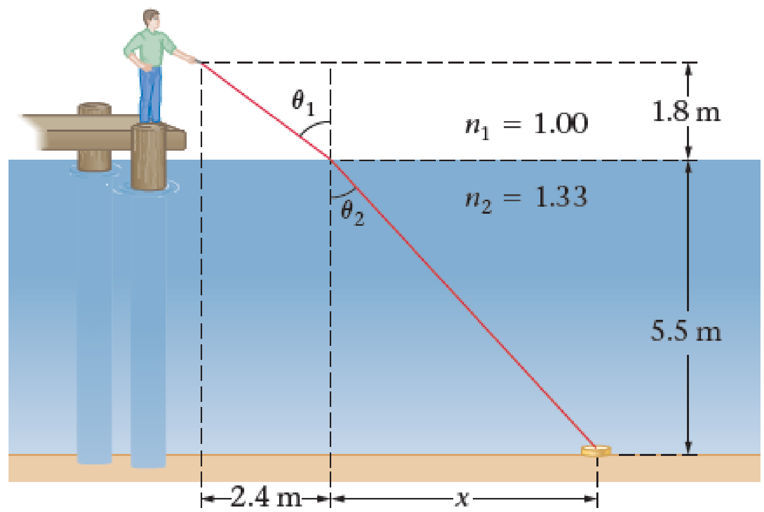
Insight: Because the image distance is negative, it follows that the image is virtual and on the same side of the lens as the object, as expected for a concave (diverging) lens. In addition, the fact that the magnification is positive means that the image is upright. These numerical values correspond to the system illustrated in **Figure 3**.

Practice Problem: An object at the distance d_o = 15 cm from a lens produces an inverted image. Is the focal length of the lens greater than, less than, or equal to 15 cm? Answer: Only a convex lens produces an inverted image, and this occurs when the object is farther from the lens than the focal point. Therefore, the focal length of the lens is less than 15 cm.

The thin-lens equation is explored further in **Examples 23.12, 23.13, and 23.14**.

Problems:

1. A PHY_105 student walks to the end of a dock and shines his laser pointer into the water. When he shines the beam of light on the water a horizontal distance of 2.4 m from the dock, he sees a glint of light from a shiny object on the sandy bottom. If the pointer is 1.8 m above the surface of the water, and the water is 5.5 m deep, what is the horizontal distance x as indicated in the figure?

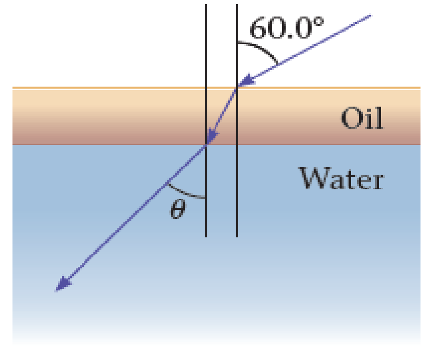


Answer: All of the known distances are indicated in the sketch, along with the angle of incidence, θ_1, and the angle of refraction, θ_2. Finally, the appropriate indices of refraction from Table 23-1 are given as well. We can use Snell's law and basic trigonometry to find the horizontal distance x. First, the information given in the problem determines the angle of incidence, θ_1. In particular, we can see from the sketch that tan θ_1 = (2.4 m)/(1.8 m). Second, Snell's law gives the angle of refraction, θ_2. Finally, we can find the distance x from θ_2, noting that tan θ_2 = x/(5.5 m).
Thus: θ_1 = 53°, θ_2 = 37°, and hence x = 4.1 m.

2. An object with a height of 2.54 cm is placed 36.3 mm to the left of a lens with a focal length of 35.0 mm. (a) Where is the image located? (b) What is the height of the image?
Answer: (a) The image is located 0.98 m to the right of the lens. (b) The image is inverted and 68 cm tall.

3. An object is located to the left of a concave lens whose focal length is -34 cm. The magnification produced by the lens is m = 1/3. (a) To decrease the magnification to m = 1/4, should the object be moved closer to the lens or farther away? (b) Calculate the distance through which the object should be moved. Answer: (a) You need to write an equation for the magnification as a function of the object distance: $m=\frac{f}{f-d_o}$. You can see from the expression that because *f* is negative, in order to decrease the magnification, the object should be moved farther away from the lens, making the denominator $f - d_o$ larger in magnitude and *m* smaller. (b) Solve for the object distance: $d_{o4} - d_{o3} = f(\frac{1}{m3} - \frac{1}{m4})$ = 34 cm.
The object should be moved 34 cm farther away from the lens.

4. A film of oil, with an index of refraction of 1.48 and a thickness of 1.50 cm, floats on a pool of water, as shown. A beam of light is incident from air on the oil at an angle θ_1= 60.0° to the vertical. (a) Find the angle θ the light beam makes with the vertical as it travels through the water. (b) How does your answer to part (a) depend on the thickness of the oil film? Explain.



Answer: (a) Because the air/oil and oil/water interfaces are parallel, the angle of refraction at the air/oil interface will equal the angle of incidence at the oil/water interface. Write Snell's law at the air/oil interface and at the oil/water interface, and then combine the two equations and solve for the angle of refraction in water:

$$\theta = \sin^{-1}\left(\frac{n_{air}}{n_{water}} \sin\theta_1\right) = \sin^{-1}\left(\frac{n_1}{n_2} \sin\theta_1\right) = 40.6°$$

(b) The answer to part (a) does not depend upon the thickness of the oil film, because θ depends only upon the original angle of incidence and the indices of refraction of air and water.

5. As shown in the figure, a real inverted image *I* of an object *O* is formed by a particular lens (not shown); the object–image separation is *d* = 40.0 cm, measured along the central axis of the lens. The image is just half the size of the object. (a) What kind of lens must be used to produce this image? (b) How far from the object must the lens be placed? (c) What is the focal length of the lens?
Answer: This time I'll give you the answer for item c only: 8.89 cm.