Conservation of Energy

- With the introduction of the kinetic energy: $K = \frac{mv^2}{2}$, $W_{F_{applied}} = \Delta K$, Eq (3) - lecture 7, and the introduction of the gravitational potential energy: $U = mgy$, $W_{F_{applied}} = \Delta U$, Eq (5) - lecture 7, we can <u>extend</u> the work-energy theorem of Lecture 7. A general statement of it becomes:

$$W_{F_{applied}} = \Delta K + \Delta U = \Delta E \quad \text{—— Eq (1)},$$

where $E \equiv$ total mechanical energy $= K + U$, (recall lecture 7 - page 10). This Eq (1) means that work done by an external force to a system can change the total energy of the system. Such system is called ~~an~~ a <u>non-isolated</u> system: a system that interacts with its environment.

- In the presence of the friction force, a further generalization of the work-energy theorem, Eq (1), is needed!

- As we have learned in lecture 6, the friction force f_K <u>always</u> points in the direction opposite to that of motion. This leads us to conclude that the work done by the friction force converts kinetic and/or potential energy into internal excitation energy, which can be vibration energy, thermal energy, deformation energy, chemical energy, or even electrical energy, depending on the two objects that exert friction on each other. Thus, the "<u>dissipation</u>" of the energy due to the friction force is always <u>negative, reducing</u> kinetic and/or potential energy and <u>converting</u> it into internal excitation energy.
- The decisive fact is that the friction force switches direction as a function of the direction of motion and causes dissipation.

- Using the symbol W_{fk} for this "dissipated" energy, Eq (1) becomes:

$$\boxed{\underset{\pm ve}{W_{F_{applied}}} + \underset{\substack{-ve \\ always}}{W_{fk}} = \Delta K + \Delta U \quad \text{--- Eq (2)}}$$

This relationship is a generalization of Eq (1) for the work-energy theorem.

(i) Eq (2) is the only equation you need to begin an energy approach to a problem solution: recall lecture 7 - page 6.

(ii) When applying Eq (2), you must select two times: a beginning and an end.

(iii) In most cases, Eq (2) reduces to a simpler one because some of the terms are zero for the specific situation.

(iv) If you analyze a system and you set all the terms on the left side of Eq (2) to zero because they do not apply to the system, then the system is an isolated system and Eq (2) reduces to the case we encountered in lecture 7 : page 9 and 10 – eq (6) : $K_i + U_i = K_f + U_f$.

By definition, an isolated system is a system of objects that interact and exert forces on one another but for which no external force causes energy changes within the system $\Rightarrow$ $\Delta E = 0$

(v) If an external force is applied to a non-isolated system and the only change in the system is in its speed, then Eq (2) reduces to Eq (3) – page 5 – lecture 7 :

$$W_{F_{applied}} = \Delta K .$$

(vi). If you analyze a block that has been set into motion on a rough incline, then Eq (2) has to be written as $[W_{F_{applied}} = 0] + W_{f_K} = \Delta K + \Delta U$; because there is a change in both the kinetic energy and the potential energy of the block.

$\Rightarrow$ **The bottom line is that you are no longer a dentist neither a cardiologist while approaching energy problem! Rather, you have to be an excellent tailor!!**

To sum up, the equality in Eq(2):

$$W_F + W_{fk} = \Delta E$$

assures that the total energy E is conserved — that is, stays constant in time — even in the presence of friction force.

This is the most important point in this chapter:

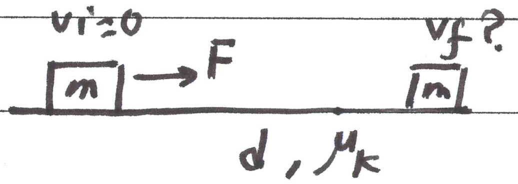
"The total energy E is always conserved in an isolated system. It can neither be created nor be annihilated [item (iv)]. This central feature of energy has been tested in countless experiments, and no experiment has ever shown this statement to be incorrect. Therefore, if the total energy of the system changes (i.e. $\Delta E \neq 0$), it can only be because energy has been transferred to/or from the system under the influence of the external force ($W_{F_{applied}}$) and/or the influence of the friction force (W_{fk})."

- A possibe scenario: $W_{F_{applied}} = +10\,J$;

$$\left\{\begin{array}{l} W_{fk} = -3\,J \\ \Delta K = +6\,J \\ \Delta U = +1\,J \end{array}\right. \Longrightarrow \underbrace{+10+(-3)}_{J} \overset{?}{=} \underbrace{+6+1}_{J}$$

- Recall the example of the block sliding on a frictionless surface [see page 6 - lecture 7].

A 6-kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of magnitude 12N. Find the block's speed after it has moved 3 m if the surface has a coefficient of kinetic friction of 0.15.



Write the appropriate reduction of Eq (2): this is the task of the "tailor"!

$$W_F + W_{f_k} = \Delta K + \Delta U$$

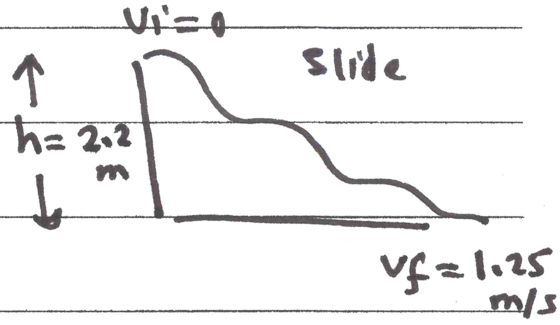
$$+Fd + -\mu_k mgd = \frac{m}{2}v_f^2 + 0 \quad \text{——— } (*)$$

$$\therefore v_f = \sqrt{2d\left(\frac{F}{m} - \mu_k g\right)}$$

Plug in the numerical values $\Rightarrow v_f = 1.8$ m/s.

- As expected, this value is less than the 3.5 m/s found in the example of lecture 7.
- Please tackle the problem using Newton's 2nd law as we did in lecture 7.
- Do Problem 6.63 : $F(d+\bar{d}) - \mu_k mg\bar{d} = \frac{m}{2}v_f^2$ ——— $(*)$
  $d = 11$m, $\bar{d} = 10$m.

- **Problem 6.41:**



m slides $\rightarrow v_i = 0,\ v_f = 1.25\,m/s$

height of the slide = 2.2 m

How much thermal energy is produced in this event (due to friction)?

Before solving the problem, we should convince ourselves about the presence of the friction force in the first place.

$\Rightarrow$ with $v_i = 0$ and $h = 2.2\,m \rightarrow$ For a free fall:

$v_f = \sqrt{2gh} = 6.6\,m/s \Rightarrow$ Thus as v_f is given to be 1.25 m/s then the change in K is due to f_K ✓.

X

The tailored form of Eq (2) is: item (vi) page 4.

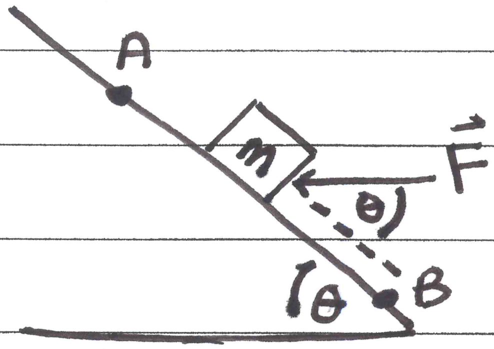
$$0 + W_{f_K} = \Delta K + \Delta U$$

$$\therefore W_{f_K} = +\frac{m v_f^2}{2} - mgh = -ve\ J \text{ as expected.}$$

- Do problems 6.43, 6.45.
- If the length of the slide is given in problem 6.41, can you estimate the magnitude of the friction force exerted on m?

Example:

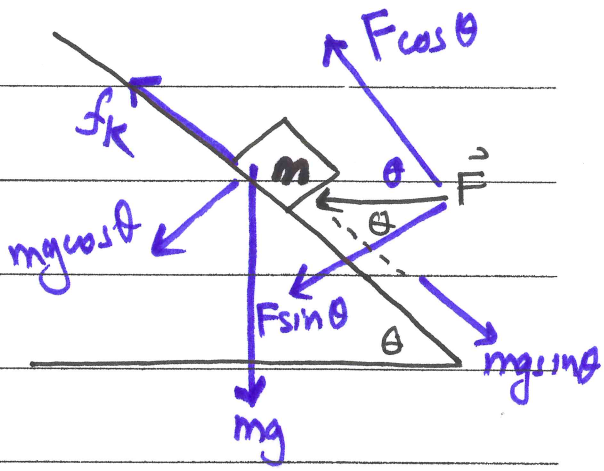
The block m is sliding down the incline from A to B.
$\vec{F}$ is applied between A and B.
K_A and K_B are given.



How much work is done on the block by f_k between A and B?

Given: $m = 4kg$, $\theta = 37°$, distance $(A \to B) = 5m$, $F = 10N$,
(d)
$K_A = 10J$, $K_B = 20J$.

As a tailor, you realize that Eq (2) fits perfectly this time!



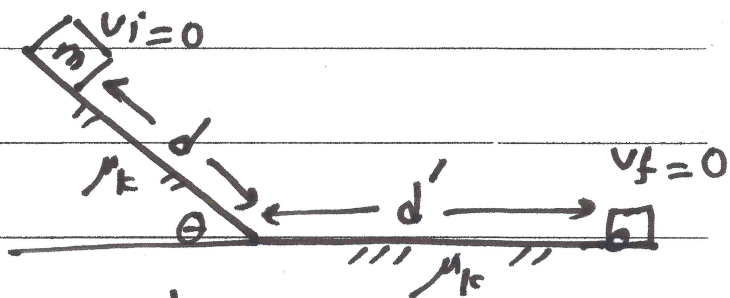
$$W_{f_k} = \Delta KE + \Delta U - W_F$$

W_{f_k} = must be negative!

$$\therefore W_{f_k} = (K_B - K_A) - mgd\sin\theta - (-F\cos\theta * d)$$
$$= -68\ J.$$

notice that the work done by $F = W_F$ is negative.

• **Example:** A box starts from rest and slides down a hill. The hill's slope makes an angle θ with the horizontal. The surface of the hill is (d) m long. When the box reaches the bottom of the hill, it continues sliding on a horizontal level. The coefficient of kinetic friction between the box and both hill and level is μ_k. How far does the box move on the horizontal level (d') before stopping? Take m of the box = 23 kg, $\theta = 35^\circ$, $d = 25\,m$, $\mu_k = 0.1$



$$W_F + W_{f_k} = \Delta K + \Delta U$$

$$0 + W_{f_k} = 0 + \Delta U$$

$$-\mu_k mg\cos\theta * d - \mu_k mg * d' = -mgh \quad , \quad h = d\sin\theta$$

$$\therefore \; d' = d\left[\frac{\sin\theta - \mu_k\cos\theta}{\mu_k}\right]$$

• notice the canceling out of mg on both sides!

• putting in the numerical values

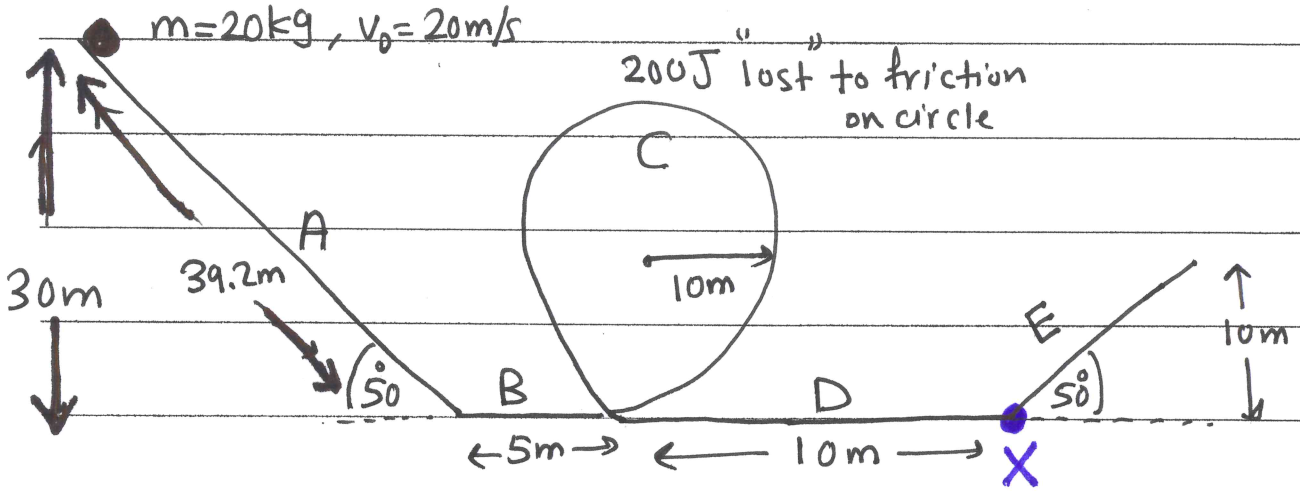
$$d' = 122.9\ m.$$

• validate: dimensional analysis:

$$d' = [\underset{\downarrow}{*}]\, d$$

pure number

■ Example: The bead shown in the figure leaves the starting point with a speed of 20m/s. Use the work-energy theorem to calculate the bead's speed at the end of the track or the maximum height it reaches if it stops before reaching the end.



Take $\mu_k = 0.25$ for all the parts of the track ⇒ from A → E .

⇒ As the bead reaches point X, it has certain amount of kinetic energy K_X. In order for the bead to reach the end of the incline E (i.e. h = 10m), it needs to have enough K_X to overcome the work due to friction along the incline W_E as well as the potential energy at the top of the incline U_E.

Therefore,

(i) if $K_x > W_E + U_E$, then the bead reaches the top and the speed can be determined as usual:

$$W_E = \Delta U + \Delta K \Rightarrow W_E = (U_E - 0) + (K_E - K_x) \Rightarrow$$

$\therefore K_E = K_x - U_E + W_E$ —— (i) do not forget that W_E is -ve!

(ii) if $K_x < W_E + U_E$, then it stops before reaching the top (i.e $h = 10m$) and the height the bead reaches can be determined by:

$W_{E^*} = \Delta U + \Delta K$, $\Delta U = mgh$, h = maximum height $< 10m$

and W_{E^*} is the work due to friction for the section of the incline up to h, $\Delta K = 0 - K_x$.

$\Rightarrow$ let's calculate the energy K_x and the "threshold" $(W_E + U_E)$ and see if K_x exceeds it or not

• $K_x \Rightarrow W_{A\to D} = \Delta K + \Delta U = (K_x - K_i) + (0 - U_i)$

$$W_{A\to D} = -\mu_k mg\cos\theta * A - \mu_k mg B - 200 - \mu_k mg D = -2169.6J$$

$$\therefore K_x = K_i + U_i + W_{A\to D} = 9880 - 2169.6 = +7710.4 J$$

- $W_E + U_E$ = as a threshold $\Rightarrow$

$|W_E| = \mu_k mg \cos\theta * (E)$, E is the length of the incline E

$\Rightarrow E = \frac{h}{\sin\theta} \Rightarrow |W_E| = \mu_k mg * \cot\theta * h = 411.2\,J$

$U_E = mgh = 1960\,J$

$\therefore |W_E| + U_E = 2371.2\,J$

Therefore, since $K_x > W_E + U_E$, the bead will reach the top of the track E, and the speed is calculated using eq (i) page 11:

$K_E = K_x - U_E + W_E$ watch out for +ve!!

$K_E = +7710.4 - 1960 - 411.2 = +5339.2\,J$

$\therefore V_{top} = \sqrt{\frac{2K_E}{m}} = 23.1\ m/s.$

- Validation: The fact that the bead reaches the top of the 2nd ramp (E) is reasonable given how much higher the 1st ramp (A) is than the 2nd.

- Can you play with the numerical values given so you assure that $K_x < W_E + U_E$ and thus calculate the maximum height h? try and enjoy!!

* Last but not least, section 6.2 and it's relevant subject in section 6.4 (elastic potential energy) are excluded from PHY 105 syllabus!

$\boxed{\text{You are expected to know the following example though!}}$

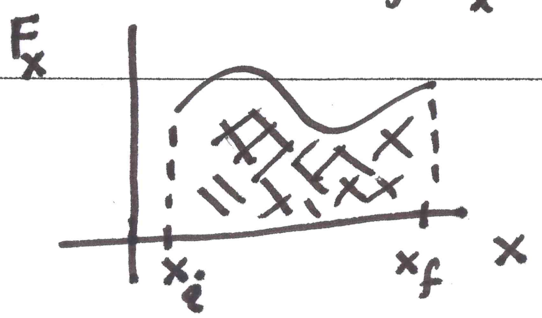
⇒ So far, we assumed that the force acting on an object is constant, thus the work done by it is simply $|\vec{F}| * |\vec{d}| \cos\theta$.

Suppose the force is not constant. What is the work done by such a force? In a case of motion in one dimension with a variable x-component of force, $F_x(x)$, the work i

$$W = \int_{x_i}^{x_f} F_x(x')\,dx'$$

notice that the integrand has x' as a dummy variable to distinguish it from the integral limits.

This equation (integral) shows that the work W is the area under the curve of $F_x(x)$.

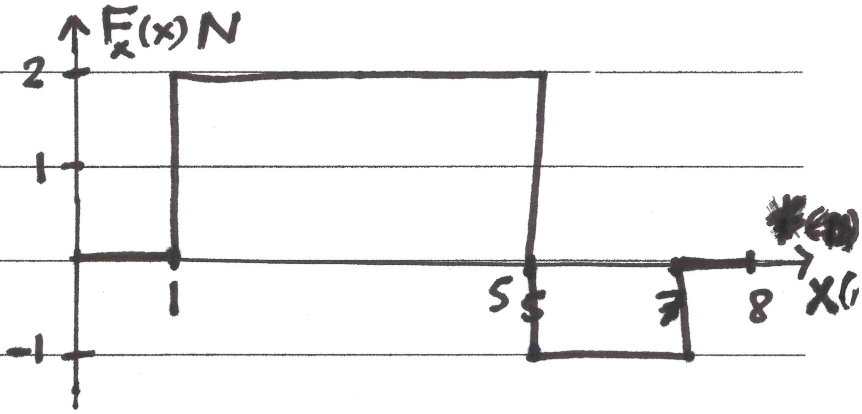


- PHY 105 "@JU" is algebra based physics not calculus based physics. That being said, $\boxed{\text{you are expected to calculate the work done by } F(x) \text{ using the area under the curve of } F(x).}$ Do not panic! We will never ask you to do integrals!

- ~~Example~~: The graph shows $F_x(x)$ that acts on a 2 kg block as it moves along a horizontal surface.

(i) Find the net work done on the block

(ii) Find the final speed of the block if it starts from rest at $t=0$.



(i) $W_{0\to 8s} = +8+(-2)$

$= +6\ J$

(ii) Work-energy theorem is valid when the force is variable.

$$W = \Delta K \rightarrow v_f = \sqrt{\frac{2W}{m}} = \sqrt{6}\ m s^{-1}$$

- ~~Do~~ problem 6.13.