1^{st}-exam : answer key

1. Given: $v(0)=0$, Δx during Δt: $t_i=1 \rightarrow t_f=2$, $\Delta t = 1 sec \rightarrow \vec{a}$?

$\vec{\Delta x} = \vec{v}_i t + \frac{\vec{a}}{2} t^2 \Rightarrow \vec{\Delta x} = \vec{v}(1) * 1 + \frac{\vec{a}}{2} * 1^2$.

$\vec{v}(1) = \vec{v}(0) + \vec{a} * (1-0) \rightarrow$ " $\vec{v}(1) = \vec{a}$ " $\rightarrow$ solve for a, $\vec{a} = \frac{2}{3} \vec{\Delta x}$.

2. Given: $\vec{\Delta x}$ during Δt and $\vec{v_f}$, $\rightarrow \vec{a}$?

Note that $\vec{v_i}$ is unknown, therefore, one can't directly get $\vec{a}$ from:

$\vec{v_f} = \vec{v_i} + \vec{a} t$ — ① nor from $\vec{\Delta x} = \vec{v_i} t + \vec{a} t^2/2$ — ②. Try:

$\vec{\Delta x} = \left(\frac{\vec{v_i} + \vec{v_f}}{2}\right) t$ and solve for $\vec{v_i} = \frac{2\vec{\Delta x}}{t} - \vec{v_f}$, and then

plug $\vec{v_i}$ into either of eq ① or ②.

* Note that if you get $\vec{v_i}$ from ① and plug it into ②, then you get the following kinematic eq $\vec{\Delta x} = \vec{v_f} t - \frac{\vec{a} t^2}{2}$ — ③.

Note the difference between eq ③ and eq ①. One can directly get $\vec{v_f}$ using eq (3)! Can you interpret the area under the curve of $v_x(t)$?

3. Given: m_1 dropped from h_1 and took t_1
Given: m_2 " " h_2 " " t_2
ratio of $\frac{h_2}{h_1}$ or $\frac{t_2}{t_1}$?

- With constant acceleration, the position Δh is always a quadratic function in t.

$h = \cancel{v_i t} - g t^2/2 \Rightarrow \frac{h_2}{h_1} = \left(\frac{t_2}{t_1}\right)^2$

- $v_f^2 = v_i^2 + 2\vec{g}\,\vec{\Delta h} \Rightarrow$ with same $|\vec{v_i}|$ (scalar) and same $\vec{\Delta h}$ (vector) $\rightarrow \vec{v_f}$ is the same.

* Note that the results are mass independent!

$\boxed{4}$ A book hangs from a wire in an elevator: given m and $T \Rightarrow \vec{a}$?

$T = mg\left(1 + \frac{\vec{a}}{g}\right)$, if $T > mg \Rightarrow \vec{a}$ must be +ve .

, if $T < mg \Rightarrow \vec{a}$ must be -ve .

$\boxed{5}$ $m_1 \xleftarrow{} F \Rightarrow a_1$, $m_2 \xleftarrow{} F \Rightarrow a_2 \Rightarrow \therefore$ $m_1 = \frac{F}{a_1}$, $m_2 = \frac{F}{a_2} \rightarrow$

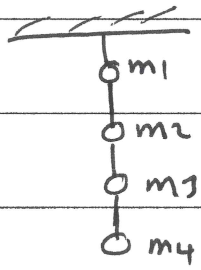
$m_2 - m_1 \leftarrow F \rightarrow a$? $a = \frac{F}{m_2 - m_1} = \frac{F}{\frac{F}{a_2} - \frac{F}{a_1}} = \frac{a_1 a_2}{a_1 - a_2}$

note : $a_{m_1} > a_{m_2 - m_1} > a_{m_2} > a_{m_1 + m_2}$.

$\boxed{6}$ The block slides down the hill with an acceleration directed up the hill

If we take +ve down the hill $\Rightarrow +mg\sin\theta - \mu_k mg\cos\theta = -ma$

$\therefore \mu_k = \frac{\sin\theta + (a/g)}{\cos\theta}$.



$\boxed{7}$ The 4 masses hang from wires.

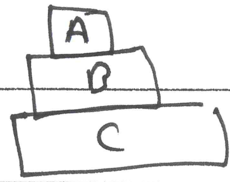
$\Sigma\vec{F}$ in the vertical direction = zero.

$\Rightarrow T$ in the wire between m1 and m2 is

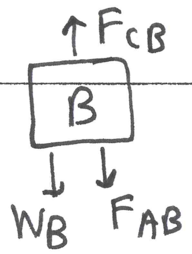
equal to weight of m2, m3 and m4 $\Rightarrow (m_2 + m_3 + m_4)g$.

~~8~~ $\boxed{m} \longrightarrow F$: $v_i = 0$, $|\Delta\vec{x}| = d$, $t \longrightarrow F$?

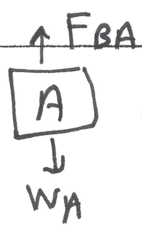
$F = ma$, $d = v_i t + \frac{at^2}{2} \longrightarrow F = \frac{2md}{t^2}$.



$\boxed{9}$ The 3 blocks are in equilibrium: F_{CB}?

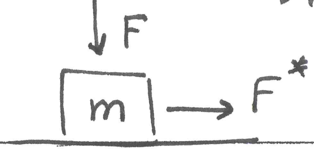


$\therefore F_{CB} = W_B + F_{AB}$, $\Rightarrow$
?



$\Rightarrow F_{BA} (= F_{AB}) = W_A$

$\therefore F_{CB} = W_B + W_A$. $\Rightarrow$ does it agree with your intuition?



10 m is on the verge of sliding:

$\Rightarrow F^* \equiv$ threshold value $= f_{s_{max}} = \mu_s (F + mg)$.

11 m is moving at constant velocity $\Rightarrow \sum \vec{F} = 0 \rightarrow F^* = f_k$.

12 $\boxed{m} \rightarrow v_i$ $\qquad v_f = 0 \qquad \mu_k$?

$\longleftarrow d \longrightarrow$

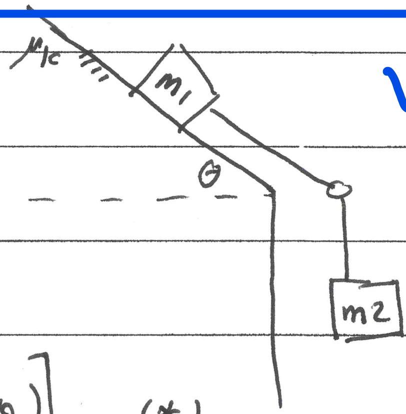
$\sum \vec{F} = -f_k = ma \rightarrow -\mu_k mg = ma \rightarrow \mu_k = -a/g$ (watch out the -ve)

$\cancel{v_f^2 = v_i^2 + 2\vec{a}\Delta\vec{x} \Rightarrow 0 = v_i^2 + 2\vec{a}(d) \Rightarrow a = -\frac{v_i^2}{2d}}$ $\Rightarrow$

$\cancel{\mu_k = +\frac{v_i^2}{2dg}}$, ~~note that v_i (# was given ✗) = 9.8 = |g|~~.
with a value of g

$\therefore \mu_k = g/2d. \Rightarrow$ $\boxed{\text{Do it using work-energy approach!}}$ ✓

13 Given: m_1, μ_k, θ and $m_2 \rightarrow a$?



Let's assume the flow tube:

$m_2 g - T = m_2 a$, and

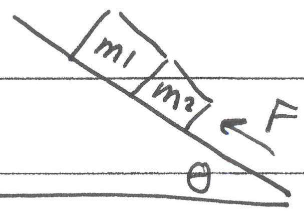
$T + m_1 g \sin\theta - \mu_k m_1 g \cos\theta = m_1 a$

Solve for $a = g \left[\frac{m_2 + m_1(\sin\theta - \mu_k \cos\theta)}{m_1 + m_2} \right]$ —— (*)

In the exam, $m_1 = 2m_2 \Rightarrow$ plug in and get the answer!

What does ~~of~~ (*) reduce to if $\theta = \mu_k = 0$? Does the algebraic expression in such case agree with your hunch?

$\boxed{14}$ Given F, θ, m_1 and m_2. (smooth incline).



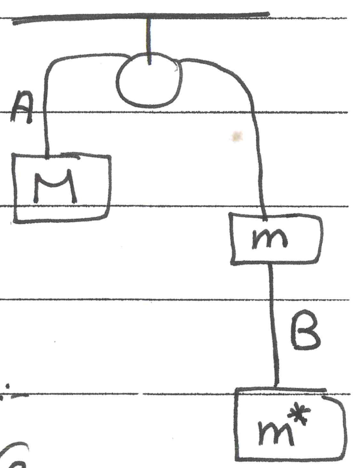
Find F_{21} : by 2 on 1

$F - (m_1+m_2)g\sin\theta = (m_1+m_2)a$ — system.

now: for m_1: $F_{21} - m_1 g\sin\theta = m_1 a \therefore F_{21} = m_1[g\sin\theta + a]$.

calculate a from the system's eq and plug it in. ✓

$\boxed{15}$ For the assembly shown,



m, m^* and T_B are given → T_A?

Let's assume the flow to be ↷

$T_A - Mg = Ma$ — ①

$T_B + mg - T_A = ma$ — ②

> 3 unknowns! ⇒ look for the 3rd eq.

$\Rightarrow m^*g - T_B = m^*a \Rightarrow a = (m^*g - T_B)/m^*$ — ③

plug a into ② [why not into ①?] ⇒ run through a little algebra, if you don't mind!!

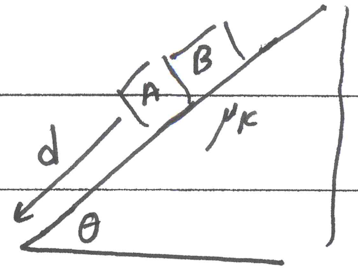
$$T_A = T_B\left[1 + \frac{m}{m^*}\right].$$

- Note that if $\underline{\underline{a}}$ is found to be -ve, this means that the actual flow of the system is opposite to our assumption; no big deal!
- Can you get M?

$\boxed{16}$ Given: m_A, m_B, μ_k and d.

Both blocks started from rest.

$\Rightarrow t$ needed to move d?



$$(m_A+m_B)g\sin\theta - \mu_k(m_A+m_B)g\cos\theta = (m_A+m_B)a$$ mass cancelled out

$$\Rightarrow a = g[\sin\theta - \mu_k\cos\theta] \Rightarrow \cancel{\text{kinematics: } d = v_0 t + at^2/2}$$

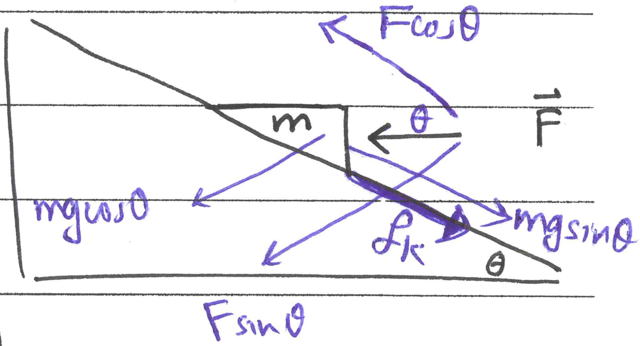
$$\therefore \cancel{t = \sqrt{\frac{2d}{a}}}.$$

Try solving the problem using work-energy approach!

$\boxed{17}$ Given: $|F|, m, \mu_k$ and $\theta \rightarrow a$?

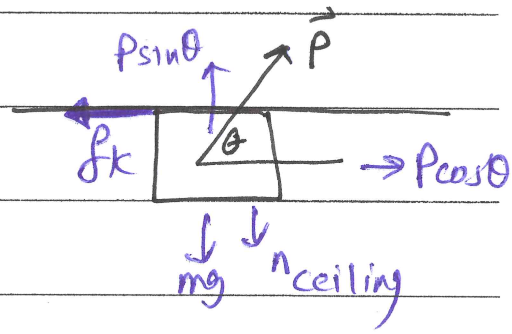
$$F\cos\theta - mg\sin\theta - \mu_k[F\sin\theta + mg\cos\theta] = ma$$

solve for a:



$$a = \frac{F}{m}[\cos\theta - \mu_k\sin\theta] - g[\sin\theta + \mu_k\cos\theta].$$

$\boxed{18}$ Given: $|\vec{P}|, \theta, \mu_k$ and $\theta \rightarrow a$?

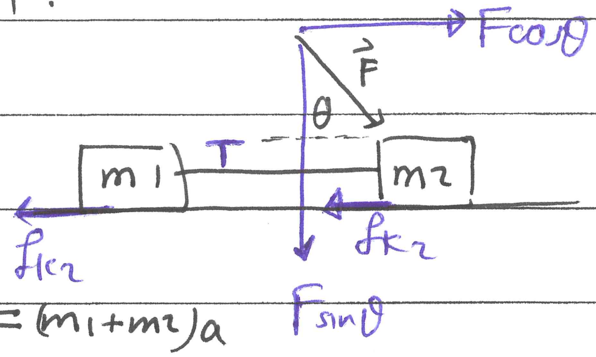


$$P\cos\theta - \mu_k[P\sin\theta - mg] = ma$$

solve for a:

$$a = \frac{P}{m}[\cos\theta - \mu_k\sin\theta] + \mu_k g.$$

$\boxed{19}$ Given: $|\vec{F}|$, θ, μ_k, m_1 and m_2 $\rightarrow$ T?



for the system:-

$$F\cos\theta - \mu_k m_1 g - \mu_k (m_2 g + F\sin\theta) = (m_1+m_2)a$$

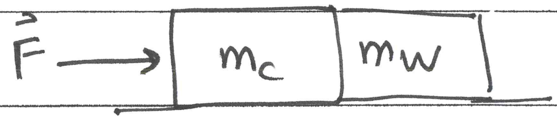
solve for a, then

for m_1, for instance, $T - \mu_k m_1 g = m_1 a$ $\rightarrow$ plug a in and solve for T.

If you run through a little algebra, you can have the expression for T:

$$T = \left[\frac{m_1(\cos\theta - \mu_k \sin\theta)}{m_1+m_2}\right]F.$$

$\boxed{20}$ Given: $|\vec{F}|$, f_{kc}, f_{kw}, m_c and m_w $\rightarrow$ F_{cw}?



$\Rightarrow$ as a system: $F - (f_{kc}+f_{kw}) = (m_c+m_w)a.$

$$\therefore\ a = \frac{F-(f_{kc}+f_{kw})}{(m_c+m_w)}.$$

for m_w $\rightarrow$ $F_{cw} - f_{kw} = m_w a$

$$\therefore\ F_{cw} = f_{kw} + \left(\frac{m_w}{m_c+m_w}\right)\left[F-(f_{kc}+f_{kw})\right].$$