## SETTING A BROKEN LEG WITH TRACTION

## EXAMPLE

A traction device employing three pulleys is applied to a broken leg, as shown in the sketch. The middle pulley is attached to the sole of the foot, and a mass $m$ supplies the tension in the ropes. Find the value of the mass $m$ if the force exerted on the sole of the foot by the middle pulley is to be 165 N .


We begin by noting that the rope supports the hanging mass $m$. As a result, the tension in the rope, $T$, must be equal in magnitude to the weight of the mass: $T=m g$.
Next, the pulleys simply change the direction of the tension without changing its magnitude. Therefore, the net force exerted on the sole of the foot by the middle pulley is the sum of the tension $T$ at $40.0^{\circ}$ above the horizontal plus the tension $T$ at $40.0^{\circ}$ below the horizontal. We will calculate the net force component by component.

1. First, consider the tension that acts upward and to the right on the middle pulley.

Resolve this tension into $x$ and $y$ components:
$\mathrm{T}_{1, \mathrm{x}}=\mathrm{T} \cos 40.0^{\circ}$
$\mathrm{T}_{1, \mathrm{y}}=\mathrm{T} \sin 40.0^{\circ}$
2. Next, consider the tension that acts downward and to the right on the middle pulley. Resolve this tension into $x$ and $y$ components. Notice the minus sign in the $y$ component:
$\mathrm{T}_{2, x}=\mathrm{T} \cos 40.0^{\circ} \quad \mathrm{T}_{2, y}=-\mathrm{T} \sin 40.0^{\circ}$
3. Sum the $x$ and $y$ components of the force acting on the middle pulley. We see that the net force acts only in the $x$ direction, as one might expect from symmetry:
$\Sigma \mathrm{F}_{\mathrm{x}}=\mathrm{T} \cos 40.0^{\circ}+\mathrm{T} \cos 40.0^{\circ}=2 \mathrm{~T} \cos 40.0^{\circ}$
$\sum F_{y}=T \sin 40.0^{\circ}-\mathrm{T} \sin 40.0^{\circ}=0$
4. Step 3 shows that the net force acting on the middle pulley is $2 T \cos 40.0^{\circ}$.

Set this force equal to 165 N and solve for $T$ :
2T $\cos 40.0^{\circ}=165 \mathrm{~N}$
$\mathrm{T}=165 \mathrm{~N} /\left[2 \cos 40.0^{\circ}\right]=108 \mathrm{~N}$
5. Solve for the mass, $m$, using $T=m g$ :
$m=T / \mathrm{g}=108 \mathrm{~N} / 9.81 \mathrm{~m} / \mathrm{s}^{2}=11.0 \mathrm{~kg}$

## INSIGHT

This pulley arrangement "magnifies the force" in the sense that a 108-N weight attached to the rope produces a $165-\mathrm{N}$ force exerted on the foot by the middle pulley. Notice that the tension in the rope always has the same value-T = 108 N -as expected with ideal pulleys, but because of the arrangement of the pulleys the force applied to the foot by the rope is $2 T \cos 40.0^{\circ}>T$. The force exerted on the foot by the middle pulley produces an opposing force in the leg that acts in the direction of the head (a cephalad force), as desired to set a broken leg and keep it straight as it heals.

## PRACTICE PROBLEM

(a) Would the required mass $m$ increase or decrease if the angles in this device were changed from $40.0^{\circ}$ to $30.0^{\circ}$ ?
(b) Find the mass $m$ for an angle of $30.0^{\circ}$.
[Answer: (a) $m$ would decrease. (b) 9.71 kg ]

