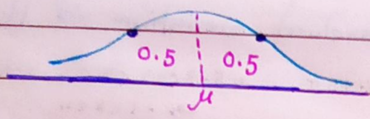
Chapter 5

* Normal Distribution

lecture "19"

* The normal distribution :

$X \sim N(\mu, \sigma^2)$

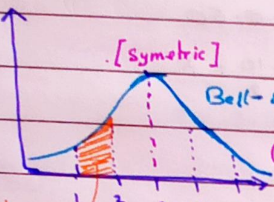


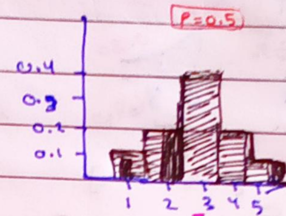
μ → ; ← P.d.f

x	1	2	3	4	5
$P(X=x)$	0.1	0.2	0.4	0.2	0.1

* $\sum_x P(X=x) = 1$

* $0 \le P(X=x) \le 1$





Integration
Area لإيجاد
$= P(1 \le x \le 2)$
المساواه مابتفرق

*Total Area Under Bell-shaped equal "1"

$\sum P(X=x) = 1$

* normal رح يذكرنا احنا ... Normal Distribution اذا كان السؤال عن

* خصائصه → ① Symetric about the mean μ.

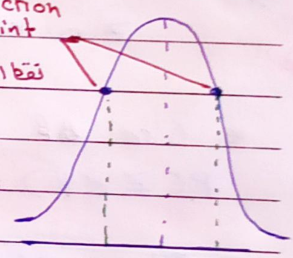
② μ = mode = median (Q_2)

③ Total Area Under the curve = 1

④ Probability = Area : percentage

⑤ $P(X=k) = 0$ & $P(X \le k) = P(X < k)$

⑥ Inflection point
نقط الانعطاف

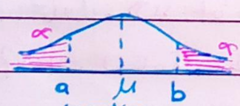


في هذا الدرس

Note 4: المساحة بين $\underset{z=-1}{\mu-\sigma}$ و $\underset{z=1}{\mu+\sigma}$: 0.68 & 0.95 = المساحة بين $\underset{z=-2}{\mu-2\sigma}$ و $\underset{z=2}{\mu+2\sigma}$

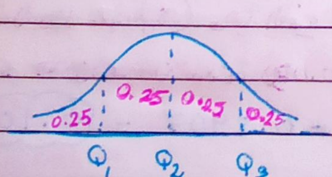
99.7% = المساحة بين $\underset{z=-3}{\mu-3\sigma}$ و $\underset{z=3}{\mu+3\sigma}$

Note ①



If $P(X \le a) = P(X \ge b) = \alpha$, then $\mu = \frac{a+b}{2}$

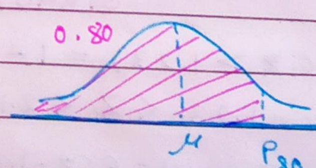
②



$P(X \le Q_1) = 0.25$

$P(X \le Q_3) = 0.75$

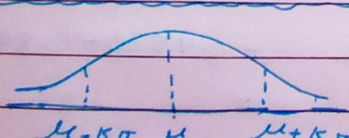
③ مهمة جداً



$P(X \le P_{80}) = 80\%$

$P(X \le P_k) = K\%$

④



The value of X that is k standard ~~deviations above~~ → the mean

below the mean

If $x \sim n(\mu, 4)$ → $\sigma^2 = 4$, $\sigma = 2$, $P(X \le 2) = 0.2$ & $P(X \ge 8) = 0.2$

Find:- i) $P(2 < X < 8)$ ii) The value of μ iii) The 20^{th} percentile

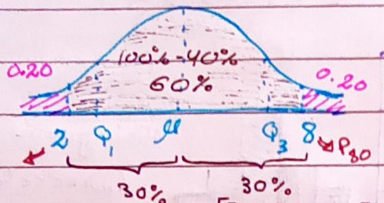
iv) the 80^{th} percentile

استخدم الرسم لتسهيل الفهم وحل السؤال.

v) The value of X that are within 2 standard deviations of the mean

vi) $P(\mu < X < 8) = 0.30$

vii) $\mu - 3\sigma < x < \mu + 3\sigma$ → $5-3(2) < x < 5+3(2)$ → $-1 < x < 11$



i) $P(2 < X < 8) = 0.60$

ii) $\mu = \frac{2+8}{2} = \frac{10}{2} = 5$

iii) $P_{20} = 2$

iv) $P_{80} = P_{100\%+20\%} = 8$

v) $\mu + 2\sigma = 5 + 4 = 9$ & $\mu - 2\sigma = 5 - 4 = 1$

(أقل من μ بـ 2σ) $1 < x < 9$ (أكثر من μ بـ 2σ)

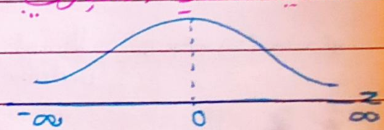
5.2

حالة خاصة من التوزيع الطبيعي

* The standard normal distribution

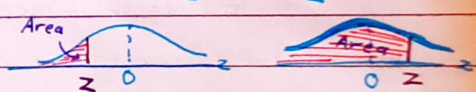
التوزيع الطبيعي المعياري

$Z \sim n(0, 1)$



* Use of tables

$P(Z \le k)$ = from the table.



بالجدول قيم Z السالبة

بالجدول قيم Z الموجبة

Z	.00	.01	0.2	0.3
0.0				
0.1			0.5478	
0.2				
0.3				
0.4				
⋮				
3.4				

* ضروري نعرف لاحقًا كيف نحول $\mu = 0$ & $\sigma^2 = 1$

حتى نقدر نستخدم الجدول. لا يمكن وضع جداول لكل حالة، لذا يتم تحويله الى توزيع طبيعي معياري، ثم إيجاد المساحة المطلوبة من الجدول.

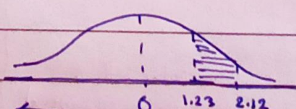
eg) If $Z \sim n(0, 1)$, then Find:

i) $P(Z \le 1.23) = 0.8907$ نأخذها من الجدول مباشرة) وهي القيمة الواقعة في الصف 1.2 مع العمود الاجزاء من مئة (0.03) التقاطع هو الجواب)) 0.8907

ii) $P(Z > 1.23) = 1 - P(Z \le 1.23) = 1 - 0.8907 = \boxed{0.1093}$

iii) $P(1.23 < Z < 2.12)$

$= P(Z < 2.12) - P(Z - 1.23)$



$= 0.9830 - 0.8907$

=

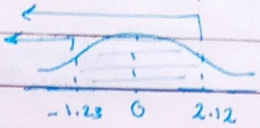
من الصف 2.1 من العمود 0.02

من الصف 1.2 والعمود 0.03

iv) $P(-1.23 < Z < 2.12)$

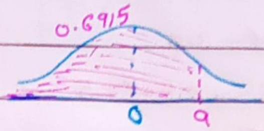
$P(Z \leq 2.12) - P(Z \leq -1.23)$

$0.9830 - 0.1093 = \ldots\ldots$



من الصف 1.2- والعمود 0.03 (جزء من مئة)

v) find a if $P(Z \leq a) = 0.6915$



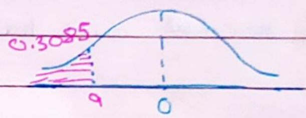
الهدف من الرسم تحديد موقع a ومعرفة اشارتها (±)

لأنها أكبر من 0.50 لذلك a تكون موجبة ⇐ ندخل الجدول ونبحث عن ناتجه من تقاطع تمثل مع تمت

Z	.00
0.0	⋮
0.1	⋮
0.2	⋮
0.3	⋮
0.4	⋮
0.5	$\boxed{.6915}$

$a = 0.50 = \boxed{0.5}$

vi) find a if $(Z < a) = 0.3085$



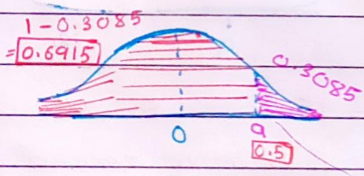
Z	.00
⋮	⋮
-0.5	$\boxed{.3085}$

$a = -0.50 = \boxed{-0.5}$

vii) a if $P(Z > a) = 0.3085$

$P(Z < a) = 0.6915$

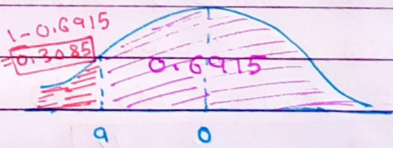
من الجدول نجد ان $\Rightarrow$ $a = 0.50 = \boxed{0.5}$



viii) a if $P(Z > a) = 0.6915$

$P(Z < a) = 0.3085$

من الجدول نجدها $\rightarrow$ $a = -0.5$



ix) a if $P(-a < Z < a) = 0.383$

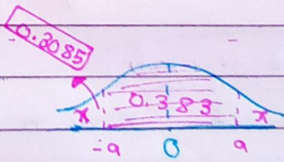
من الجدول $\leftarrow$ $P(Z < -a) = 0.3085$

او من خلال هذه مما الجدول $\leftarrow$ $P(Z < a) = 0.3085 + 0.383$

$= 0.6915$

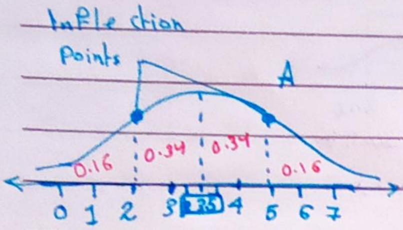
$-a = -0.5$ نجد ان

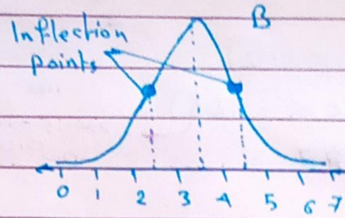
$\Rightarrow \boxed{a = 0.5}$



$2x + 0.383 = 1$

$x = \frac{1 - 0.383}{2} = 0.3085$





لاحظ أن A أكثر تشتت من B من الشكل لذلك أعلى σ

Mean: $\mu = 3.5$

$\mu + \sigma = 5 \rightarrow 3.5 + \sigma = 5$

$\rightarrow \boxed{\sigma = 1.5}$

Mean: $\mu = 3.5$

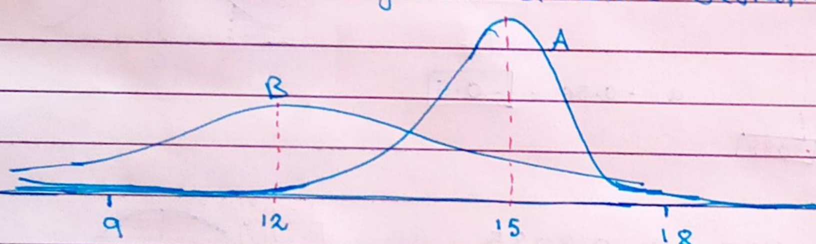
$\sigma + \mu = 4.5$

$\sigma + 3.5 = 4.5 \rightarrow \boxed{\sigma = 1}$

EXAMPLE 1 Understanding Mean and Standard Deviation:

① Which normal curve has a greater mean?

② Which normal cure has a greater standard deviation.



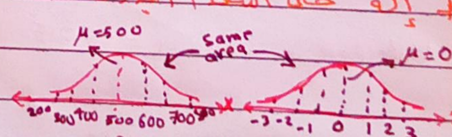
A: $\mu = 15$ B: $\mu = 12$, so curve A has a greater mean.

Curve B is more spread out than A, so, curve B has a greater standard deviation.

الآن خلانا ع standard normal distribution لأن من الجدول مباشرة الآن لازم أحول لأنه ما بقدر استعمل الجدول.

If $x \sim n(\mu, \sigma^2)$, then

$$\boxed{Z = \frac{x - \mu}{\sigma}} \sim n(0, 1)$$



$\Rightarrow x = \mu + z\sigma$

* If $X = \mu + K\sigma \rightarrow Z = K$

eg: If $x \sim n(\overset{\mu}{5}, \overset{\sigma^2}{4})$, then find

* If $K = \mu - K\sigma \rightarrow Z = -K$

i) $P(X \leq 6)$

$Z = \frac{6-5}{2} = \frac{1}{2} = 0.5$ $\therefore P\left(\frac{x-5}{2} \leq \frac{6-5}{2}\right)$

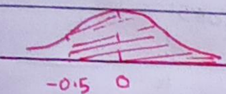
$\rightarrow P(Z \leq 0.5) = 0.6915$: the cumulative area to a z-score of 0.5

Note

* Find the Cumulative Area: هي نعطيا المساحة على يسار الرقم

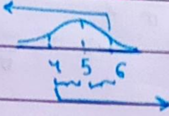
ii) $P(X > 4) = P\left(Z > \frac{4-5}{2}\right) = P(Z > -0.5) = P(Z < 0.5)$

أو من خلال ←



$= 1 - P(Z \le -0.5) \rightarrow 1 - 0.3085 = \boxed{0.6915}$

$P(X \le 6) = P(X > 4)$



لاحظ أنه

لانا 4 و 6 تبعدان نفس البعد عن $\mu = 5$

iii) $P(4 < X < 6) = P\left(\frac{4-5}{2} < \frac{X-5}{2} < \frac{6-5}{2}\right)$

$\rightarrow P(-0.5 < Z < 0.5) = P(Z \le 0.5) - P(Z \le -0.5)$ → او من خلال الرسم نعرفها

الكبير — الصغير

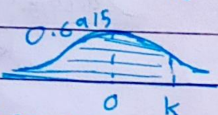
$0.6915 - 0.3085 = 0.383$

iv) a if $P(X \le a) = 0.6915$

$P\left(Z \le \frac{a-5}{2}\right) = 0.6915$

* منحول الى Z

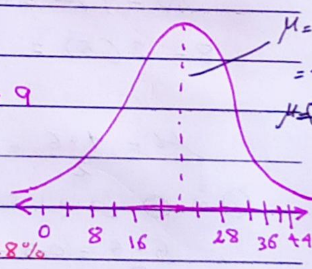
سنفرض أن $\boxed{\frac{a-5}{2} = k}$



موجبه لان المساحة الأقل منها أكبر من 50%

$k = 0.5 \Rightarrow \frac{a-5}{2} = 0.5 \rightarrow a - 5 = 1 \rightarrow \boxed{a = 6}$

eg) Find the probability that the student works for less than 4 hours per week, $\sigma = 9$



sol) The figure shows a normal curve with $\mu = 22$

$\mu = \frac{16+28}{2} = \frac{44}{2}$, $\mu = 22$

$z = \frac{x-\mu}{\sigma} = \frac{4-22}{9} = \boxed{-2}$

$P(X < 4) = P(Z < -2) = 0.0228 \times 100\% = 2.28\%$

2.28% is less than 5% this is unusual event

((المثال من الكتاب))

وارجع للكتاب للمثال الذي يليه

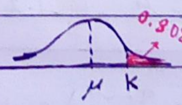
eg) $X \sim n(\mu, 4)$ & $P(X \ge 6) = 0.3085$. Find μ

sol) $P\left(Z \ge \frac{6-\mu}{2}\right) = 0.3085$

نفرض ان $\frac{6-\mu}{2} = k$

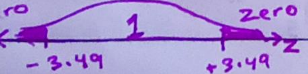
$P(Z \ge k) = 0.3085 \rightarrow P(Z < k) = 1 - 0.3085$

$\boxed{k = 0.5}$ من الجدول $P(Z < k) = 0.6915$



$\frac{6-\mu}{2} = 0.5 \rightarrow 6 - \mu = 1 \rightarrow \mu = 5$

Note :-



* دائما المساحة اقل من -3.49 واكبر من 3.49 بيساوي zero وبينهم واحد

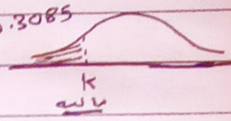
eg) $P(Z > 4) = 0$

eg) If $x \sim n(5, \sigma^2)$ & $P(X \geq 4) = 0.6915$. Find σ

sol) $P\left(Z \geq \frac{4-5}{\sigma}\right) = 0.6915$

$P\left(Z < \frac{4-5}{\sigma}\right) = 1 - 0.6915 = 0.3085$

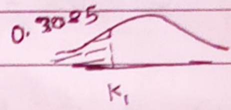
$\frac{4-5}{\sigma} = -0.5 \rightarrow \frac{-1}{\sigma} = \frac{-1}{2} \rightarrow \boxed{\sigma = 2}$



eg) If $x \sim n(\mu, \sigma^2)$, $P(X \geq 4) = 0.6915$ & $P(X \leq 6) = 0.6915$, Find μ & σ.

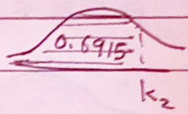
sol) $P(X \leq 4) = 1 - 0.6915 \rightarrow P\left(Z \leq \frac{4-\mu}{\sigma}\right) = 0.3085$

$\boxed{\frac{4-\mu}{\sigma} = -0.5}$ --- ①



$P(X \leq 6) = 0.6915 \rightarrow P\left(Z \leq \frac{6-\mu}{\sigma}\right) = 0.6915$

$\boxed{\frac{6-\mu}{\sigma} = 0.5}$ --- ②



بالحذف

$8 - 2\mu = -\sigma$ --- ①

$12 - 2\mu = \sigma$ --- ②

① + ②

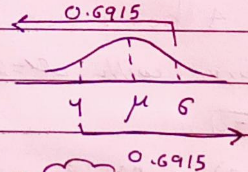
$20 - 4\mu = 0 \rightarrow 20 = 4\mu \rightarrow \boxed{\mu = 5}$

نعوضها في ①

$8 - 2(5) = -\sigma \rightarrow 8 - 10 = -\sigma \rightarrow -2 = -\sigma \rightarrow \boxed{\sigma = 2}$

* أما بطريقة أسهل :

$\mu = \frac{4+6}{2} = 5$



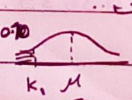
وبالتعويض في ① أو ②

$\frac{6-5}{\sigma} = 0.5 \rightarrow \frac{1}{\sigma} = \frac{1}{2} \rightarrow \sigma = 2$

eg) If $x \sim n(\mu, \sigma^2)$, $P(X \leq 2) = 0.10$ & $P(X \geq 9) = 0.20$, Find μ & σ.

غير متساويات ذلك نفصل للحل بالطريقة الأولى..

$P\left(Z \leq \frac{2-\mu}{\sigma}\right) = 0.10$



نلاحظ ان 0.1000 غير موجودة بالجدول لكن هناك قيم قريبة منها حوليها

0.0985 ، 0.1003 الأقرب

$z = -1.29$ ، -1.28

$\boxed{\frac{2-\mu}{\sigma} = -1.28}$ --- ①

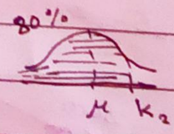
$P\left(Z \leq \frac{9-\mu}{\sigma}\right) = 1 - 0.20 = 0.8000$

$\boxed{\frac{9-\mu}{\sigma} = 0.84}$ --- ②

.7995 ، .8023 من حوليها

$z = 0.84$ ، $z = 0.85$

الأقرب



بحل المعادلتين نجد ان $\mu = \ldots$ & $\sigma = \ldots$

eg) Suppose that the grades in a general examination are normally distributed with mean 68 & standard deviation equal to 10.

Find: a) The proportion of grades that are more than 85.

b) The proportion of grades that are between 60 & 90

c) The 95^{th} percentile. sol) $\mu = 68$ & $\sigma = 10$

$X \sim n(68, 10^2)$

a) $P(X > 85) = P\left(Z > \frac{85-68}{10}\right) = 1 - P(Z < 1.7) = 1 - 0.9554$

$= 0.0446$

b) $P(60 < X < 90) = P\left(\frac{60-68}{10} < Z < \frac{90-68}{10}\right) = P(-0.8 < Z < 2.2)$

$P(Z < 2.2) - P(Z < -0.8) = 0.9861 - .2119 = 0.7742$

c) $P_{95} = P(X < P_{95}) = 0.95 \rightarrow P\left(Z < \frac{P_{95} - 68}{10}\right) = 0.9500$

$P\left(Z < \frac{P_{95} - 68}{10}\right) = 0.9500 \rightarrow \frac{P_{95} - 68}{10} = 1.64$

$P_{95} = 84.4$

0.9495 → $\boxed{Z = 1.64}$ نختار الاصغر

0.9505 → $Z = 1.65$

نفس البعد

eg) If the heights of students are normally distributed with mean 170 cm & standard deviation 10 cm.

a) a student is selected at random, What is the prob. that this student is shorter than 175 cm?

b) 2 students are selected at random, What is the prob. that exactly 1 of them is shorter than 175 cm (with replacement).?

c) 10 students are selected one by one with replacement. What is the prob. that exactly 4 of them are shorter than 175 cm.

sol) X: height $\sim n(170, 10^2)$

a) $P(X < 175) = P\left(Z < \frac{175-170}{10}\right) = P(Z < 0.5) = \boxed{0.6915}$

b) Tree di.. اكثر من مرة منرسم

- 0.6915 S
  - 0.6915 S → (S, S)
  - 0.3085 T → (S, T) *
- 0.3085 T
  - 0.6915 S → (T, S) *
  - 0.3085 T → (T, T)

$2(0.6915)(0.3085) = \ldots$

0.42666

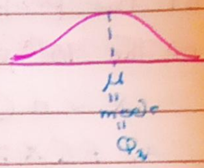
c) Binomial ← 3 كزمن، Y: no. of students that are shorter than 175 cm

$Y \sim Bin(10, 0.6915)$ // $P(Y=4) = \binom{10}{4}(0.6915)^4(0.3085)^6 = \ldots$

اما من الجدول مباشرة
او من القانون

0.0414

eg) If $X \sim n(50, 4)$ ($\mu = 50$, $\sigma^2 = 4$) Find: a) The mean b) The mode c) The median d) The IQR e) The 85^{th} percentile f) The standard deviation & variance.



$* \ \mu = \text{mode} = Q_2$

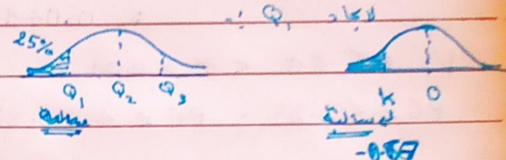
Sol) a) 50 b) 50 c) 50

d) $IQR = Q_3 - Q_1$

لإيجاد Q_1:

$* \ P(X < Q_1) = 0.2500$

$P\left(Z < \left(\frac{Q_1 - 50}{2}\right)\right) = 0.2500$



$\frac{Q_1 - 50}{2} = -0.67$

$Q_1 = 48.66 \approx Q_1 = \boxed{48.7}$

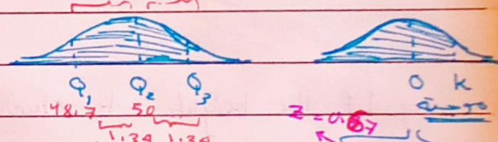
لإيجاد Q_3:

$* \ P(X < Q_3) = 0.7500$

$P\left(Z < \frac{Q_3 - 50}{2}\right) = 0.7500$

$\frac{Q_3 - 50}{2} = 0.67$

$\boxed{Q_3 = 51.34}$



أو من خلال $Q_2 + 1.3 = 50 + 1.34 = 51.34$

2.68

1.34 1.34

$Z = 0.67$

0.7486 0.7517

بعد 0.0014 ، بعد 0.0017

(الأقرب)

إذا تساووا في البعد نوجد الوسط أو الأقل

$\therefore IQR = Q_3 - Q_1 = 51.34 - 48.66 = 2.68$

e) $P_{85} \Rightarrow P(X < P_{85}) = 0.8500$

Z نحولها $P\left(Z < \frac{P_{85} - 50}{2}\right) = 0.8500$

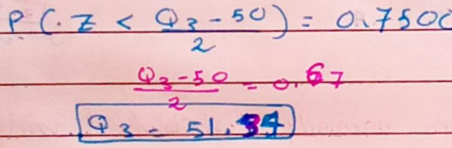
$\frac{P_{85} - 50}{2} = 1.04 \rightarrow P_{85} = 52.08$

85%

موجبة

$\boxed{k = 1.04}$ أقرب قيمة

f) $\sigma^2 = 4$ & $\sigma = 2$



Using the Empericals you know that about 68% of the scores are between $(\mu - \sigma)$ and $(\mu + \sigma)$, about 95% of the scores are between $(\mu - 2\sigma)$ and $(\mu + 2\sigma)$, and about 99.7% of the scores are between $(\mu - 3\sigma)$ and $(\mu + 3\sigma)$

eg) If $P(X > \mu + \sigma) = 0.16$, then Find $P(X > \mu - \sigma)$

0.16

$\boxed{1 - 0.16 = 0.84}$

5.5

The normal approximation to the binomial distribution

Lecture 22

If n is large & p is small or moderate, then let ((($np \geq 5$ & $nq \geq 5$)))

$n = 25$ في Table لها — $X \sim Bin(n,p)$, then $X \sim n(np, npq)$

continuous ← normal & Discrete ← Binomial   ولأنت μ σ^2

فلو حطينا $x \approx 5$ مثلاً 4, ...   4   4.5   5   5.5   6   اعداد صحيحة من 0 الى n

لاحظ ان اي رقم بين (4.5, 5.5) يحول بعد التقريب الى (5). مختلف تحول الاعداد من Discrete

*** Continuity correction.**   اي Interval اسمه :

If $X = 1 \longrightarrow X \in (0.5, 1.5)$ & If $X = 5 \longrightarrow x \in (4.5, 5.5]$ يساوي او لا يتغير ( ) ، [ ]

* eg) If $X \sim Bin(100, 0.2)$ find approximate values for the following probabilites: $np = 20 > 5$ & $nq = 80 > 5$ لاحظ أن

a) $P(X < 26)$ b) $P(X \leq 26)$ c) $P(18 < X \leq 26)$ d) $P(18 \leq X < 26)$ e) $P(18 \leq X \leq 26)$

sol) $n = 100$ (كثيرة كبيرة) , $p = 0.2$

$\mu = n \cdot p = 20$ & $\sigma^2 = n \cdot p \cdot q = 20 \times (0.8) = 16 \Rightarrow X \sim n(20, 4^2)$

* a) $P(X < 26)$ 26 لا تشمل   25.5   26   26.5 $\therefore P(X \leq 25.5)$

$P(Z \leq \frac{25.5 - 20}{4})$ $P(Z \leq 1.38) = 0.9162$ من الجدول

* b) $P(X \leq 26)$ 26 كلها تعبر عن   25.5   26   26.5 $P(X \leq 26.5)$

$P(Z \leq \frac{26.5 - 20}{4})$ $P(Z \leq 1.63) = 0.9484$ من الجدول

* c) $P(18 < X \leq 26)$ 18   26   17.5   18   18.5   25.5   26   26.5 $\therefore P(18.5 < X \leq 26.5)$

$P(\frac{18.5 - 20}{4} < Z < \frac{26.5 - 20}{4}) = P(0.38 < Z < 1.63)$

$P(Z \leq 1.63) - P(Z \leq -0.38) = 0.9484 - 0.3520 = .5964$

* d)   18   26   17.5   18   18.5   25.5   26   26.5 $: P(17.5 \leq X \leq 25.5)$

$= P(\frac{17.5 - 20}{4} \leq Z \leq \frac{25.5 - 20}{4})$

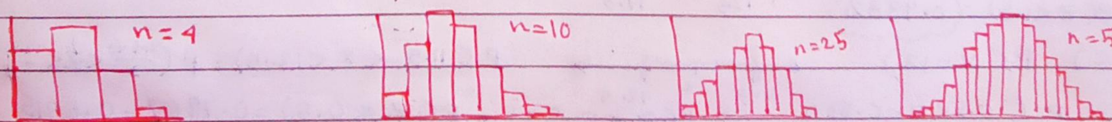
$\therefore P(-0.63 \leq Z \leq 1.38) = P(Z \leq 1.38) - P(Z \leq -0.63)$

$= 0.9162 - 0.2643 = 0.6519$

* e) $P(18 \leq X \leq 26)$ 17.5   18   18.5   25.5   26   26.5 $P(17.5 \leq X \leq 26.5)$

$P(\frac{17.5 - 20}{4} \leq Z \leq \frac{26.5 - 20}{4}) = P(-0.63 \leq Z \leq 1.63)$

$\therefore P(Z \leq 1.63) - P(Z \leq -0.63) = 0.9484 - 0.2643 = 0.6841$



Notice that as (n) increases, the shape of a binomial becomes more similar [to normal approximation]

Shown below are several cases of binomial probabilities involving the number c and how to convert each to a normal distribution probability.

Binomial	⟷	Normal	Notes
Exactly c	مغلق c C−0.5 C C+0.5	$P(c-0.5<X<c+0.5)$	Includes c
At most c	مغلق c C−0.5 C C+0.5	$P(X<c+0.5)$	Includes c
Fewer than c	c C−0.5 C C+0.5	$P(X<c-0.5)$	Does not include (c)
At least c	[illegible] c C−0.5 C C+0.5	$P(X>c-0.5)$	Includes c
More than c.	C−0.5 C C+0.5	$P(X>c+0.5)$	Does not include c

eg) Suppose that 10% of heavy smokers will suffer from Lung cancer after the age forty. In a sample of 100 heavy smokers, what is the probability that;

Independent [illegible]

$n \geq 3$ [illegible]

a) at least 12 will have lung cancer?

b) no more than 14 will have lung cancer? (At most 14)

c) exactly 12 will have lung cancer?

$n = 100$ & $p = .10$ & $q = .90$

so $np = 10 > 5$ & $nq = 90 > 5$; So normal approxim...

sol) $X \sim Bin(100, 0.10) \Rightarrow X \sim n(10, 3^2)$

a) $P(X \geq 12)$ — 11.5 12 12.5 — $P(X \geq 11.5) = P\left(Z \geq \frac{11.5-10}{3}\right)$

$= P(Z \geq 0.5) = 1 - P(Z \leq 0.5) \longrightarrow 1 - 0.6915 = 0.3085$

b) $P(X \leq 14)$ — 13.5 14 14.5 — $P(X \leq 14.5) = P\left(Z \leq \frac{14.5-10}{3}\right)$

$P(Z \leq 1.5) = 0.9332$

c) $P(X = 12)$ — 11.5 12 12.5 — $P(11.5 < X < 12.5) = P\left(\frac{11.5-10}{3} < Z < \frac{12.5-10}{3}\right)$

$P(0.5 < Z < 0.83) \longrightarrow P(Z \leq 0.83) - P(Z \leq 0.5) = 0.7967 - 0.6915$

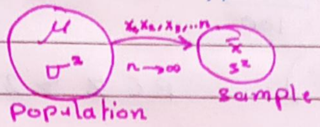
$= 0.1052$

The Central Limit Theorem "lecture 23"

The Central limit Theorem (C.L.T):

If a random Sample $X_1, X_2, \dots X_n$ is drawn from a population with mean μ & variance σ^2, then for large n, the distribution of sample mean $\bar{x}$ is approximately normal with mean μ & variance $\frac{\sigma^2}{n}$.

$$?\leftarrow \bar{x} = \frac{\sum_{i}^{n} X_i}{n} \rightarrow ?$$



$\bar{X} \sim n(\mu, \frac{\sigma^2}{n})$ for $n \geq 30$

eg) Suppose that a random sample of size $n=100$ is drawn from a population with mean 70 & standard deviation 20. What is the probability that the sample mean $\bar{x}$ will be : a) more than 70, b) less than 73

sol) $\boxed{n=100}$, $\boxed{\mu=70}$, $\boxed{\sigma=20}$ $\quad X_1, X_2, \dots X_{100} \sim n(70, \frac{20^2}{100})$

$\bar{x} \sim n(70, 2^2)$

a) $P(\bar{x} > 70) = P(Z > \frac{70-70}{2}) = P(Z > 0) = 1 - P(Z \leq 0)$

$1 - 0.5 = \boxed{0.5}$

b) $P(\bar{x} < 73) = P(Z < \frac{73-70}{2}) = P(Z < 1.5) = 0.9332$

eg) Suppose that the mean weight & standard deviation of orange boxes are 10 & 2 kgs, respectively. If 100 boxes are to be loaded in a car with threshold 1000kgs. What is the prob. that the car will break down?

sol) $X_1, X_2, \dots X_{100} \sim n(10, 2^2)$

$\bar{x} \sim n(10, \frac{4}{100}) \rightarrow (0.2)^2$

انا عندي 1000 ky تلك الـ Orange boxes فلازم تكون السيارة تتحمل اكثر منه عليه حتى ما تتعطل

فاحتمال انها تتعطل لما يكون وزن تلك الصناديق اكبر من (1000)kg

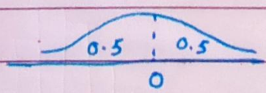
$P(X_1 + X_2, X_3 + \dots X_{100} > 1000)$: $P(\sum_{i=1}^{100} x_i > 1000)$

$P(\frac{\sum_{i=1}^{100} x_i}{100} > \frac{1000}{100})$: $P(\bar{x} > 10) \longrightarrow P(Z > \frac{10-10}{0.2}) = P(Z > 0)$

$1 - P(Z < 0) = 1 - 0.5 = 0.5$

من المنحنى لان منحنى z=0 يكون

Area اليمين منها 0.5 والايسر منها 0.5



eg.) Solve the previous example if the threshold of the car is 1050 kg.

$$P\left(\frac{\sum X_i}{100} > \frac{1050}{100}\right) \;\; P(\bar{X} > 10.5)$$

$$P\left(Z > \frac{10.5-10}{0.2}\right) = P(Z > 2.5) = 1 - P(Z < 2.5)$$

$1 - 0.9938 = 0.0062$

لاحظ انه كل ما كانت السيارة تتحمل حمولة اكبر (الحمل المسموح به) كلما كانت احتمالية أن تتعطل اقل (انخفض الاحتمالية)

Sampling Distribution.

Population

Parameters
1) Mean: μ
2) Variance: σ^2
3) Proportion: P

$x_1, x_2, \dots x_n \rightarrow$

Sample

Estimator
1) mean: $\bar{X}$ ~
2) Variance: S^2 ~
3) proportion: $\hat{P}$ ~

كيفا توزيعهم ?? حتى اقدر احسب عليهم probability

هدول ال ⇑   هدول كلهم Estimator تقريبي ⇑

بالسكشن الي فاته درسنا العلاقة بين ال mean ال population (x) Values of a random variable

هون السكشن هاد بدرس العلاقة بين ال population mean وال random samples من ال population

A **Sampling distribution** is the probability distribution of a sample statistic that is formed when random samples of size n are repeatedly taken from a population. If the sample statistic is a sample mean, then the distribution is the **sampling distribution of a sample means**. Every sample statistic has a sampling distribution.

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

$\mu_{\bar{x}} = \mu$

mean of the sample = mean of the Population

standard deviation of the sample means $\sigma_{\bar{x}}$ is equal to the population standard deviation σ devided by the square root of the sample size n. → $var(\bar{x}) = \dfrac{\sigma^2}{n}$

The standard deviation of the sampling distribution of the sample means is called the **standard error of the mean**.

If $x_1, \dots x_n \sim n(\mu, \sigma^2)$ then $\bar{x} \sim n(\mu, \frac{\sigma^2}{n})$

for any distribution أي normal مو شرط

eg) Suppose that the weights of a certain population are normally distributed with mean 70 Kgs & standard deviation 10 Kgs, If a sample of size $n = 25$ persons is to be drawn, What is the prob. that: a) The average weight will be less than 75 Kgs

b) The total weight exceeds 1800 Kgs

sol) $x_1, x_2, \dots, x_{25} \sim n(70, 10^2)$ normal

$\bar{x} \sim n(70, \frac{10^2}{25})$

$\bar{x} \sim n(70, 2^2)$ normal

a) $P(\bar{x} < 73) = P(Z < \frac{73-70}{2})$

$P(Z < 1.5) = 0.9333$

b) $P(\frac{\sum_{i=1}^{25} x_i}{n} > \frac{1800}{25}) = P(\bar{x} > 72) = P(Z > \frac{72-70}{2})$

$1 - P(Z \leq 1) = 1 - 0.8413 = 0.1587$

من الجدول.

* The distribution of the sample proportion $\hat{p} = \frac{x}{n}$:-

lecture 25
Part "2"

مثلاً جبنا عينة ، 100 شخص ، وطلعت نسبة المدخنين طلعوا 60 مدخن

فنسبة الـ proportion = 0.60 = $\frac{60}{100}$ عشان استخدم $\hat{p}$ لازم اعرف ان n Large

$$\boxed{\hat{p} \sim n\left(p, \frac{pq}{n}\right)} \quad \text{or} \quad Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \sim n(0,1)$$

eg. Suppose that 10% of a certain production are defective. If 400 items are drawn from the production, what is the prob. that the sample proportion will be:

a) more than 12%   b) between 9% & 11%

sol) $n = 400$, $p = 0.10$, $q = 0.90$

$$\hat{p} \sim n\left(0.10, \frac{(0.10)(0.90)}{400}\right) \Rightarrow \hat{p} \sim n\left(0.10, \left(\frac{3}{200}\right)^2\right)$$

a) $P(\hat{p} > 0.12) = P\left(Z > \frac{0.12 - 0.10}{3/200}\right) = P(Z > 1.33)$

$= 1 - P(Z \leq 1.33) = 1 - 0.9082 = 0.0918$

b) $P(0.09 < \hat{p} < 0.11) = P\left(\frac{0.09 - 0.10}{3/200} < Z < \frac{0.11 - 0.10}{3/200}\right)$

$P(-0.67 < Z < 0.67) = P(Z < 0.67) - P(Z < -0.67)$

$= 0.7486 - 0.2514 = 0.4972$

p: نسبة المعيبين defective في الـ population

$\hat{p}$: نسبة المعيبين defective في الـ sample

COLLEGE

eg) Suppose that 90% of the university students pass Calculus 101. In a sample of 200 students taking Calculus 101, What is the prob. that the proportion of those who will pass is less than 85%.

$n = 200 \qquad p = 0.90 \qquad q = 0.10$

$$\hat{p} \sim n\left(0.90, \frac{(0.90)(0.10)}{200}\right)$$

$$P(\hat{p} < 0.85) = P\left(Z < \frac{0.85 - 0.90}{\sqrt{(0.90)(0.10)/200}}\right)$$

$$P(Z < -2.36) = 0.0091$$