Chapter (4)

Random variables and distributions

Sheet (1)

Q1 Suppose you want to play a carnival game that costs \$5 each time you play it, if you win, you get \$100 while the probability of winning is 0.1, the expected amount of money the player can win is:

A)-2 B)100 C)7 D)5 E) 0

<u>Solution:</u>

	Win	Lose
x	95	-5
$P(x = x_i)$	0.1	0.9

$$E(x) = \Sigma x.\, P(x = x_i) = 0.1(95) + (0.9)(-5) = 5 \rightarrow D$$

Q2 Let X be a discrete random variable with values 0, 1 & 2, if P(X=1) =0.5 & E(X)= 1.2, then P(X=2) =

A)0.2 B) 0.4 C)0.15 D)0.35 E) 0.1

<u>Solution:</u>

x	0	1	2
$P(x = x_i)$	a	0.5	b

$$E(x) = \Sigma x.\, P(x = x_i) = 0.9 + 1(0.5) + 2b = 1.2 \rightarrow 0.5 + 2b = 1.2 \rightarrow b = 0.35$$

$$\rightarrow P(x = 2) = b = 0.35 \rightarrow D$$

Q3 If the E(X)=5, Var(X) = 9, then E(X^2+5X-2) =

A) 56 B) 41 C) 52 D) 57 E) none

<u>Solution:</u>

$$E(x^2 + 5x - 2) = E(x^2) + 5\,E(x) - E(2) = \left[Var(x) + \left(E(x)\right)^2\right] + 5\,E(x) - 2$$

$$= [9 + (5)^2] + 5(2) - 2 = 57 \rightarrow D$$

Q4 suppose a game is played with one six-sided die, if the die is rolled and landed on (1,2,3), the player wins nothing, if the die lands on 4 or 5, the player wins \$3, if the die land on 6, the player wins \$12, the expected value is:

A) \$4 **B) \$12** **C) \$17** **D) \$36** **E) \$3**

Solution:

	1,2,3	4,5	6
x	0	3	12
$P(x = x_i)$	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$

$E(x) = \Sigma x. P(x = x_i) = 0\left(\frac{3}{6}\right) + \frac{2}{6}(3) + 12\left(\frac{1}{6}\right) = 3 \rightarrow E$

Q5 Suppose the random variable X has the distribution given below:

X	**1**	**10**	**15**	**20**
$P(X=x_i)$	**0.15**	**0.2**	**0.3**	**0.35**

A) μ = 13.65 & σ = 227.65 **B) μ =13.65 & σ = 41.3275**

C) μ = 6.43 & σ =13.65 **D) μ =13.65 & σ = 6.43**

Solution:

$E(x) = \Sigma x. P(x = x_i) = 1(0.15) + 10(0.2) + 15(0.3) + 20(0.35) \rightarrow E(x) = 13.65$

$E(x^2) = \Sigma x^2. P(x = x_i) = 1(0.15) + 100(0.2) + 225(0.3) + 400(0.35) = 227.65$

$Var(x) = E(x^2) - \left(E(x)\right)^2 = 227.65 - (13.65)^2 = 41.3275$

$\sigma = \sqrt{41.3275} = 6.43 \rightarrow D$

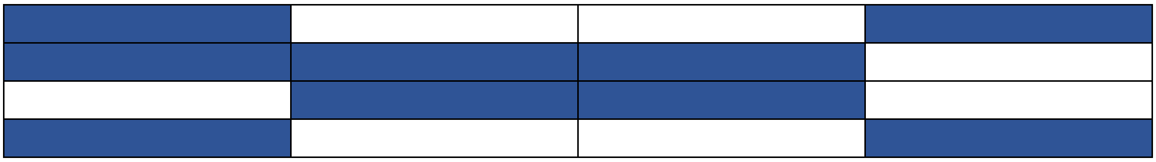
Q6 If X is a random variable such that E(X)=10 and Var(X)=4, then E(X^2+X) =

A) 114 **B) 124** **C) 24** **D) 14** **E)91**

Solution:

$E(x^2 + x) = E(X^2) + E(X) = [\ 4 + 10^2\ ] + 10 = 114 \rightarrow A$

Q7 A game consists of throwing a dart at the target Shown, the player pays \$24 to play the game, if the dart hits the shaded region, the player wins \$44, otherwise the player receives nothing, the expected value of money the player can win.



A) -\$0.5 **B) \$0.25** **C) -\$0.1** **D) \$0.75** **E) none**

Solution:

	Hit the shaded region	Hit the unshaded region
x	$44 - 24 = 20$	-24
$P(x = x_i)$	$\frac{9}{16}$	$\frac{7}{16}$

$E(x) = \Sigma x.P(x = x_i) = 20\left(\frac{9}{16}\right) + (-24)\left(\frac{7}{16}\right) = 0.75 \rightarrow D$

Q8 A discrete random variable X takes the values 1 and 2. If $E(X^2) = 1.9$. Then

$P(X = 2) =$

A) 0.2 **B) 0.1** **C) 0.4** **D) 0.5** **E) 0.3**

Solution:

X	1	2
$P(X=x_i)$	a	b

$E(X^2) = \sum X^2 * P(X = x_i) = 1 * a + 4 * b = 1.9$ …..(1)

$\sum P(X = x_i) = 1 \rightarrow a + b = 1$ ……(2)

$\therefore a = 0.7$ and $b = 0.3 \rightarrow P(X=2) = 0.3 \rightarrow E$

Q9 Let X be a random variable such that E(X) =8. If the standard deviation of X is 2 then E(X^2) equals?

A) 60 **B) 64** **C) 56** **D) 62** **E) 68**

<u>Solution:</u>

$\sigma = 2 \quad , \quad E(x) = 8$

$E(x^2) = \sigma^2 + \mu^2 = 4 + 64 = 68 \rightarrow E$

Q10 Consider the following distribution of a random variable X

X	0	1	2	3
P(X)	0.1	0.2	0.2	0.5

Find P(X<1)

A) 0.1 **B)0.2** **C)0.3** **D)0.4** **E)0.5**

<u>Solution:</u>

$P(x < 1) = P(x = 0) = 0.1 \rightarrow A$

Q11 Let x be a discrete random variable with the following probability density then E (x^2) is:

P	-2	0	5
f(x)	C	0.2	0.3

A) 9.5 **B) 6.8** **C) 4.7** **D) 5.2** **E)7.1**

<u>Solution:</u>

Since it's p.d.f $\rightarrow \Sigma f(x) = 1 \rightarrow C + 0.2 + 0.3 = 1 \rightarrow C = 0.5$

x	-2	0	5
f(x)	0.5	0.2	0.3

$E(x^2) = \Sigma x^2 . f(x) = (-2)^2 . (0.5) + 0(0.2) + (5)^2 (0.3) = 9.5 \rightarrow A$

Q12 A discrete random variable has mean 20 and standard deviation 4, then E(X(X+1)) =

A)426 **B)436** **C)416** **D)446**

Solution:

$E(x) = 20 \text{ and } \sigma = 4$

$E(x^2 + x) = E(x^2) + E(x) = Var(x) + \left(E(x)\right)^2 + E(x) = (4)^2 + (20)^2 + 20$

$= 436 \rightarrow B$

Q13 let X be a random variable, E(X) =10 & Var(X) = 9, then $E(3X^2-4X+7)$?

Solution:

$E(3X^2-4X+7) = 3E(X^2) - 4E(X) + E(7) = 3*(9+10^2) - 4*10 + 7 = 294$

Q14 Consider the following distribution of a random variable X

X	-2	-1	0	1
P(X)	0.2	0.3	0.2	0.3

Find E(3X+3)

A) 1.4 **B)1.2** **C)1** **D)1.8** **E)1.1**

Solution:

$E(3x) + E(3) = 3E(x) + 3$

$\rightarrow E(x) = \Sigma x.P(x = x_i) = -2(0.2) + (-1)(0.3) + 0(0.2) + 1(0.3) = -0.4$

$\rightarrow E(3x + 3) = 3(-0.4) + 3 = 1.8 \rightarrow D$

Q15 In a game of chance, three fair coins are tossed simultaneously. If all three coins show heads, Then the player wins $15. If all three coins show tails, then the player wins $10. Any other combination will result in not winning any money. If it costs $5 to play, what is the player's expected not gain or loss after two games?

A) The player can expect to gain $15 after two games.

B) The player can expect to gain $1.88 after two games.

C) The player can expect to gain $3.75 after two games.

D) The player can expect to lose $1.88 after two games.

E) The player can expect to lose $3.75 after two games.

Solution:

	HHH	TTT	Otherwise
X	15 – 5 = 10	10 – 5 = 5	0 – 5 = -5
$P(X=x_i)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{6}{8}$

$E(x) = \Sigma x. P(x = x_i) = 10 * \frac{1}{8} + 5 * \frac{1}{8} + -5 * \frac{6}{8}$ = -1.875

E(2x) = 2* -1.875 = -3.75 → E

Sheet (2)

Q1 A box contains 13 white balls & 7 black, the experiment of drawing one ball from this box is repeated 10 times with replacement. let X be the number of times a white ball is drawn in the 10 trials, then the distribution is?

A)Bin(10 , 0.7) **B)Bin(3 , 0.3)** **C)Bin(10 , 0.3)**

D)Bin(10 , 0.65) **E)none**

Number of trials is 10 and X is the number of white balls

So P = $\frac{13}{20}$ = 0.65 → X~ Bin(10 , 0.65) → D

Q2 If a family has 7 children, what is the probability that three of them are boys?

X ~ Bin (7 , 0.5)

P(X=3) = $\binom{7}{3}$ $*0.5^{3} * 0.5^{7-3}$ = 0.273 OR by tables: P(X = 3) = 0.273

Q3 The number of industrial injuries per working week in a particular factory is known to follow a Poisson distribution with mean 0.5, find the probability that in a particular week, there will be more than two accidents?

X~ $POi(0.5)$

P(X>2) = 1 – P(X≤ 2) = 1 – 0.986 = 0.0144

Q4 Let X be the number of Radioactive particles passing through a counter during 1 millisecond, Assume X is Poisson random variable with mean 4, Find the following:

1) P(X ≤ 4) **2) E($2X^2$-1)**

X~ Poi (4)

1) P(X ≤ 4) = 0.629 “from tables”

2) E($2x^2 - 1$)= 2E(x^2) – E(1) = 2 [Var(X) + $(E(X))^2$] – E(1)

= 2 [4 + 16] – 1 = 39

Q5 A multiple choice exam consists of 20 questions, each question has five choices one of them is correct, A person is answering by guessing, then the probability that he\she will answer at least one question correctly equals:

A) 0.988 B) 0.0115 C) 0.930 D) 0.0577 E) 0.012

First, we must know our distribution n=20, independent, & we have two outcomes: correct and incorrect → X~Bin(20,$\frac{1}{5}$)

P(X≥1) = 1 – p(X<1) = 1 – P(X≤0) = 1 – 0.012 = 0.988 → A

Q6 A mail-order company receives an average 10 orders per day, in a given two days, Find the prob that orders is at most 15?

X~ Poi (10) "per day" → X~ Poi (20) "because they are two days"

P(X≤15) = 0.157

Q7 Let X~ Binomial (50, p), if E(X) =10 then Var(X) equals?

A)8 B)10 C)6 D)12 E)4

E(X) = n*p = 10 → 10 = 50*P → P = 0.2 & q = 0.8

Var(X) = n*p*q = 50*0.2*0.8 = 8 → A

Q8 A multiple-choice exam consists of 8 independent questions, each question has 5 choices and only one choice is correct. If a student answers all 8 questions randomly by guessing. Find the expected number of questions that will be answered?

X~Bin(8 , $\frac{1}{5}$)

E(X) = n*p = 8 * $\frac{1}{5}$ = 1.6

Q9 Let X~Binomial (10,0.2). Then E(X^2 – 3) =

A)5.6 B)4.5 C)3.6 D)2.6 E)1.6

$E(x^2 - 3) = E(x^2) - E(3) = [n * p * q + (n * p)^2 - 3 =$

$[10 * 0.2 * 0.8 + (10 * 0.2)^2 - 3 = 2.6 \rightarrow$ D

Q10 If X ~ Bin (n, p) such that E(X) = 2 and Var(X) = 1.6, then P (X< 6):

A) 0.678 B) 0.967 C) 0.879 D) 0.994 E)0.006

E(X) = n*p = 2 ….(1) and Var(x) = n*p*q =1.6 ….(2)

$2 * q = 1.6 \rightarrow q = 0.8$ and $p = 0.2$

$\therefore n = 10$

$X \sim Bin(10,0.2)$

$P(X < 6) = P(X \leq 5) = 0.994 \rightarrow D$

Q11 let X be Binomial (n, 0.1) with mean 10 , then the variance =

A)4.8 B) 2.75 C) 7.2 D) 9 E) none

$E(x) = n \times P = n \times 0.1 = 10 \Rightarrow n = 100$

$q = 1 - P = 1 - 0.1 = 0.9 \rightarrow q = 0.9$

$var(x) = n \times q \times p = 100 \times 0.1 \times 0.9 = 9 \rightarrow D$

Q12 If X ~ Bin(20 , 0.2), then μ & σ are :

A) 4 , 3.2 B)4 , 1.8 C)0.766 , 0.899 D) 9 E) NONE

$E(x) = n \times P = 20 \times 0.2 = 4$

$var(x) = n \times q \times p = 20 \times 0.2 \times 0.8 = 3.2$

$\sigma = \sqrt{3.2} \rightarrow \sigma = 1.8 \rightarrow B$

Q13 A box contains 4 white balls and 6 black balls. the experiment of drawing one ball from this box is repeated 10 times with replacement. let X be the number of times a black ball is drawn in the 10 draws, find E(X^2):

A) 36 B) 2.4 C)34.8 D)38.4 E)12.5

$$x \sim \mathrm{Bin}\left(10, \frac{6}{10}\right)$$

$$E(x^2) = \mathrm{var}(x) + \left(E(x)\right)^2 = n.p.q + (n.p)^2 = 10 \times \frac{6}{10} \times \frac{4}{10} + \left(10 \times \frac{6}{10}\right)^2 = 38.4 \rightarrow D$$

Q14 A box contains 80 white and 20 black balls. One ball is drawn from this box with replacement 10 times. The probability that a black ball shows up at most 3 times equals:

A) 0.121 B)0.823 C)0.879 D)0.492

$$x \sim \mathrm{Bin}(10, 0.20)$$

$$P(x \leq 3) = 0.879 \rightarrow C$$

Q15 Which one of the following variables is a binomial random variable:

A)The number of textbooks a randomly selected university student bought this semester

B) The number of a computer CDs a randomly selected person owns.

C) The number of men taller than 170 cm in a random sample of 5 men.

D)The time it takes a randomly student to complete a multiple-choice exam.

The answer is C

Q16 Let X ~ Bin (5, 0.5), then P(x > 3 |x > 2)is:

A) 0.727 B) 0.615 C) 0.375 D) 0.188 E) 0.381

$$X \sim \mathrm{Bin}(5, 0.5)$$

$$P(X>3 \setminus X>2) = \frac{P(X=4)+P(X=5)}{P(X>2)} = \frac{P(X=4)+P(X=5)}{1-P(X\leq 2)}$$

$$\frac{\binom{5}{4}0.5^4 0.5^1 + \binom{5}{5}0.5^5 0.5^0}{1-0.500} = 0.375 \rightarrow C$$

Q17 Let X ~ Bin (20, 0.7), then P(5 ≤ x < 10) is closest to:

A) 0.0595 B) 0.278 C) 0.0593 D) 0.141 E) 0.017

$P(5 \leq x < 10) = P(x \leq 9) - P(x \leq 4) = \ 0.017 - 0.00 = 0.017 \ \rightarrow E$

Q18 Which of the following is NOT a property of binomial experiment?

A) It consists of a fixed number of trials, n.

B) Trails are independent.

C) Each trail has two outcomes.

D) The probability of success is constant for each trail.

E) Trails are independent, and the probability of success is constant for each trail.

The answer is A

Q19 A box contains 4 white balls and 6 black balls. The experiment of drawing one ball from this box is repeated 10 times with replacement. Let X be the number of times a black ball is drawn in the 10 draws. Find E(X^2):

A) 38.4 B) 2.6 C) 8.4 D) 34.8 E) None

$X \sim Bin(10, \frac{6}{10})$

$E(X^2) = Var(X) + \left(E(X)\right)^2 = n * p * q + \ (n * p)^2 = 10 * 0.6 * 0.4 + (10 * 0.6)^2 = 38.4 \rightarrow \ A$

Q20 Let X be a binomial random variable with a mean of 20 and variance of 4. Then E($3X^2$ - 3 X + 2) is:

A) -46 B) 1154 C) 1526 D) 1142

E($3X^2$ - 3 X + 2) = 3E(X^2) – 3E(X) + E(2) = 3* (4 + 20^2) – 3*(20) +2 = 1154 → B