Chapter (8)

Test of hypotheses

Sheet (1)

Q2 If a two tailed hypotheses test for a proportion has a test statistic of Z= -2.39. What is the corresponding p-value?

A) 0.017 B) 0.034 C) 0.483 D) 0.0084

$P(Z < -2.39) = 0.0084 \rightarrow$ D

Q3 The hypotheses H_0: $\mu = \mu_0$ is to be tested against H_1: $\mu \neq \mu_0$. Suppose that (10, 20) is a 95% C.I. for μ . Then, H_0 is accepted at significance level $\alpha = 0.05$ only if:

A) $\bar{X}$<10 or $\bar{X}$ >20 B) μ <20 C) $\bar{X}$>20

D) $10 \leq \mu_0 \leq 20$ E) $\mu_0 < 10 \ or \ \mu_0 > 20$

H_0 is accepted if $\mu_0 \in (10 , 20) \rightarrow$ D

Q4 Test the claim that for the adult population of a certain town, the mean annual salary is given by $\mu = \$30,000$. sample data are summarized as n=17, $\bar{X}$=\$22,298 and s=\$14,200. Use a significance level of α=0.05, the test statistic is?

A)t = -2.24 B)Z = -2.24 C)t = -2.43 D)Z = -2.54 E)NONE

$\bar{X} = 22{,}298 \quad , \quad S = 14{,}200 \quad , \quad n = 17 \quad , \quad \mu_0 = 30{,}000$

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{22298 - 30000}{14200/\sqrt{17}} = -2.24 \rightarrow A$$

Q5 The grades of students in a certain population are normally distributed with variance 64. A random sample of 9 students has mean 69. Using α=0.05 significance level we want to test that the population mean is smaller than 74 . In such case:

A) The test statistic is -1.88 and we reject H_0.

B) The test statistic is -1.88 and we can not reject H_0.

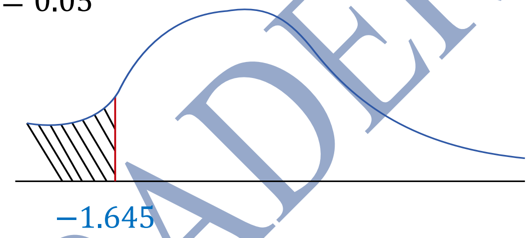
C) The test statistic is -1.72 and we cannot reject H_0.

D) The test statistic is -1.5 and we reject H_0.

$n = 9 \quad , \quad \sigma^2 = 64 \quad , \quad \overline{X} = 69 \quad , \quad \alpha = 0.05$

$H_0: \quad \mu = 74 \quad \text{Vs} \quad H_1: \ \mu < 74$

$\text{Test stat } Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{69 - 74}{8/\sqrt{9}} = -1.88$



$\text{Now } Z_{0.05} = -1.645$

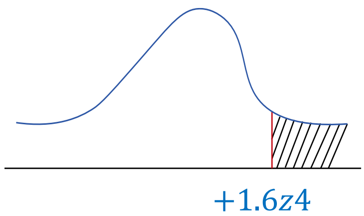
$\text{Since} - 1.88 < -1.645 \text{ We rej } H_0 \quad \to A$

Q6 In a hypothesis test for a proportion, the null hypothesis is H_0: P = 0.7 and the alternative hypothesis is H_1: P > 0.7 with α = 0.05. Then we cannot reject H_0 if the test statistic Z is.

A) less than 1.64 and reject H0 otherwise.

B) between -1.96 and 1.96 and reject H0 otherwise.

C) between -1.28 and 1.28 and reject H0 otherwise.



$Z_\alpha = -1.64 \ \to A$

Q7 A sample of size 16 has mean 20 and standard deviation 5 is randomly selected from a normally distributed population. We used this sample to test H_0: μ= μ0 versus an alternative hypothesis. If the test statistic equals 5 then μ0 equals:

A) 19.1 B) 14.56 C) 13.75 D) 17.7 E) 18

$n = 16 \quad , \quad \overline{X} = 20 \quad , \quad S = 5$

$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \to 5 = \frac{20 - \mu_0}{5/\sqrt{16}} \to \mu_0 = 13.75 \to C$

Q8 Write the claim as a mathematical statement. State the null and alternative hypotheses and identify which represents the claim.

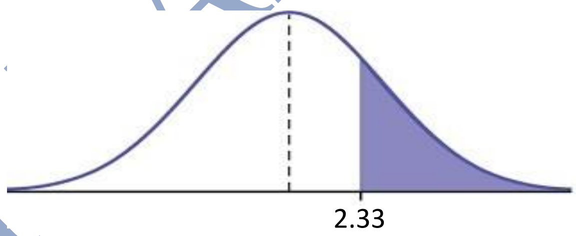
1. **The standard deviation of the sale price of a bike is no more than $225.**
2. **According to a recent survey, 73% of college students did not use student loans to pay for college.**

1) H_0: $\sigma = 225$ Vs. H_1: $\sigma < 225$.
2) H_0: $P = 0.73$ Vs. H_1: $P \neq 0.73$.

Q9 A random sample of 100 medical school applicants at a university has a mean total score of 502 on the MCAT. According to a report, the mean total score for the school's applicants is more than 499. Assume the population standard deviation is 10.6. At $\alpha = 0.01$, is there enough evidence to support the report's claim?

$n = 100$, $\bar{x} = 502$, $\mu > 499$, $\sigma = 10.6$, $\alpha = 0.01$

$H_0: \mu = 499$ vs $H_1: \mu > 499$



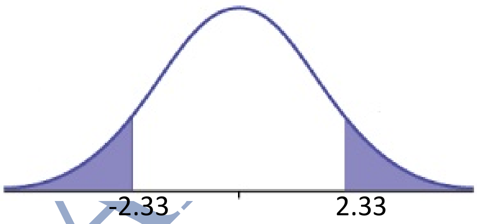
$Z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{502-499}{10.6/\sqrt{100}} = 2.83$

$Z_\alpha = Z_{0.01} = 2.33 \rightarrow$ We reject H_0

Q10 A random sample of 8 observations was taken from a normal population. The sample mean is 70 and the sample standard deviation is 20. When testing at 5% significance level

H_0: $\mu = 80$ vs. H_1: $\mu \neq 80$, we have:

A) The test statistics is Z = -1.41, and we don't reject H_0.
B) The test statistics is t = 1.41, and we cannot reject H_0.
C) The test statistics is t = -1.41, and we cannot reject H_0.
D) The test statistics is t = -1.41, and we reject H_0.



$n = 8 , \bar{x} = 70 , S = 20 , \alpha = 0.05$

$H_0 : \mu = 80 \quad \text{Vs } H_1 : \mu \neq 80$

$t = \frac{\bar{x}-\mu_0}{S/\sqrt{n}} = \frac{70-80}{20/\sqrt{8}} = -1.41$

$\alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025 \rightarrow t_{0.025}^{(7)} = \pm 2.365$

$\therefore$ We Can't rej $H_0 \rightarrow$ C

Test the claim that for the adult population of a certain town, the mean annual salary is given by $\mu = \$30,000$. sample data are summarized as n=17, $\bar{X}$=$22,298 and s=$14,200. Use a significance level of α=0.05, the test statistic is?

A) -2.24 B) -2.30 C) -2.43 D) -2.54 E) NONE

Solution:

σ unknown, n $<$ 30 $\rightarrow$ t − test

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{22{,}298 - 30{,}000}{14{,}200/\sqrt{17}} = -2.24 \rightarrow \quad A$$

Sheet (2)

Q1 The weights of (4) patient before and after taking a medication are given below. For testing whether their mean of difference (di = before – after) is different from 0, the test statistics is:

Before	**85**	**77**	**99**	**84**
0After	**85**	**74**	**96**	**86**

A) 0.72 **B) -0.82** **C) 0.82** **D) -0.72** **E) 1.98**

<u>Solution:</u>

$H_0 : \mu_d = 0$ Vs $H_1 : \mu_d \neq 0$

d_i	0	3	3	-2
d_i^2	0	9	9	4

$\sum d_i = 4$, $\sum d_i^2 = 22$

$$\bar{d} = \frac{\sum d_i}{n} = \frac{4}{4} = 1 \quad \& \quad \text{S.d} = \sqrt{\frac{\sum d_i^2}{n-1} - \frac{(\sum d_i)^2}{n*(n-1)}} = \sqrt{\frac{22}{4-1} - \frac{16}{4*(4-1)}} = 2.45$$

$$t = \frac{\bar{d} - \mu_d}{s.d/\sqrt{n}} = \frac{\bar{d}}{s.d/\sqrt{n}} = \frac{1}{2.45/\sqrt{4}} = 0.82 \rightarrow \text{C}$$

Q2 To compare the dry braking distances from 60 to 0 miles per hour for two makes of automobiles, a safety engineer conducts braking tests for 23 models of Make A and 24 models of Make B. The mean braking distance for Make A is 137 feet. Assume the population standard deviation is 5.5 feet. The mean braking distance for Make B is 132 feet. Assume the population standard deviation is 6.7 feet. At α = 0.10, find the test statistics?

<u>Solution:</u>

Make (A)

$n = 23$

$\bar{x} = 137$

$\sigma_1 = 5.5$

Make (B)

$m = 24$

$\bar{Y} = 132$

$\sigma_2 = 6.7$

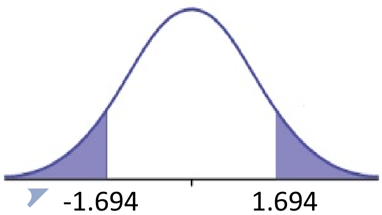
$$Z = \frac{\bar{x} - \bar{y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} = \frac{137 - 132 - (0)}{\sqrt{\frac{(5.5)^2}{23} + \frac{(6.7)^2}{24}}} = 2.8$$

Q3 A pet association claims that the mean annual costs of routine veterinarian visits for dogs and cats are the same. The results for samples of the two types of pets are shown at the table below. At $\alpha = 0.10$, can you reject the pet association's claim? Assume the population variances are equal.

Dogs	**Cats**
$\bar{X}_1 = \$263$ $S_1 = \$30$ n1=16	$\bar{X}_2 = \$183$ $S_2 = \$27$ n2=18

<u>Solution:</u>

$H_0: \mu_1 - \mu_2 = 0$ vs $H_1: \mu_1 \neq \mu_2: \mu_1 - \mu_2 \neq 0$



$$Sp = \sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(15)30^2+(17)27^2}{16+18-2}} = 28.45$$

$$t = \frac{\bar{x_1}-\bar{x_2}-(\mu_1-\mu_2)}{Sp\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{283-183-0}{28.45\cdot\sqrt{\frac{1}{16}+\frac{1}{18}}} = 8.18$$

$$\alpha = 0.10 \rightarrow \frac{\alpha}{2} = 0.05, d.f = 16+18-2 = 32 \rightarrow t_{0.05}^{(32)} = 1.694$$

$\therefore$ We reject H_0

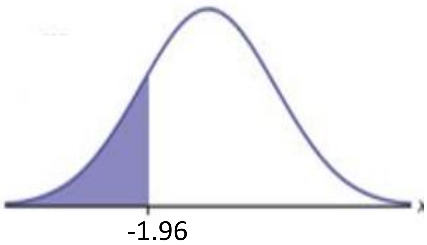
Q4 Test the claim about the difference between two population proportions P_1 and P_2 at the level of significance a. Assume the samples are random and independent.

Claim: $P_1 < P_2$; $\alpha = 0.05$ Sample statistics: $x_1 = 471$, $n_1 = 785$ and $x_2 = 372$, $n_2 = 465$

<u>Solution:</u>

$H_0: p_1 - p_2 = 0$ vs $H_1: p_1 - p_2 < 0$

$$\widehat{p_1} = \frac{x_1}{n_1} = \frac{471}{786} = 0.6 \text{ and } \widehat{p_2} = \frac{x_2}{n_2} = \frac{372}{465} = 0.8$$



$$p^* = \frac{x_1+x_2}{n_1+n_2} = \frac{471+372}{785+465} = 0.6744$$

$$Z = \frac{\widehat{p_1}-\widehat{p_2}-(p_1-p_2)}{\sqrt{p^*(1-p^*)}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{0.6-0.8}{\sqrt{0.6744(0.3256)}\sqrt{\frac{1}{785}+\frac{1}{465}}} = -7.29$$

$$\alpha = 0.05 \rightarrow Z_{0.05} = -1.96$$

$\therefore$ We reject H_0

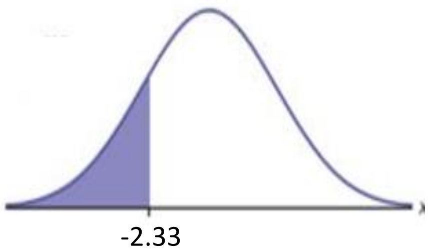
Q5 A non-governmental organization wants to choose between two regions in a state to initiate a campaign for rainwater harvesting. A researcher claims that Region A receives lesser rainfall than Region B. To test the regions, the average rainfall is calculated for 60 days in each region. The mean rainfall in Region A is 700 millimeters. Assume the population standard deviation is 60 millimeters. The mean rainfall in Region B is 725 millimeters. Assume the population standard deviation is 66 millimeters. At $\alpha = 0.01$, can the organization support the researcher's claim?

Solution:

Region (A)	Region (B)
$n = 60$	$m = 60$
$\bar{x} = 700$	$\bar{Y} = 700$
$\sigma_1 = 60$	$\sigma_2 = 66$

$H_0: \mu_A - \mu_B = 0$ vs $H_1: \mu_A < \mu_B$

$H_0: \mu_A - \mu_B = 0$ vs $H_1: \mu_A - \mu_B < 0$



$$Z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} = \frac{700 - 725}{\sqrt{\frac{60^2}{60} + \frac{66^2}{60}}} = -2.17$$

$\alpha = 0.01 \rightarrow Z_{0.01} = -2.33$

$\therefore$ We fail to reject H_0

Q6 A demographics researcher claims that the mean household income in a recent year is greater in Cuyahoga County, Ohio, than it is in Wayne County, Michigan. In Cuyahoga County, a sample of 19 residents has a mean household income of \$45,600 and a standard deviation of \$2,800. In Wayne County, a sample of 15 residents has a mean household income of \$41,500 and a standard deviation of \$1,310. At α = 0.05, fins the pooled standard deviation?

Solution:

Cuyahoga Country

$n = 19$

$S_1 = 2{,}800$

Wayne Country

$m = 15$

$S_2 = 1{,}310$

$$Sp = \sqrt{\frac{(n-1)S_1^2+(m-1)S_2^2}{n+m-2}} = \sqrt{\frac{(18)(2{,}800)^2+(14)(1{,}310)^2}{19+15-2}} = 2271.74$$

Q7 In a survey of 1000 drivers from the West, 934 wear a seat belt. In a survey of 1000 drivers from the Northeast, 909 wear a seat belt. At a = 0.05, can you support the claim that the proportion of drivers who wear seat belts is greater in the West than in the Northeast.

Solution:

West ($\hat{P}_1$)

n= 1000

x=934

Northwest ($\hat{P}_2$)

m= 1000

y=909

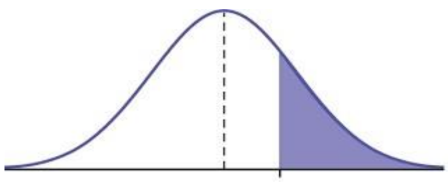
$\alpha = 0.05,\ H_1: P_1 > P_2$

$H_0: P_1 - P_2 = 0$ vs $H_1: P_1 - P_2 > 0$

$$\widehat{P_1} = \frac{x}{n} = \frac{934}{1000} = 0.934$$

$$\widehat{P_2} = \frac{y}{m} = \frac{909}{1000} = 0.909$$

$$P^* = \frac{x+y}{n+m} = \frac{934+909}{1000+1000} = 0.9215$$



$$Z = \frac{\widehat{P_1}-\widehat{P_2}}{\sqrt{P^*(1-P^*)}\sqrt{\frac{1}{n}+\frac{1}{m}}} == \frac{0.934-0.909}{\sqrt{0.9215(0.0785)}\sqrt{\frac{1}{1000}+\frac{1}{1000}}} = 2.08$$

$\alpha = 0.05 \to Z_{0.05} = 1.645$

$\therefore$ We reject H_0