# *t*-test

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# Review 6 Steps for Significance Testing

1. Set alpha (p level).
2. State hypotheses, Null and Alternative.
3. Calculate the test statistic (sample value).
4. Find the critical value of the statistic.
5. State the decision rule.
6. State the conclusion.

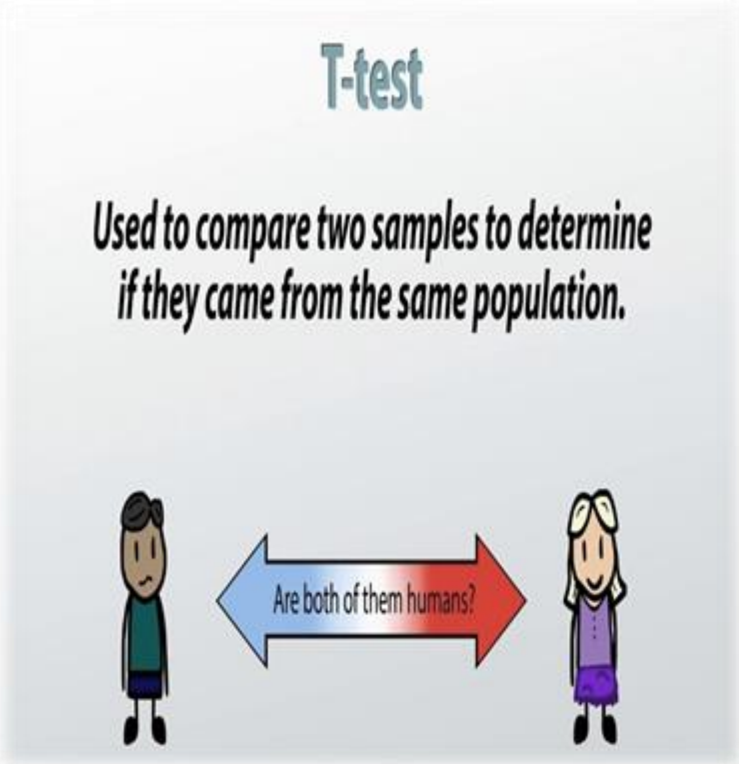
# *t*-test

t –test is about means: distribution and evaluation for group distribution

Withdrawn form the normal distribution

The shape of distribution depend on sample size and, the sum of all distributions is a normal distribution

t- distribution is based on sample size and vary according to the degrees of freedom



# What is the t -test

*t* test is a useful technique for comparing mean values of two sets of numbers.

The comparison will provide you with a statistic for evaluating whether the difference between two means is statistically significant.

*t* test is named after its inventor, William Gosset, who published under the pseudonym of student.

*t* test can be used either :

1. to compare two independent groups (independent-samples *t* test)
2. to compare observations from two measurement occasions for the same group (paired-samples *t* test).

# What is the t -test

The null hypothesis states that any difference between the two means is a result to difference in distribution.

Remember, both samples drawn randomly form the same population.

Comparing the chance of having difference is one group due to difference in distribution.

***Assuming that both distributions came from the same population, both distribution has to be equal***.

# What is the t -test

Then, what we intend:

**“To find the difference due to chance”**

Logically, The larger the difference in means, the more likely to find a significant *t* test.

But, recall:

1. Variability

More variability = less overlap = larger difference

2. Sample size

Larger sample size = less variability (pop) = larger difference

# Types

1. The ***independent-sample t test*** is used to compare two groups' scores on the same variable. For example, it could be used to compare the salaries of dentists and physicians to evaluate whether there is a difference in their salaries.

2. The ***paired-sample t test*** is used to compare the means of two variables within a single group. For example, it could be used to see if there is a statistically significant difference between starting salaries and current salaries among the general physicians in an organization.

# Assumption

1. Dependent variable should be continuous (I/R)
2. The groups should be randomly drawn from normally distributed and independent populations

e.g. Male X Female

Dentist X Physician

Manager X Staff

**NO OVER LAP**

# Assumption

3. the independent variable is categorical with two levels
4. Distribution for the **two independent** variables is normal
5. Equal variance (homogeneity of variance)
6. large variation = less likely to have sig t test = accepting null hypothesis (fail to reject) = Type II error = a threat to power

Sending an innocent to jail for no significant reason

# Independent Samples *t*-test

Used when we have two independent samples, e.g., treatment and control groups.

Formula is:

$$t_{\bar{X}_1-\bar{X}_2} = \frac{\bar{X}_1 - \bar{X}_2}{SE_{diff}}$$

Terms in the numerator are the sample means.

Term in the denominator is the standard error of the difference between means.

# Independent samples *t*-test

The formula for the standard error of the difference in means:

$$SE_{diff} = \sqrt{\frac{SD_1^2}{N_1} + \frac{SD_2^2}{N_2}}$$

Suppose we study the effect of caffeine on a motor test where the task is to keep a the mouse centered on a moving dot. Everyone gets a drink; half get caffeine, half get placebo; nobody knows who got what.

# Independent Sample Data (Data are time off task)

| Experimental (Caff) | Control (No Caffeine) |
|---|---|
| 12 | 21 |
| 14 | 18 |
| 10 | 14 |
| 8 | 20 |
| 16 | 11 |
| 5 | 19 |
| 3 | 8 |
| 9 | 12 |
| 11 | 13 |
| | 15 |
| $N_1$=9, $M_1$=9.778, $SD_1$=4.1164 | $N_2$=10, $M_2$=15.1, $SD_2$=4.2805 |

# Independent Sample Steps(1)

1. Set alpha. Alpha = .05
2. State Hypotheses.

   Null is $H_0$: $\mu_1 = \mu_2$.

   Alternative is $H_1$: $\mu_1 \neq \mu_2$.

# Independent Sample Steps(2)

3. Calculate test statistic:

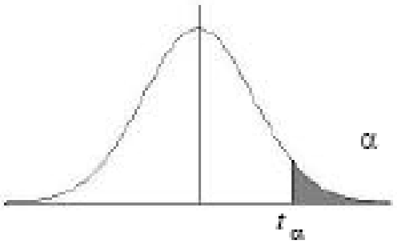
$$t = \frac{\overline{X}_1 - \overline{X}_2}{SE_{diff}} = \frac{9.778 - 15.1}{1.93} = \frac{-5.322}{1.93} = -2.758$$

$$SE_{diff} = \sqrt{\frac{SD_1^2}{N_1} + \frac{SD_2^2}{N_2}} = \sqrt{\frac{(4.1164)^2}{9} + \frac{(4.2805)^2}{10}} = 1.93$$

# Independent Sample Steps (3)

4. Determine the critical value. Alpha is .05, 2 tails, and df = N1+N2-2 or 10+9-2 = 17. The value is 2.11.
5. State decision rule. If |-2.758| > 2.11, then reject the null.
6. Conclusion: Reject the null. the population means are different. Caffeine has an effect on the motor pursuit task.

Table 4: Percentage Points of the *t* distribution




| df | α 0.250 | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 |
|---|---|---|---|---|---|---|
| 1 | 1.000 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 0.816 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 0.765 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 0.741 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 0.727 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 0.718 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 0.711 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 0.706 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 0.703 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 0.700 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 0.697 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| ⋮ | | | | | | |
| 29 | 0.683 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 0.683 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 40 | 0.681 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 60 | 0.679 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 120 | 0.677 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 |
| ∞ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |

# Using SPSS

Open SPSS

Open file “SPSS Examples” for Lab 5

Go to:

- “Analyze” then “Compare Means”
- Choose “Independent samples t-test”
- Put IV in “grouping variable” and DV in “test variable” box.
- Define grouping variable numbers.
  - E.g., we labeled the experimental group as “1” in our data set and the control group as “2”

# Independent Samples Exercise

| Experimental | Control |
|---|---|
| 12 | 20 |
| 14 | 18 |
| 10 | 14 |
| 8 | 20 |
| 16 | |

Work this problem by hand and with SPSS.

You will have to enter the data into SPSS.

# SPSS Results

**Group Statistics**

| | GROUP | N | Mean | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|---|
| TIME | experimental group | 5 | 12.0000 | 3.1623 | 1.4142 |
| | control group | 4 | 18.0000 | 2.8284 | 1.4142 |

**Independent Samples Test**

| | | Levene's Test for Equality of Variances | | t-test for Equality of Means | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | 95% Confidence Interval of the Difference | |
| | | F | Sig. | t | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | Lower | Upper |
| TIME | Equal variances assumed | .130 | .729 | -2.958 | 7 | .021 | -6.0000 | 2.0284 | -10.7963 | -1.2037 |
| | Equal variances not assumed | | | -3.000 | 6.857 | .020 | -6.0000 | 2.0000 | -10.7493 | -1.2507 |

# Dependent Samples t-tests

# Dependent Samples *t*-test

Used when we have dependent samples – matched, paired or tied somehow

- Repeated measures
- Brother & sister, husband & wife
- Left hand, right hand, etc.

Useful to control individual differences. Can result in more powerful test than independent samples *t*-test.

# Dependent Samples *t*

Formulas:

$$t_{\overline{X}_D} = \frac{\overline{D}}{SE_{diff}}$$

*t* is the difference in means over a standard error.

$$SE_{diff} = \frac{SD_D}{\sqrt{n_{pairs}}}$$

The standard error is found by finding the difference between each pair of observations. The standard deviation of these difference is $SD_D$. Divide $SD_D$ by sqrt (number of pairs) to get $SE_{diff}$.

# Another way to write the formula

$$t_{\bar{X}_D} = \frac{\bar{D}}{SD_D \Big/ \sqrt{n_{pairs}}}$$

# Dependent Samples *t* example

| Person | Painfree (time in sec) | Placebo | Difference |
|---|---|---|---|
| 1 | 60 | 55 | 5 |
| 2 | 35 | 20 | 15 |
| 3 | 70 | 60 | 10 |
| 4 | 50 | 45 | 5 |
| 5 | 60 | 60 | 0 |
| M | 55 | 48 | 7 |
| SD | 13.23 | 16.81 | 5.70 |

# Dependent Samples *t* Example (2)

1. Set alpha = .05
2. Null hypothesis: $H_0$: $\mu_1 = \mu_2$.
   Alternative is $H_1$: $\mu_1 \neq \mu_2$.
3. Calculate the test statistic:

$$SE_{diff} = \frac{SD}{\sqrt{n_{pairs}}} = \frac{5.70}{\sqrt{5}} = 2.55$$

$$t = \frac{\overline{D}}{SE_{diff}} = \frac{55-48}{2.55} = \frac{7}{2.55} = 2.75$$

# Dependent Samples t Example (3)

4. Determine the critical value of t.

   Alpha =.05, tails=2

   df = N(pairs)-1 =5-1=4.

   Critical value is 2.776

5. Decision rule: is absolute value of sample value larger than critical value?

6. Conclusion. Not (quite) significant. Painfree does <u>not</u> have an effect.

# Using SPSS for dependent t-test

Open SPSS

Open file “SPSS Examples” (same as before)

Go to:

- “Analyze” then “Compare Means”
- Choose “Paired samples t-test”
- Choose the two IV conditions you are comparing. Put in “paired variables box.”

# Dependent t- SPSS output

**Paired Samples Statistics**

| | | Mean | N | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|---|
| Pair 1 | PAINFREE | 55.0000 | 5 | 13.2288 | 5.9161 |
| | PLACEBO | 48.0000 | 5 | 16.8077 | 7.5166 |

**Paired Samples Correlations**

| | | N | Correlation | Sig. |
|---|---|---|---|---|
| Pair 1 | PAINFREE & PLACEBO | 5 | .956 | .011 |

**Paired Samples Test**

| | | Paired Differences | | | | | t | df | Sig. (2-tailed) |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | 95% Confidence Interval of the Difference | | | | |
| | | Mean | Std. Deviation | Std. Error Mean | Lower | Upper | | | |
| Pair 1 | PAINFREE - PLACEBO | 7.0000 | 5.7009 | 2.5495 | -7.86E-02 | 14.0786 | 2.746 | 4 | .052 |

# Relationship between t Statistic and Power

$$t = \frac{\overline{X} - \mu}{SE_M}$$

To increase power:

- Increase the difference between the means.
- Reduce the variance
- Increase N
- Increase α from α = .01 to α = .05

# To Increase Power

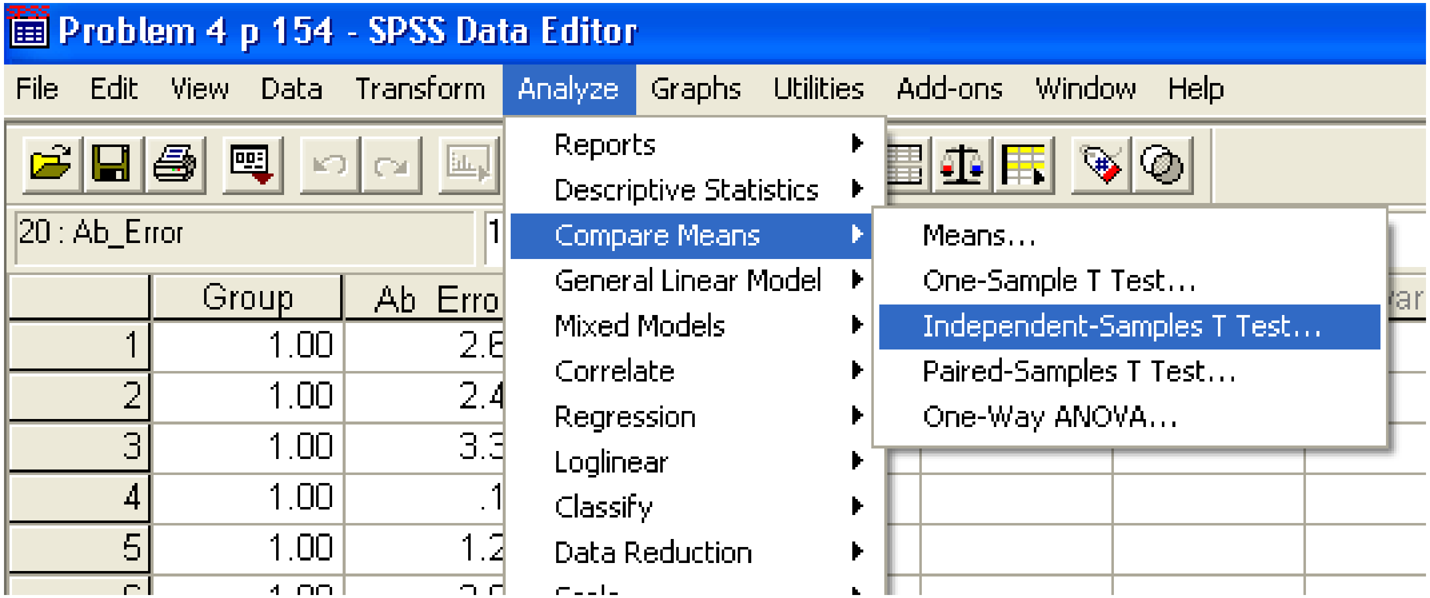
Increase alpha, Power for $\alpha = .10$ is greater than power for $\alpha = .05$

Increase the difference between means.

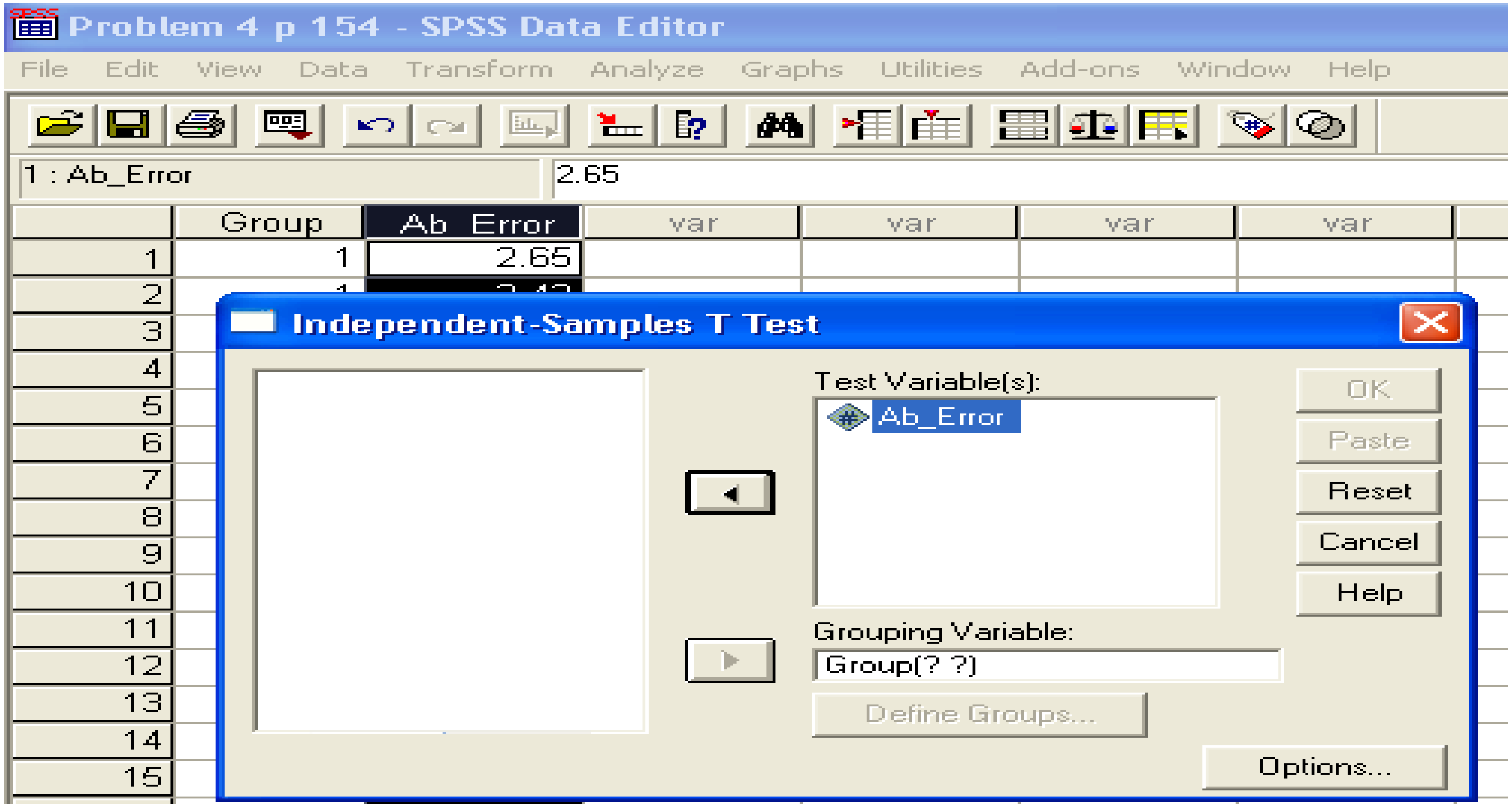
Decrease the sd's of the groups.

Increase N.

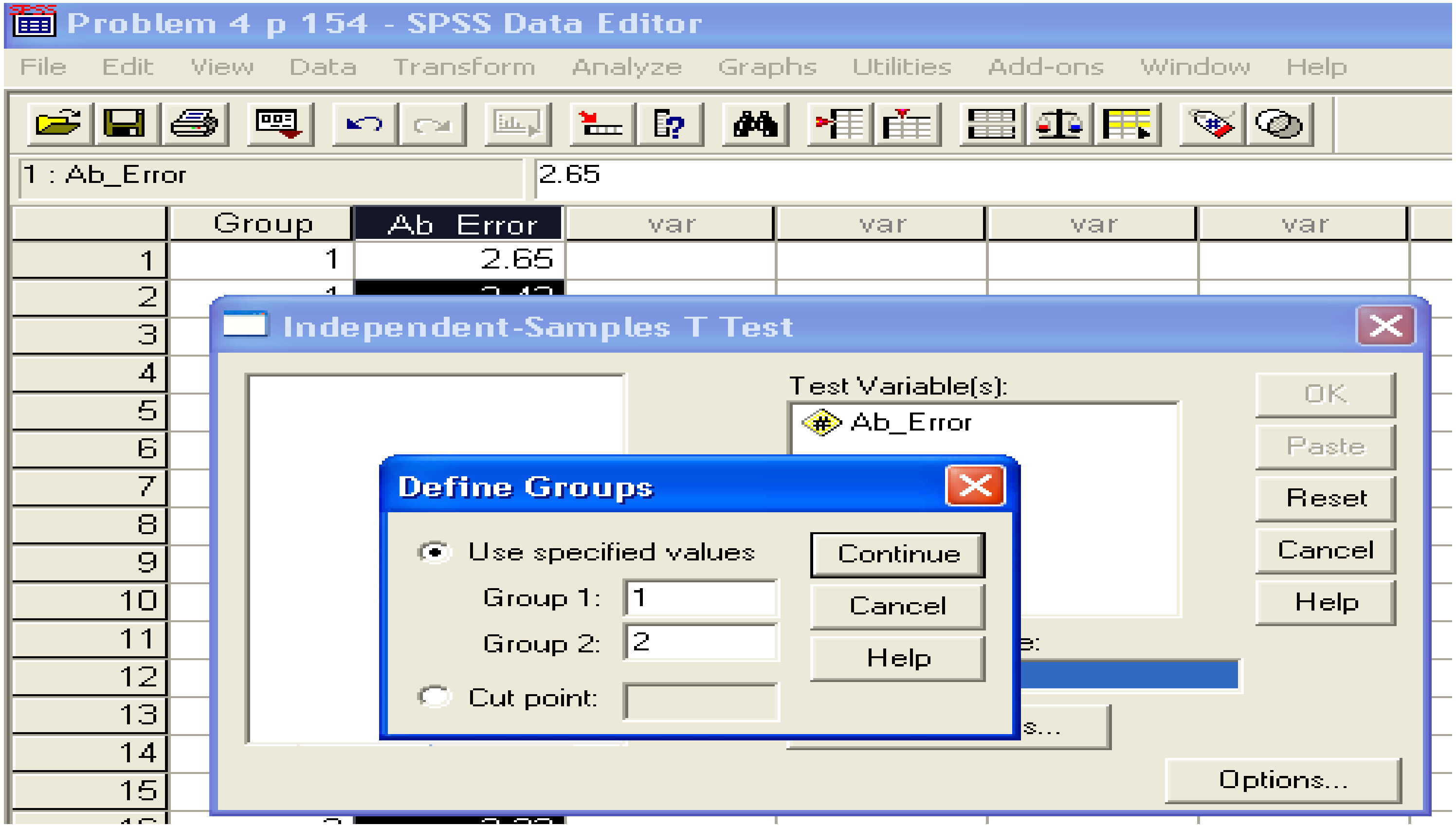
# Independent t-Test



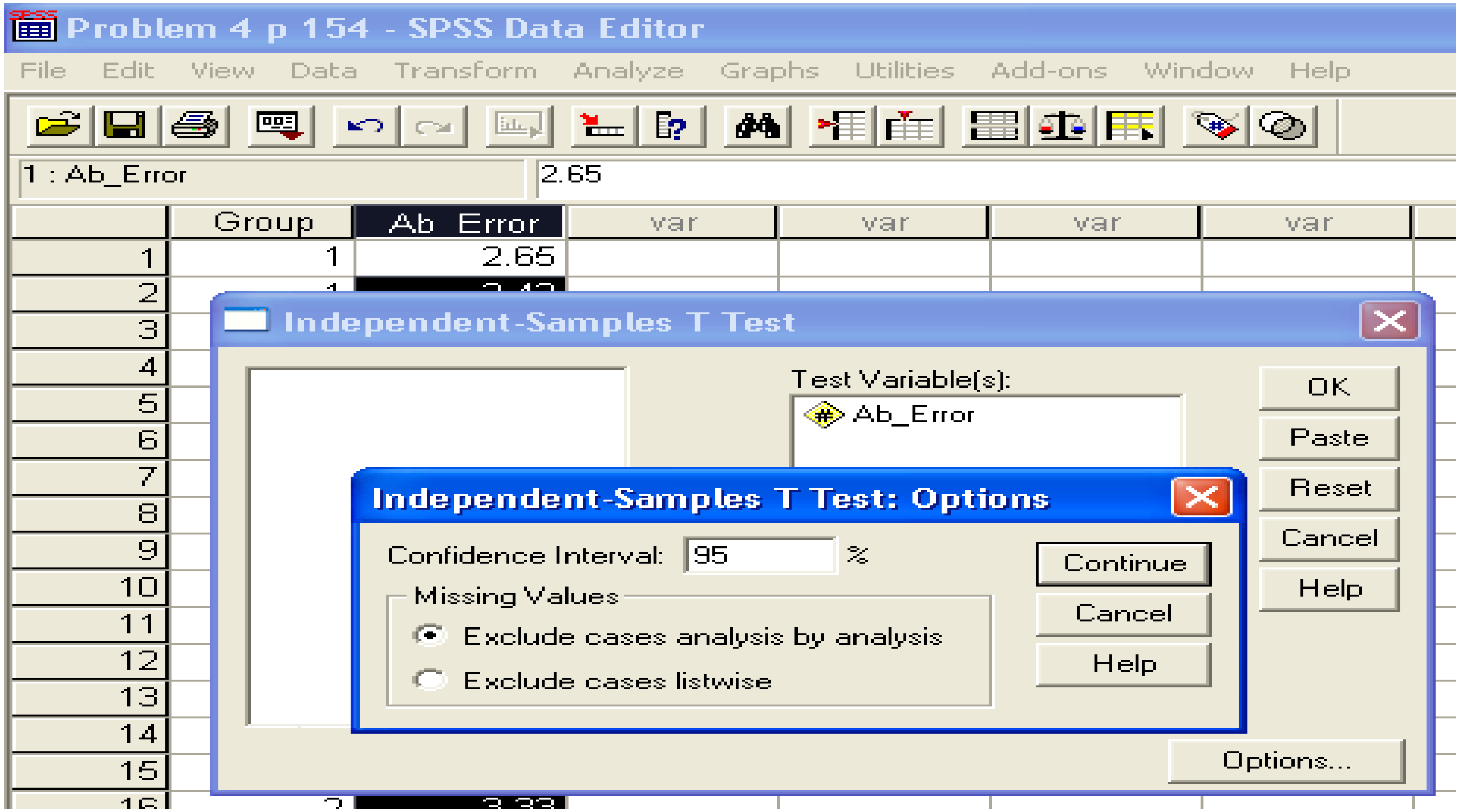
# Independent t-Test: Independent & Dependent Variables



# Independent t-Test: Define Groups



# Independent t-Test: Options



## Independent t-Test: Output

**Group Statistics**

| | Group | N | Mean | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|---|
| Ab_Error | Active | 10 | 2.2820 | 1.24438 | .39351 |
| | Passive | 10 | 1.9660 | 1.50606 | .47626 |

**Independent Samples Test**

| | | Levene's Test for Equality of Variances | | t-test for Equality of Means | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | 95% Confidence Interval of the Difference | |
| | | F | Sig. | t | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | Lower | Upper |
| Ab_Error | Equal variances assumed | .513 | .483 | .511 | 18 | .615 | .31600 | .61780 | -.98194 | 1.61394 |
| | Equal variances not assumed | | | .511 | 17.382 | .615 | .31600 | .61780 | -.98526 | 1.61726 |

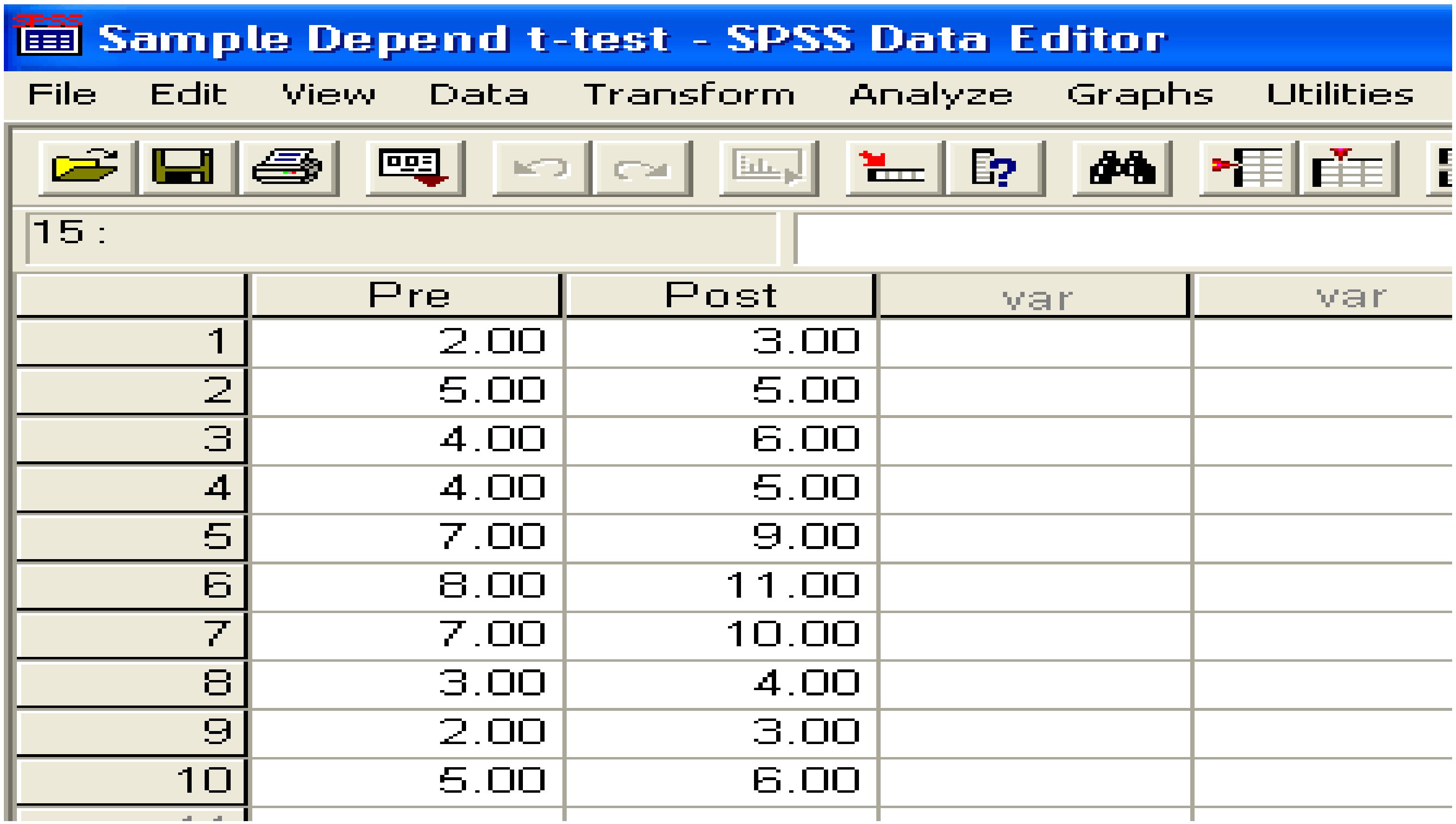
Are the groups different?

t(18) = .511, p = .615

NO DIFFERENCE

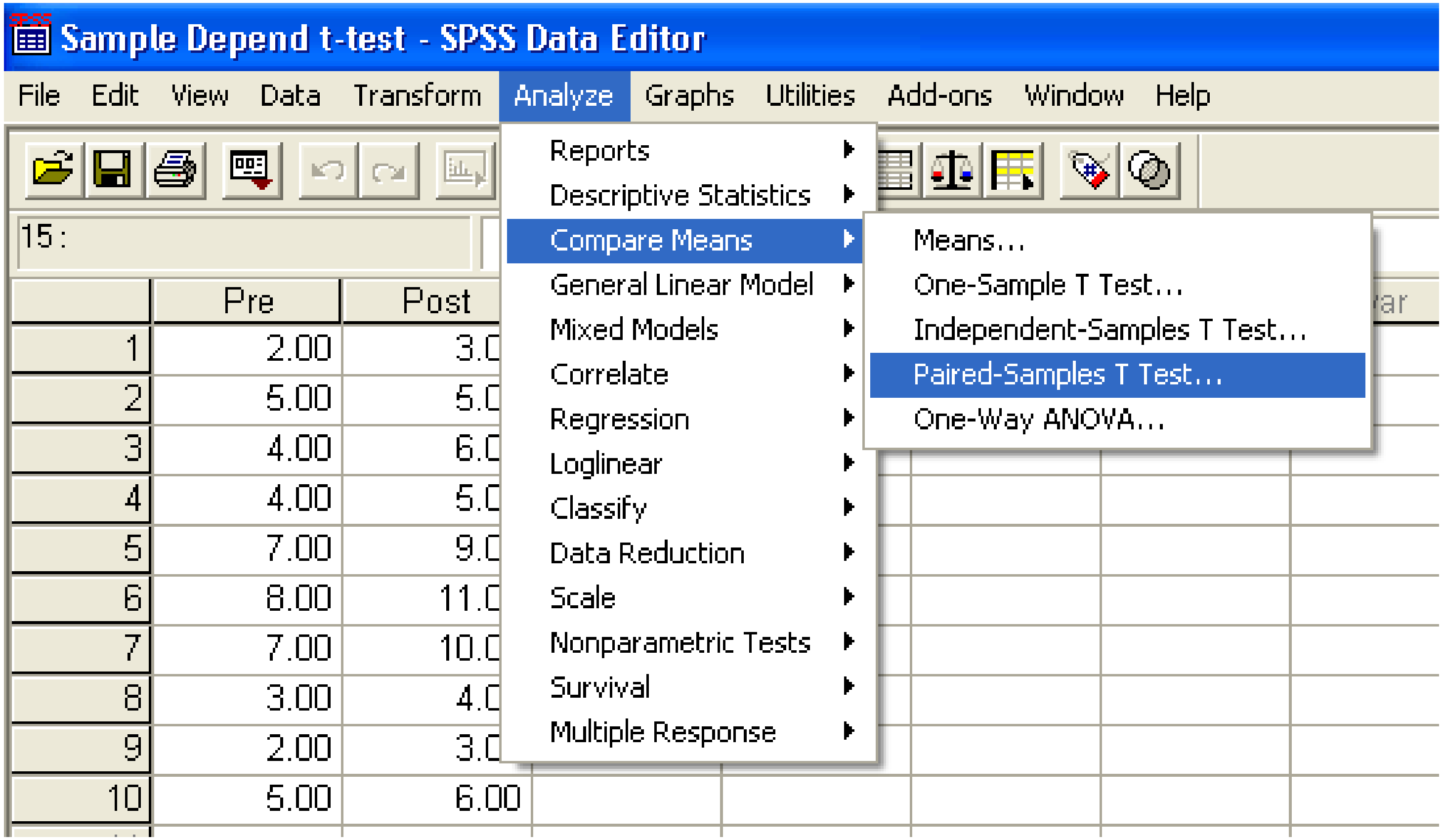
2.28 is not different from 1.96

# Dependent or Paired t-Test: Define Variables

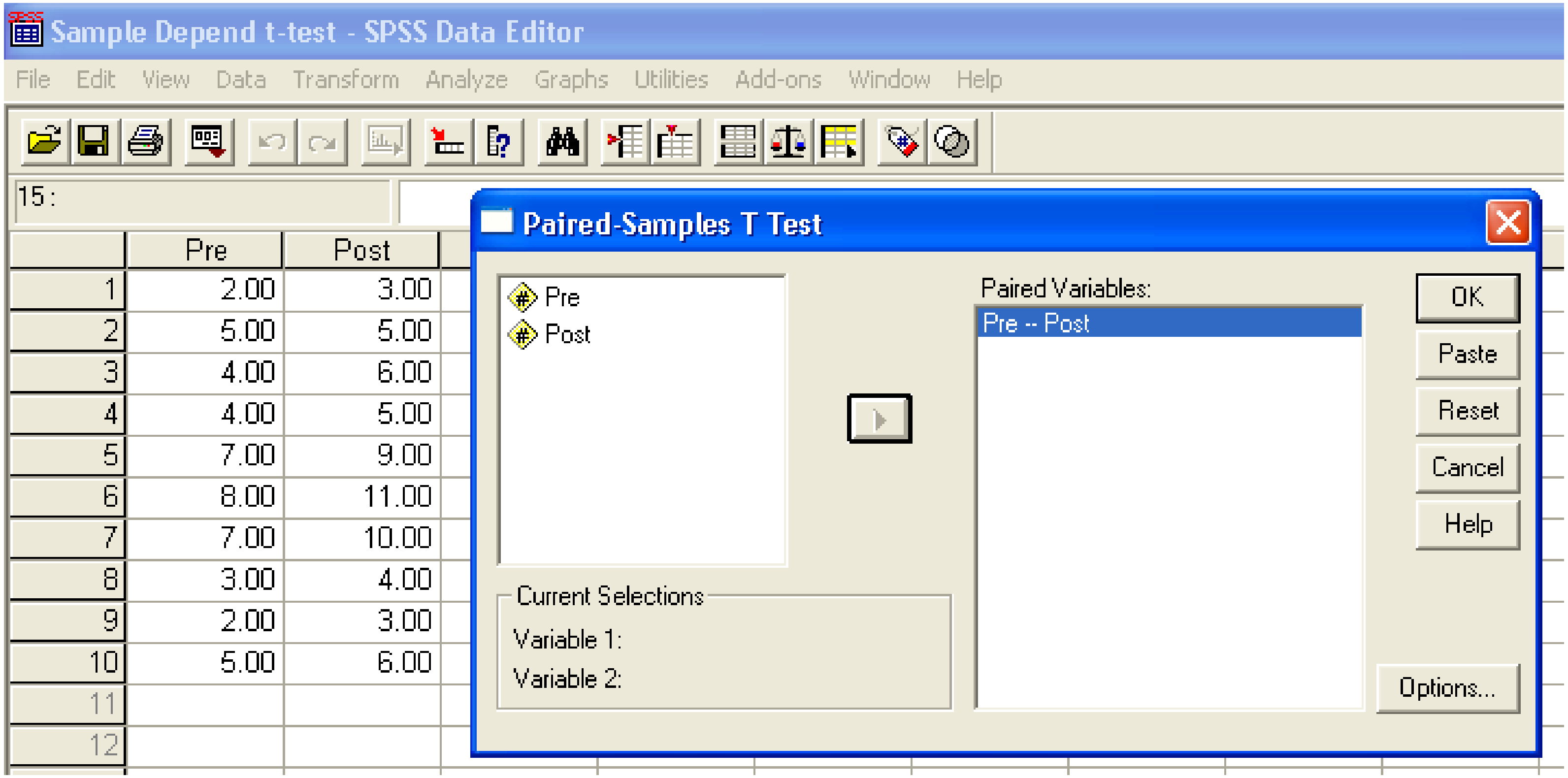



| | Pre | Post | var | var |
|---|---|---|---|---|
| 1 | 2.00 | 3.00 | | |
| 2 | 5.00 | 5.00 | | |
| 3 | 4.00 | 6.00 | | |
| 4 | 4.00 | 5.00 | | |
| 5 | 7.00 | 9.00 | | |
| 6 | 8.00 | 11.00 | | |
| 7 | 7.00 | 10.00 | | |
| 8 | 3.00 | 4.00 | | |
| 9 | 2.00 | 3.00 | | |
| 10 | 5.00 | 6.00 | | |

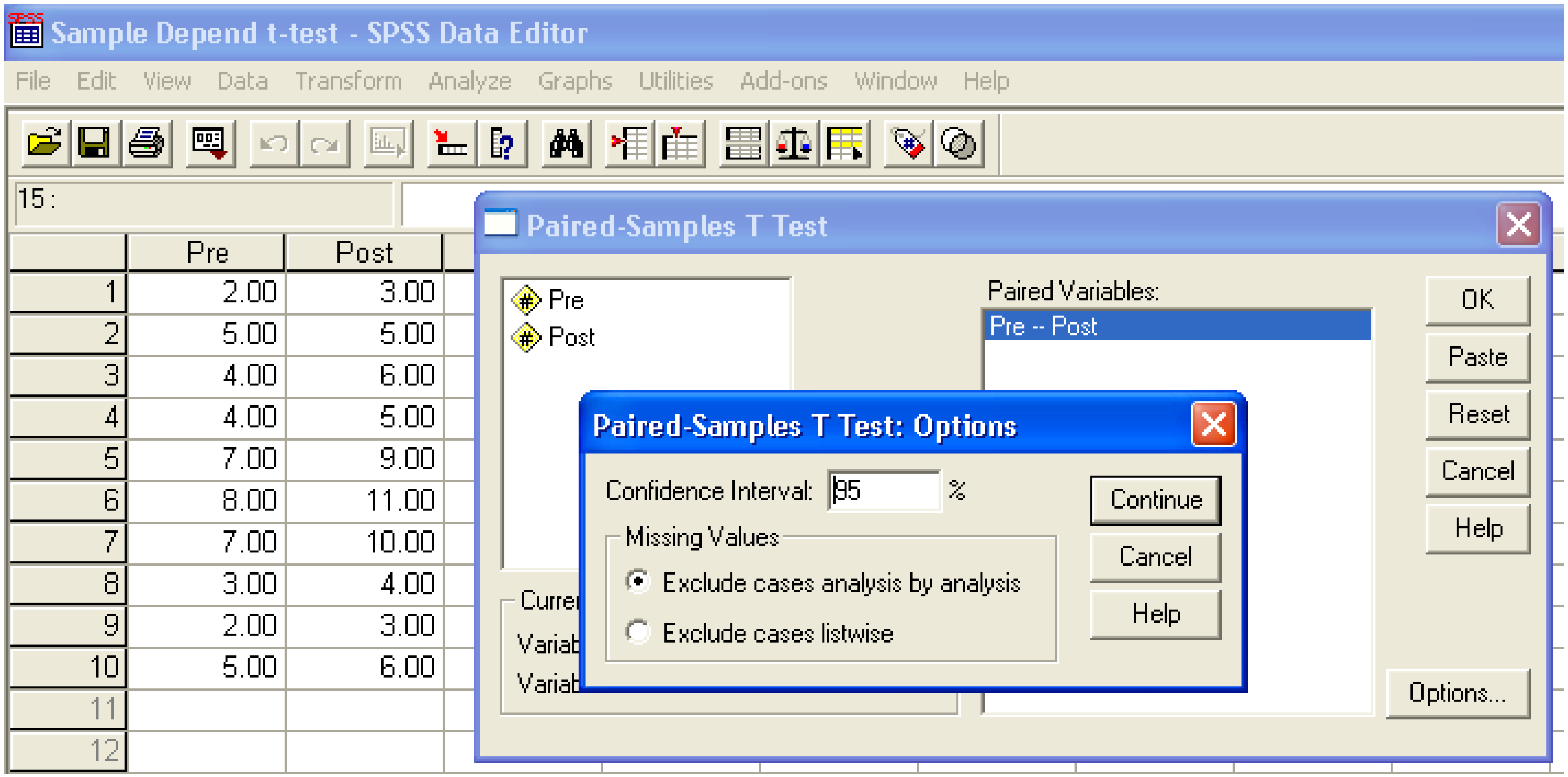
# Dependent or Paired t-Test: Select Paired-Samples



# Dependent or Paired t-Test: Select Variables



# Dependent or Paired t-Test: Options



## Dependent or Paired t-Test: Output

**Paired Samples Statistics**

| | | Mean | N | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|---|
| Pair 1 | Pre | 4.7000 | 10 | 2.11082 | .66750 |
| | Post | 6.2000 | 10 | 2.85968 | .90431 |

**Paired Samples Correlations**

| | | N | Correlation | Sig. |
|---|---|---|---|---|
| Pair 1 | Pre & Post | 10 | .968 | .000 |

**Paired Samples Test**

| | | Paired Differences | | | | | t | df | Sig. (2-tailed) |
|---|---|---|---|---|---|---|---|---|---|
| | | Mean | Std. Deviation | Std. Error Mean | 95% Confidence Interval of the Difference | | | | |
| | | | | | Lower | Upper | | | |
| Pair 1 | Pre - Post | -1.50000 | .97183 | .30732 | -2.19520 | -.80480 | -4.881 | 9 | .001 |

Is there a difference between pre & post?

$t(9) = -4.881, p = .001$

Yes, 4.7 is significantly different from 6.2