

# *t*-test

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# Review 6 Steps for Significance Testing

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1. Set alpha ( $p$  level).
2. State hypotheses, Null and Alternative.
3. Calculate the test statistic (sample value).
4. Find the critical value of the statistic.
5. State the decision rule.
6. State the conclusion.

# *t*-test

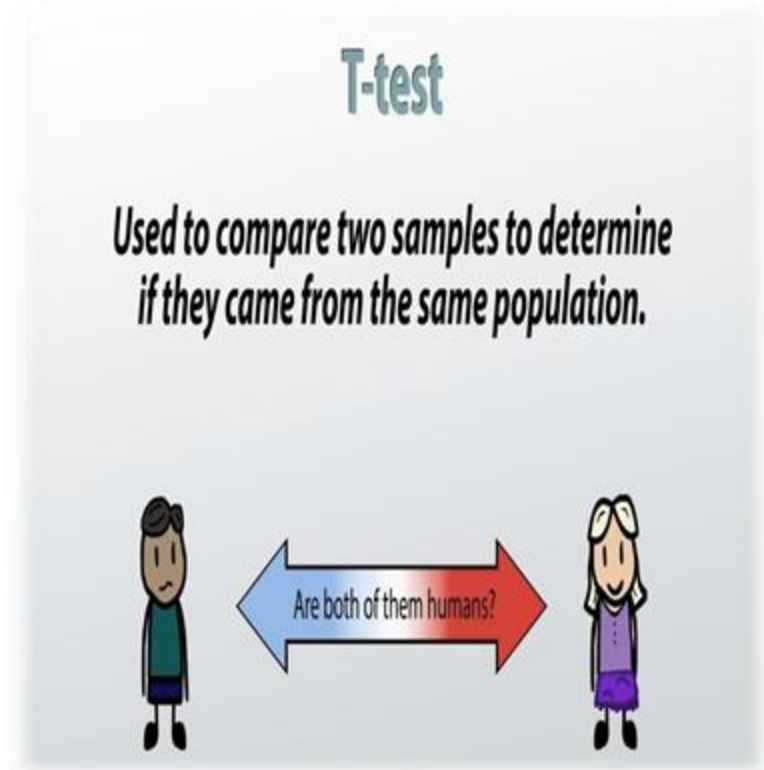
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t –test is about means:  
distribution and evaluation  
for group distribution

Withdrawn form the normal  
distribution

The shape of distribution  
depend on sample size and,  
the sum of all distributions is  
a normal distribution

t- distribution is based on  
sample size and vary  
according to the degrees of  
freedom



# What is the $t$ -test

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$t$  test is a useful technique for comparing mean values of two sets of numbers.

The comparison will provide you with a statistic for evaluating whether the difference between two means is statistically significant.

$t$  test is named after its inventor, William Gosset, who published under the pseudonym of student.

$t$  test can be used either :

1. to compare two independent groups (independent-samples  $t$  test)
2. to compare observations from two measurement occasions for the same group (paired-samples  $t$  test).

# What is the t -test

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The null hypothesis states that any difference between the two means is a result to difference in distribution.

Remember, both samples drawn randomly form the same population.

Comparing the chance of having difference is one group due to difference in distribution.

***Assuming that both distributions came from the same population, both distribution has to be equal.***

# What is the t -test

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Then, what we intend:

**“To find the difference due to chance”**

Logically, The larger the difference in means, the more likely to find a significant  $t$  test.

But, recall:

## 1. Variability

More variability = less overlap = larger difference

## 2. Sample size

Larger sample size = less variability (pop) = larger difference

# Types

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1. The *independent-sample t test* is used to compare two groups' scores on the same variable. For example, it could be used to compare the salaries of dentists and physicians to evaluate whether there is a difference in their salaries.
2. The *paired-sample t test* is used to compare the means of two variables within a single group. For example, it could be used to see if there is a statistically significant difference between starting salaries and current salaries among the general physicians in an organization.

# Assumption

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1. Dependent variable should be continuous (I/R)
2. The groups should be randomly drawn from normally distributed and independent populations

e.g. Male X Female

Dentist X Physician

Manager X Staff

**NO OVER LAP**



# Assumption

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3. the independent variable is categorical with two levels
4. Distribution for the two independent variables is normal
5. Equal variance (homogeneity of variance)
6. large variation = less likely to have sig t test = accepting null hypothesis (fail to reject) = Type II error = a threat to power

Sending an innocent to jail for no significant reason

# Independent Samples $t$ -test

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Used when we have two independent samples, e.g., treatment and control groups.

Formula is: 
$$t_{\bar{X}_1 - \bar{X}_2} = \frac{\bar{X}_1 - \bar{X}_2}{SE_{diff}}$$

Terms in the numerator are the sample means.

Term in the denominator is the standard error of the difference between means.

# Independent samples *t*-test

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The formula for the standard error of the difference in means:

$$SE_{diff} = \sqrt{\frac{SD_1^2}{N_1} + \frac{SD_2^2}{N_2}}$$

Suppose we study the effect of caffeine on a motor test where the task is to keep a the mouse centered on a moving dot. Everyone gets a drink; half get caffeine, half get placebo; nobody knows who got what.

# Independent Sample Data (Data are time off task)

Experimental (Caff)	Control (No Caffeine)
12	21
14	18
10	14
8	20
16	11
5	19
3	8
9	12
11	13
	15
$N_1=9, M_1=9.778, SD_1=4.1164$	$N_2=10, M_2=15.1, SD_2=4.2805$

# Independent Sample Steps(1)

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1. Set alpha. Alpha = .05

2. State Hypotheses.

Null is  $H_0: \mu_1 = \mu_2$ .

Alternative is  $H_1: \mu_1 \neq \mu_2$ .

# Independent Sample Steps(2)

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3. Calculate test statistic:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{SE_{diff}} = \frac{9.778 - 15.1}{1.93} = \frac{-5.322}{1.93} = -2.758$$

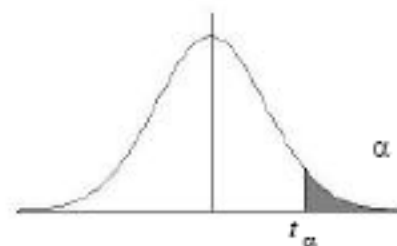
$$SE_{diff} = \sqrt{\frac{SD_1^2}{N_1} + \frac{SD_2^2}{N_2}} = \sqrt{\frac{(4.1164)^2}{9} + \frac{(4.2805)^2}{10}} = 1.93$$

# Independent Sample Steps (3)

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4. Determine the critical value. Alpha is .05, 2 tails, and  $df = N_1 + N_2 - 2$  or  $10 + 9 - 2 = 17$ . The value is 2.11.
5. State decision rule. If  $|-2.758| > 2.11$ , then reject the null.
6. Conclusion: Reject the null. the population means are different. Caffeine has an effect on the motor pursuit task.

Table 4: Percentage Points of the  $t$  distribution



df	$\alpha$					
	0.250	0.100	0.050	0.025	0.010	0.005
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
			•			
			•			
			•			
29	0.683	1.311	1.699	2.045	2.462	2.756
30	0.683	1.310	1.697	2.042	2.457	2.750
40	0.681	1.303	1.684	2.021	2.423	2.704
60	0.679	1.296	1.671	2.000	2.390	2.660
120	0.677	1.289	1.658	1.980	2.358	2.617
$\infty$	0.674	1.282	1.645	1.960	2.326	2.576



# Using SPSS

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Open SPSS

Open file “SPSS Examples” for Lab 5

Go to:

- “Analyze” then “Compare Means”
- Choose “Independent samples t-test”
- Put IV in “grouping variable” and DV in “test variable” box.
- Define grouping variable numbers.
  - E.g., we labeled the experimental group as “1” in our data set and the control group as “2”

# Independent Samples Exercise

Experimental	Control
12	20
14	18
10	14
8	20
16	

Work this problem by hand and with SPSS.  
You will have to enter the data into SPSS.

# SPSS Results

## Group Statistics

	GROUP	N	Mean	Std. Deviation	Std. Error Mean
TIME	experimental group	5	12.0000	3.1623	1.4142
	control group	4	18.0000	2.8284	1.4142

## Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
TIME	Equal variances assumed	.130	.729	-2.958	7	.021	-6.0000	2.0284	-10.7963	-1.2037
	Equal variances not assumed			-3.000	6.857	.020	-6.0000	2.0000	-10.7493	-1.2507

# Dependent Samples t-tests

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# Dependent Samples $t$ -test

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Used when we have dependent samples – matched, paired or tied somehow

- Repeated measures
- Brother & sister, husband & wife
- Left hand, right hand, etc.

Useful to control individual differences. Can result in more powerful test than independent samples  $t$ -test.

# Dependent Samples $t$

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Formulas:

$$t_{\bar{X}_D} = \frac{\bar{D}}{SE_{diff}}$$

$t$  is the difference in means over a standard error.

$$SE_{diff} = \frac{SD_D}{\sqrt{n_{pairs}}}$$

The standard error is found by finding the difference between each pair of observations. The standard deviation of these difference is  $SD_D$ . Divide  $SD_D$  by sqrt (number of pairs) to get  $SE_{diff}$ .

# Another way to write the formula

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$$t_{\bar{X}_D} = \frac{\bar{D}}{SD_D / \sqrt{n_{pairs}}}$$

# Dependent Samples $t$ example

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Person	Painfree (time in sec)	Placebo	Difference
1	60	55	5
2	35	20	15
3	70	60	10
4	50	45	5
5	60	60	0
M	55	48	7
SD	13.23	16.81	5.70



# Dependent Samples $t$ Example (2)

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1. Set alpha = .05
2. Null hypothesis:  $H_0: \mu_1 = \mu_2$ .  
Alternative is  $H_1: \mu_1 \neq \mu_2$ .
3. Calculate the test statistic:

$$SE_{diff} = \frac{SD}{\sqrt{n_{pairs}}} = \frac{5.70}{\sqrt{5}} = 2.55$$

$$t = \frac{\bar{D}}{SE_{diff}} = \frac{55 - 48}{2.55} = \frac{7}{2.55} = 2.75$$

# Dependent Samples t Example (3)

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4. Determine the critical value of t.  
Alpha = .05, tails=2  
df = N(pairs)-1 = 5-1=4.  
Critical value is 2.776
5. Decision rule: is absolute value of sample value larger than critical value?
6. Conclusion. Not (quite) significant. Painfree does not have an effect.

# Using SPSS for dependent t-test

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Open SPSS

Open file “SPSS Examples” (same as before)

Go to:

- “Analyze” then “Compare Means”
- Choose “Paired samples t-test”
- Choose the two IV conditions you are comparing. Put in “paired variables box.”

# Dependent t- SPSS output

## Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	PAINFREE	55.0000	5	13.2288	5.9161
	PLACEBO	48.0000	5	16.8077	7.5166

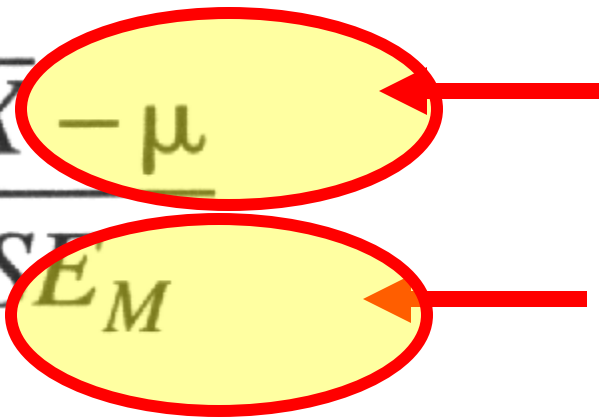
## Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	PAINFREE & PLACEBO	5	.956	.011

## Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	PAINFREE - PLACEBO	7.0000	5.7009	2.5495	-7.86E-02	14.0786	2.746	4	.052

## Relationship between t Statistic and Power

$$t = \frac{\bar{X} - \mu}{SE_M}$$


To increase power:

- Increase the difference between the means.
- Reduce the variance
- Increase N
- Increase  $\alpha$  from  $\alpha = .01$  to  $\alpha = .05$

# To Increase Power

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Increase alpha, Power for  $\alpha = .10$  is greater than power for  $\alpha = .05$

Increase the difference between means.

Decrease the sd's of the groups.

Increase N.

# Independent t-Test

The screenshot shows the SPSS Data Editor interface. The title bar reads "Problem 4 p 154 - SPSS Data Editor". The menu bar includes File, Edit, View, Data, Transform, Analyze, Graphs, Utilities, Add-ons, Window, and Help. The Analyze menu is open, showing options: Reports, Descriptive Statistics, Compare Means, General Linear Model, Mixed Models, Correlate, Regression, Loglinear, Classify, Data Reduction, and Scale. The "Compare Means" option is selected, and its submenu is open, showing: Means..., One-Sample T Test..., Independent-Samples T Test... (highlighted), Paired-Samples T Test..., and One-Way ANOVA... The data grid below the menu shows a variable named "Ab\_Error" with values for groups 1 through 5.

	Group	Ab_Error
1	1.00	2.6
2	1.00	2.4
3	1.00	3.3
4	1.00	.1
5	1.00	1.2

# Independent t-Test: Independent & Dependent Variables

The screenshot shows the SPSS Data Editor window titled "Problem 4 p 154 - SPSS Data Editor". The data table has the following structure:

	Group	Ab_Error	var	var	var	var
1	1	2.65				
2	1	2.42				
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						

The "Independent-Samples T Test" dialog box is open, showing the following settings:

- Test Variable(s): Ab\_Error
- Grouping Variable: Group(? ?)
- Buttons: OK, Paste, Reset, Cancel, Help, Define Groups..., Options...



# Independent t-Test: Define Groups

The screenshot shows the SPSS Data Editor interface with a data file named "Problem 4 p 154". The main window displays a data grid with columns for "Group", "Ab Error", and several "var" columns. The first row shows a value of 2.65 for "Ab Error".

Overlaid on the data grid is the "Independent-Samples T Test" dialog box. The "Test Variable(s)" list contains "Ab\_Error". The "Define Groups" sub-dialog is open, showing the "Use specified values" option selected. The "Group 1" field is set to "1" and the "Group 2" field is set to "2". The "Cut point" option is unselected.

Group	Ab Error	var	var	var	var
1	2.65				
2	2.42				
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					

# Independent t-Test: Options

The screenshot shows the SPSS Data Editor interface. The main window title is "Problem 4 p 154 - SPSS Data Editor". The menu bar includes File, Edit, View, Data, Transform, Analyze, Graphs, Utilities, Add-ons, Window, and Help. The toolbar contains various icons for file operations and analysis. The data grid shows a variable named "Ab\_Error" with a value of 2.65 for the first row. The "Independent-Samples T Test" dialog box is open, with "Ab\_Error" selected as the Test Variable(s). The "Independent-Samples T Test: Options" sub-dialog box is also open, showing a Confidence Interval of 95% and the "Exclude cases analysis by analysis" option selected.

Group	Ab_Error	var	var	var	var
1	2.65				
2	3.42				
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16	3.33				

**Independent-Samples T Test**

Test Variable(s):  
# Ab\_Error

**Independent-Samples T Test: Options**

Confidence Interval: 95 %

Missing Values

Exclude cases analysis by analysis

Exclude cases listwise

Buttons: OK, Paste, Reset, Cancel, Help, Continue, Cancel, Help, Options...

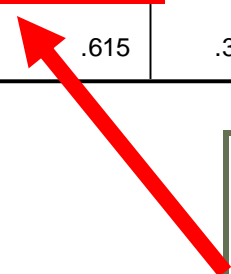
### Group Statistics

	Group	N	Mean	Std. Deviation	Std. Error Mean
Ab_Error	Active	10	2.2820	1.24438	.39351
	Passive	10	1.9660	1.50606	.47626

Independent t-Test: Output

### Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Ab_Error	Equal variances assumed	.513	.483	.511	18	.615	.31600	.61780	-.98194	1.61394
	Equal variances not assumed			.511	17.382	.615	.31600	.61780	-.98526	1.61726



Are the groups different?

$t(18) = .511, p = .615$

**NO DIFFERENCE**

2.28 is not different from 1.96

# Dependent or Paired t-Test: Define Variables

15 :

	Pre	Post	var	var
1	2.00	3.00		
2	5.00	5.00		
3	4.00	6.00		
4	4.00	5.00		
5	7.00	9.00		
6	8.00	11.00		
7	7.00	10.00		
8	3.00	4.00		
9	2.00	3.00		
10	5.00	6.00		

# Dependent or Paired t-Test: Select Paired-Samples

The screenshot shows the SPSS Data Editor window titled "Sample Depend t-test - SPSS Data Editor". The menu bar includes File, Edit, View, Data, Transform, Analyze, Graphs, Utilities, Add-ons, Window, and Help. The Analyze menu is open, showing options like Reports, Descriptive Statistics, Compare Means, General Linear Model, Mixed Models, Correlate, Regression, Loglinear, Classify, Data Reduction, Scale, Nonparametric Tests, Survival, and Multiple Response. The Compare Means submenu is also open, highlighting "Paired-Samples T Test...".

	Pre	Post
1	2.00	3.00
2	5.00	5.00
3	4.00	6.00
4	4.00	5.00
5	7.00	9.00
6	8.00	11.00
7	7.00	10.00
8	3.00	4.00
9	2.00	3.00
10	5.00	6.00

# Dependent or Paired t-Test: Select Variables

The screenshot shows the SPSS Data Editor window titled "Sample Depend t-test - SPSS Data Editor". The menu bar includes File, Edit, View, Data, Transform, Analyze, Graphs, Utilities, Add-ons, Window, and Help. The toolbar contains various icons for file operations and analysis. The data table has columns "Pre" and "Post" and rows 1 through 12. The "Paired-Samples T Test" dialog box is open, showing "Pre" and "Post" selected in the "Paired Variables" list. The "Current Selections" section shows "Variable 1:" and "Variable 2:".

	Pre	Post
1	2.00	3.00
2	5.00	5.00
3	4.00	6.00
4	4.00	5.00
5	7.00	9.00
6	8.00	11.00
7	7.00	10.00
8	3.00	4.00
9	2.00	3.00
10	5.00	6.00
11		
12		

# Dependent or Paired t-Test: Options

The screenshot shows the SPSS Data Editor window titled "Sample Depend t-test - SPSS Data Editor". The data table contains the following values:

	Pre	Post
1	2.00	3.00
2	5.00	5.00
3	4.00	6.00
4	4.00	5.00
5	7.00	9.00
6	8.00	11.00
7	7.00	10.00
8	3.00	4.00
9	2.00	3.00
10	5.00	6.00
11		
12		

The "Paired-Samples T Test" dialog box is open, showing "Pre" and "Post" as paired variables. The "Paired Variables:" list contains "Pre -- Post". The "Options" sub-dialog box is also open, showing a "Confidence Interval" of 95% and the "Exclude cases analysis by analysis" option selected under "Missing Values".

### Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Pre	4.7000	10	2.11082	.66750
	Post	6.2000	10	2.85968	.90431

Dependent or Paired t-Test:  
Output

### Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	Pre & Post	10	.968	.000

### Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Pre - Post	-1.50000	.97183	.30732	-2.19520	-.80480	-4.881	9	.001

Is there a difference between pre & post?

$$t(9) = -4.881, p = .001$$

Yes, 4.7 is significantly different from 6.2